Calibration, Gravity Signals, and Model Uncertainties Relating to the Apache Point Observatory Lunar Laser-ranging Operation (APOLLO)

Tom Murphy[1], Jan Kodet[2], Yan Liang[1,3] Fanzhi Ni[1], James Battat[4], John Chandler[1,5], Nick Colmenares[1]
Nathan Johnson[1], Robert Reasenberg[1,5], Sanchit Sabhlok[1], Ulrich Schreiber[2], Irwin Shapiro[5]

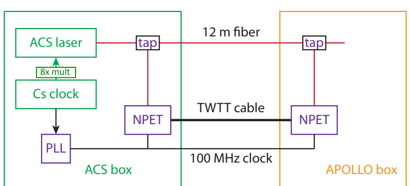
1: University of California, San Diego; Center for Astrophysics and Space Sciences (CASS)
2: Fundamentalstation Wettzell, Germany
3: University of California, Berkeley
4: Wellesley College; Department of Physics; Wellesley, MA, USA
5: Harvard/Smithsonian Center for Astrophysics, Cambridge, MA, USA

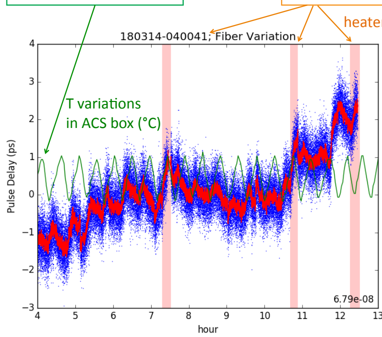
tmurphy@physics.ucsd.edu

A pache
P oint
O bservatory
L unar
L aser–ranging
O peration

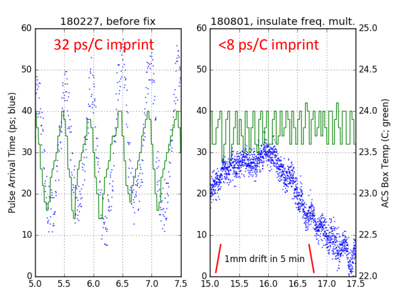
Calibration
(JK, TM, SS, US, NC, JB)

APOLLO installed an Absolute Calibration System (ACS) in 2016 based on an 80 MHz 10 ps laser locked to a cesium standard (CQG **34**, 245008). In 2018, visiting instrumentation from Wettzell validated calibration fiber delivery at the few-ps level.





Variations are well under 1 mm (6.7 ps). Differential use of the ACS means we only care about variations over 5–10 minute periods. We did find a thermal dependence of the absolute ACS pulse times, which we then were able to suppress, as seen below.



The ACS 8x clock multiplier was thermally susceptible. Better temperature regulation and insulation keeps drift below millimeter scales in relevant 5 minute intervals.

Gravity Signals
(YL, TM)

Numerically integrating the Einstein Infeld Hoffmann (EIH) Eq. of motion, we explore the imprints of physical influences on the lunar orbit. A least-squares fit removes distracting changes to initial conditions, leaving an irreducible physics signal, $\Delta r(t)$.

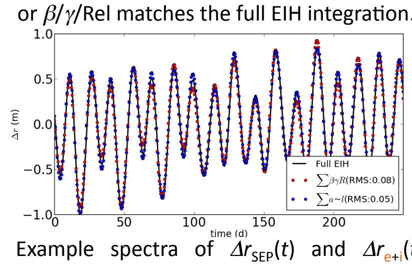
$$
\begin{aligned}
\ddot{\mathbf{r}}_{i\,\mathrm{point\,mass}} &= \sum_{j\neq i}\frac{\mu_j(\mathbf{r}_j-\mathbf{r}_i)}{r_{ij}^3}\Bigg\{1-\overset{a}{\frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}}}-\overset{b}{\frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}}}+\overset{c}{\gamma\left(\frac{v_i}{c}\right)^2} \\
&+\overset{d}{(1+\gamma)\left(\frac{v_j}{c}\right)^2}-\overset{e}{\frac{2(1+\gamma)}{c^2}\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j}-\frac{3}{2c^2}\left[\frac{(\mathbf{r}_i-\mathbf{r}_j)\cdot\dot{\mathbf{r}}_j}{r_{ij}}\right]^2 \text{ f} \\
&+\overset{g}{\frac{1}{2c^2}(\mathbf{r}_j-\mathbf{r}_i)\cdot\ddot{\mathbf{r}}_j}\Bigg\}+\frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\times\left\{[\mathbf{r}_i-\mathbf{r}_j]\cdot\dot{\mathbf{r}}_i\overset{h}{(2+2\gamma)}\ \overset{i}{(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j)}\right\} \\
&-\frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\times\left\{[\mathbf{r}_i-\mathbf{r}_j]\cdot\dot{\mathbf{r}}_j\overset{j}{(1+2\gamma)}\ \overset{k}{(\dot{\mathbf{r}}_i-\dot{\mathbf{r}}_j)}\right\}+\frac{3+4\gamma}{2c^2}\sum_{j\neq i}\frac{\mu_j\ddot{\mathbf{r}}_j}{r_{ij}} \text{ l}
\end{aligned}
$$

The EIH package can be run as a whole, as individual pieces (a–l), or decomposed into β, γ, or 'relativistic' (β=γ=0) terms. The table shows example synodic (29.53 d) signals.

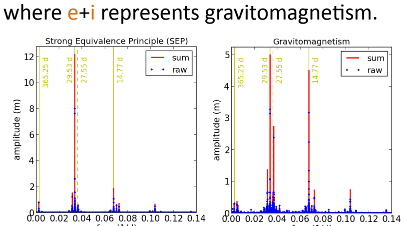
Example Table of 29.53 d

Term	Ampl (m)	Phase (°)	Term	Ampl (m)	Phase (°)
a	0.87	-2.6	j	8.55	-1.6
b	0.00	-4.3	k	0.38	21.7
c	-3.08	-0.8	l	-1.12	-0.6
d	-0.01	-55.3	Σa…l	**0.11**	**14.6**
e+i	5.01	-2.9	β	0.44	-2.6
f	0.03	-0.5	γ	-0.08	11.3
g	-0.05	-0.6	Rel	-0.26	-5.4
h	-10.46	-2.0	Σ$\beta\gamma$Rel	**0.11**	**10.7**
Full EIH run: Ampl = **0.10 m**; Phase = **11.1°**					

Summing individual irreducible series of a–l or β/γ/Rel matches the full EIH integration.

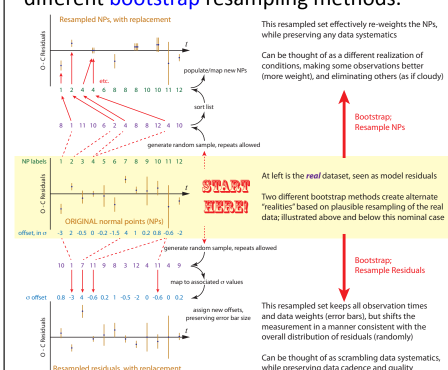


Example spectra of $\Delta r_{\mathrm{SEP}}(t)$ and $\Delta r_{\mathrm{e+i}}(t)$; where e+i represents gravitomagnetism.

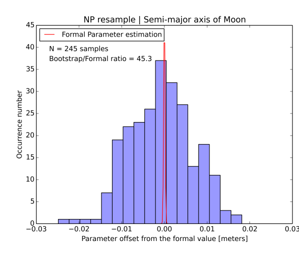


Model Uncertainties
(FN, NJ, JC, TM, RR, IS, JB)

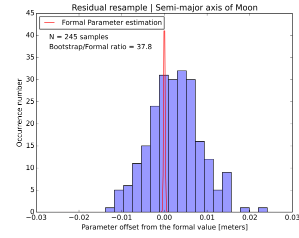
Observed-minus-calculated (O–C) residuals for APOLLO tend to exceed estimated data uncertainties by factors of ten or more, for all models. Formal parameter uncertainties therefore tend to be unrealistically small. Scaling factors are necessary. Here, we systematically scale errors based on two different bootstrap resampling methods.



A multitude of such "fake," but plausible, data sets can be generated and used to estimate parameters in the model. The resulting distributions of parameter estimates suggest realistic uncertainties.



μ_{formal} = 381291.345265 km
σ_{formal} = 0.16 mm
$\mu_{\mathrm{bootstrap}}$ = 381291.345264 km
$\sigma_{\mathrm{bootstrap}}$ = 7.22 mm
σ ratio = 45.3



μ_{formal} = 381291.345265 km
σ_{formal} = 0.16 mm
$\mu_{\mathrm{bootstrap}}$ = 381291.345267 km
$\sigma_{\mathrm{bootstrap}}$ = 6.02 mm
σ ratio = 37.8

Few-millimeter uncertainties are more likely realistic than the 0.16 mm formal error.