On the principles of satellite-based Gravity Field Determination with special focus on the Satellite Laser Ranging technique

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GFZ Helmholtz Centre POTSDAM

GRGS GROUPE DE RECHERCHE DE GEODESIE SPATIALE

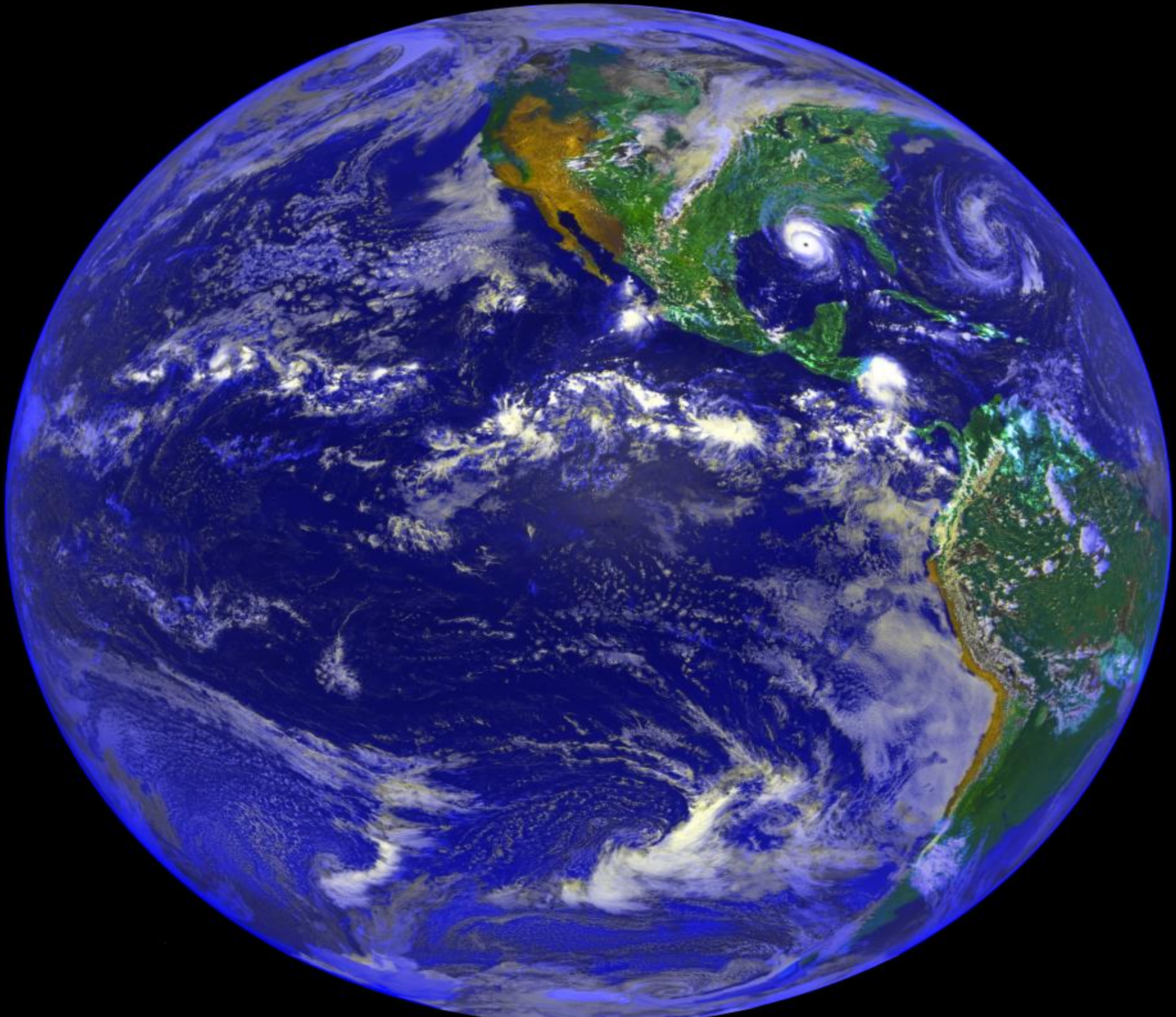
Outline:

- On the Gravity Field of the Earth
- The principles of global gravity field estimation using satellites
- Recent gravity satellite missions and some results
- About the contribution of the SLR technique to global gravity field determination

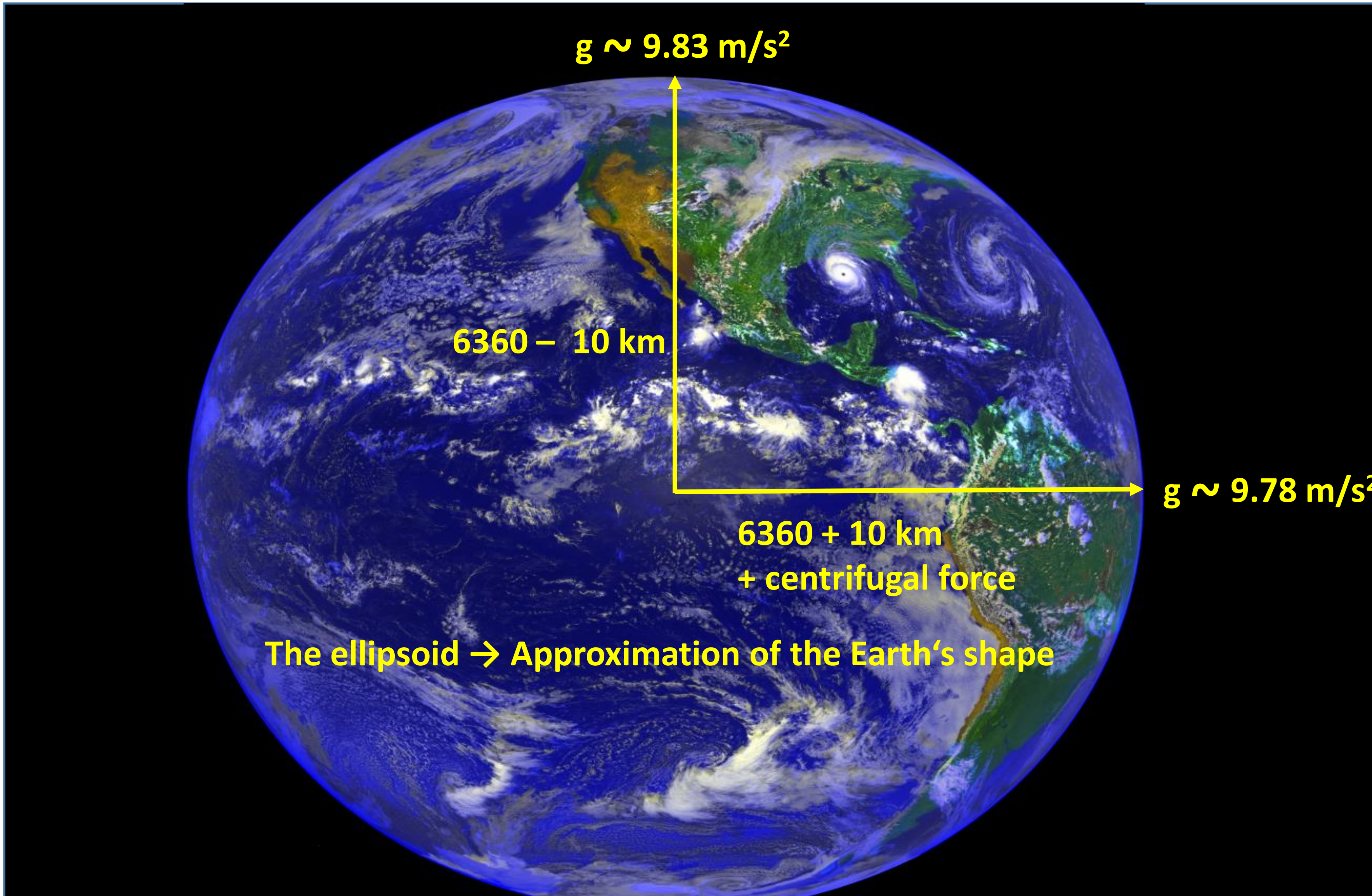
On the Gravity Field of the Earth


At first glance the shape of the Earth seems to be a perfect sphere

When looking more in detail → The Earth is flattened



→ the gravitational attraction is larger at the poles than at the equator



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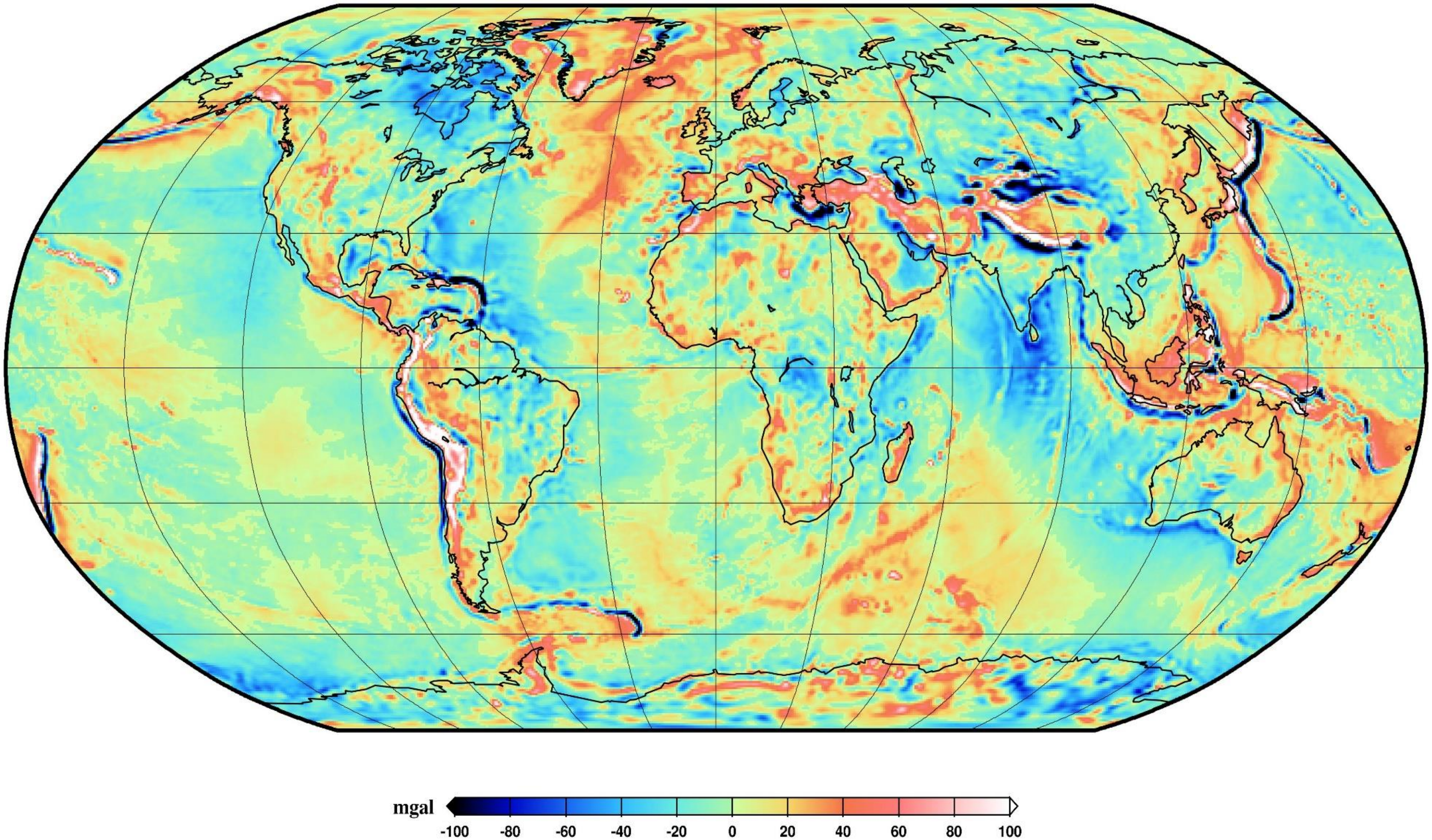
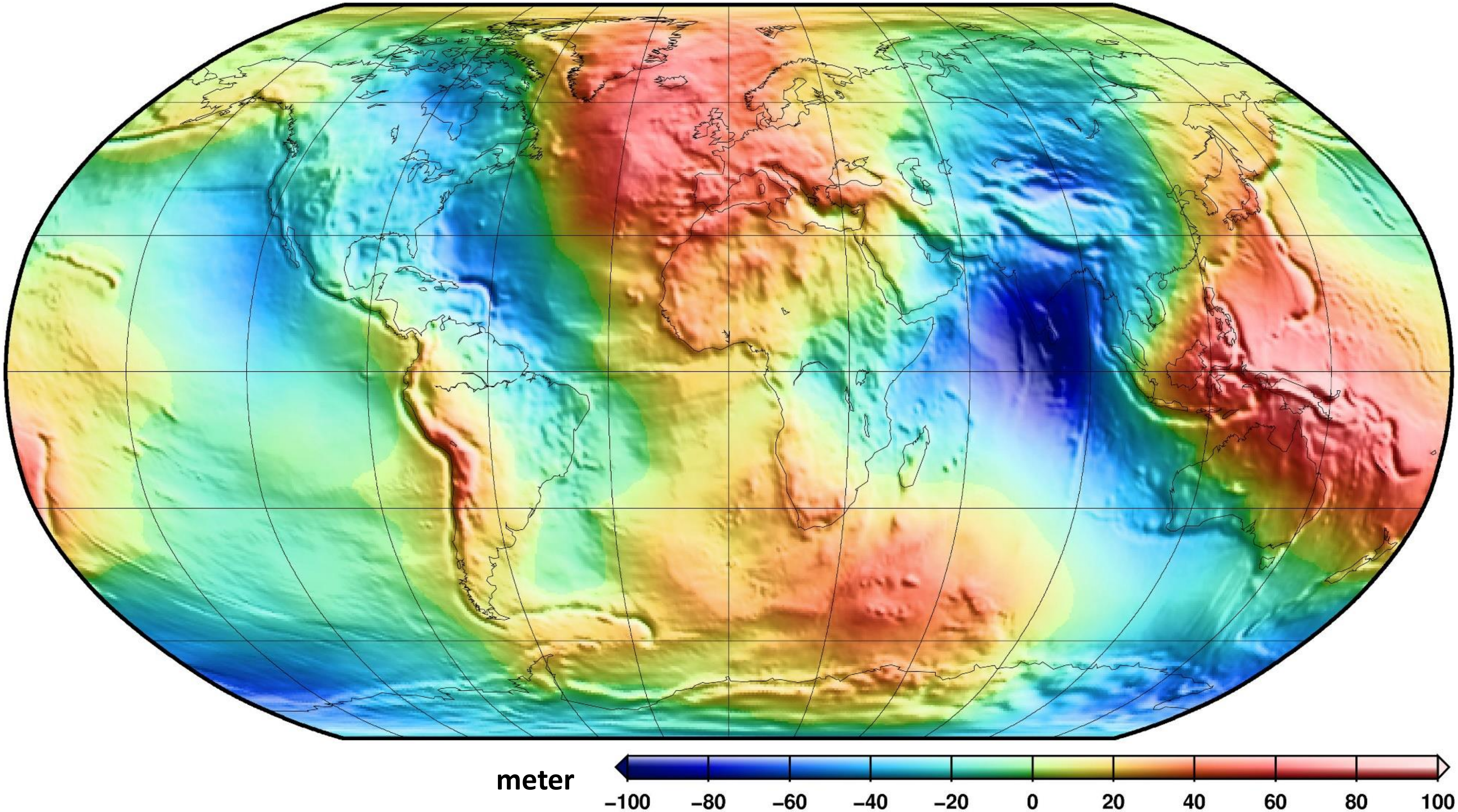
The Gravity Field of the Earth (in terms of Gravity Anomalies)



Deviation of the gravitational acceleration w.r.t. the gravity on the reference ellipsoid (simplified): The values are between –100 (black) to +100 (white) millionths of the mean gravitational acceleration on the Earth‘s surface (**mgal**). Individual values are up to three times larger

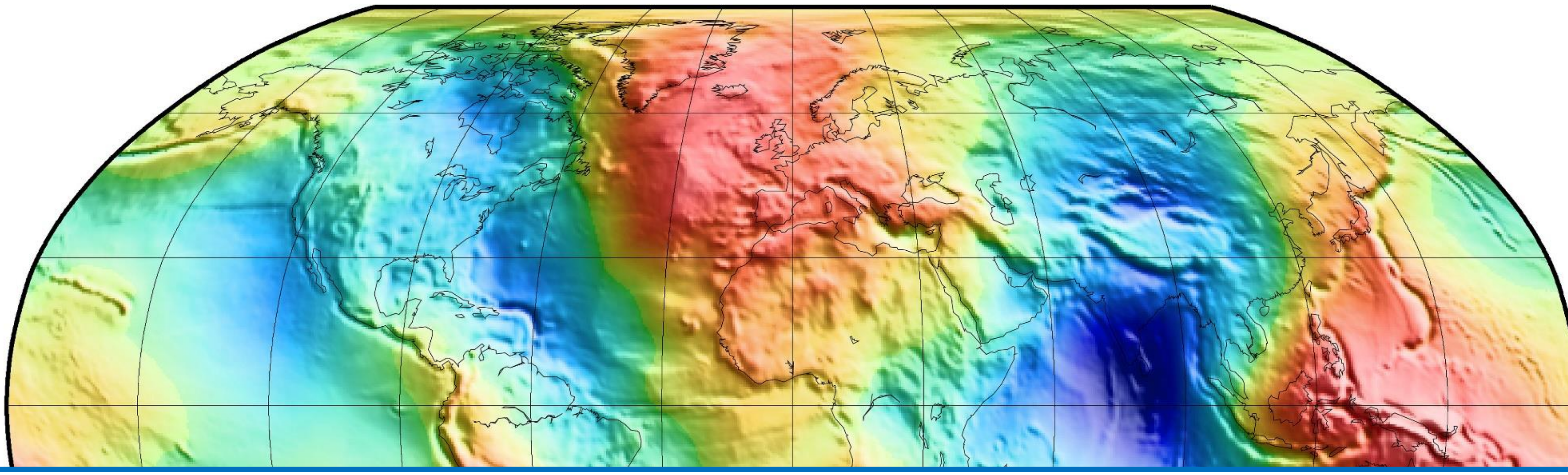
The gravity field of the Earth (in terms of Geoid* height)



Geographical distribution of the height differences between the Geoid* and the reference ellipsoid („bulges“ and „depressions“):
The values are between –110 m (dark blue) up to + 90 m (dark red).

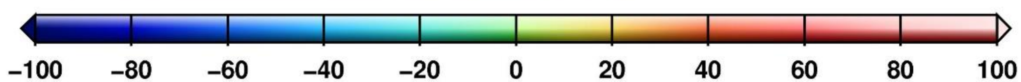
*Geoid = Equipotential surface of the Earth gravity field which coincides with the undisturbed sea surface)

The gravity field of the Earth (in terms of Geoid height)



Reasons for the fine structure resp. deviations of the gravity field:

- Various mass / density inhomogeneities in the Earth interior (mantle, lithosphere …) and near resp. on the surface
- Continental and ocean bottom topography
- Ocean currents
- .....



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The most common mathematical representation of global gravity field models:

Expressed as expansion (superposition) of spherical harmonic functions

Spherical harmonic functions = three-dimentional spatial waves of spherical symmetry

$$V(r,\varphi,\lambda)=\frac{GM}{r}\left(C_{00}+\sum_{l=1}^{\infty}\left(\frac{R}{r}\right)^{l}\sum_{m=0}^{l}\left[C_{lm}P_{lm}(\sin\varphi)\cos m\lambda+S_{lm}P_{lm}(\sin\varphi)\sin m\lambda\right]\right)$$

surface spherical harmonics

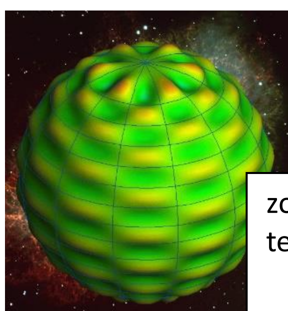
V = Earth's gravitational potential expresssed in spherical harmonics

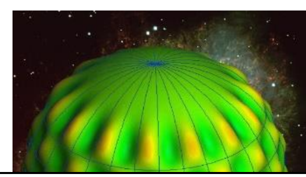
- Gravity field determination means estimation of spherical harmonic coefficients C_{lm} and S_{lm} for the gravity field of the Earth
- A „Gravity fiel model“ is a data set of spherical harmonic coefficients generated from measurements, which expresses the Earth gravity field in a certain accuracy

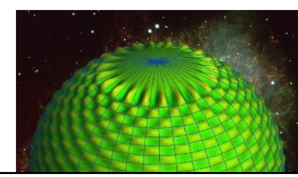
C_{lm}, S_{lm} - spherical harmonic (or Stokes') coefficients

$P_{lm}(\sin\varphi)$ - Legendre polynomials (m=0), associated Legendre polynomials ($m \neq 0$)

Tesseral harmonics



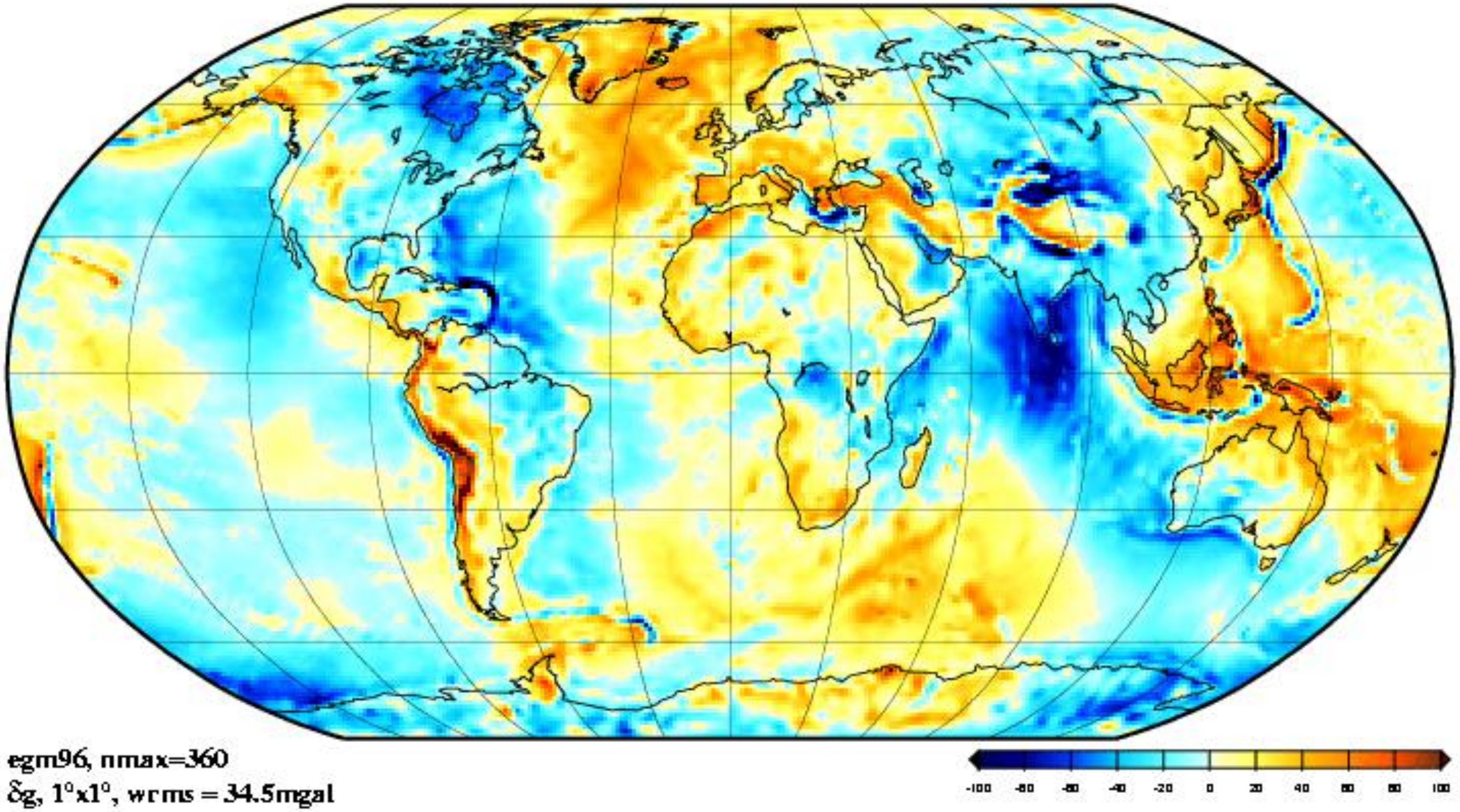




zonal harmonics, coefficients:	m = 0	(l 'oscillations' along latitude)
tesseral harmonics, coefficients:	m ≠ l, m>0	(l-m 'oscillations' along latitude, m 'oscillations' along longitude)
sectorial harmonics, coefficients:	m = l	(l 'oscillations' along longitude)

Composition of a gravity field model (gravity anomalies)
Example for the successive superposition of spherical hamormics

EGM96 (Lemoine et al. 1998) , maximum degree/order 360 → spatial resolution of 50 km at the Earth's surface



On the principle of gravity field determination using satellites

The main issue of global gravity field determination:

- Local (terrestrial) gravity measurements have a high accuracy (Gravimetry, Airborne gravimetry, Superconducting gravimetry, Absolute gravimetry) → Applications i.e. in Geophysics, Geology, Exploration and Hydrology

- But: It's up to now impossible to link/combine ground based gravity measurements accurately over long and global distances

- → The accuracy of long-scale and global gravity field components based on local resp. ground-based data is poor (height problem, height reference between continents, accuracy of sea surface)

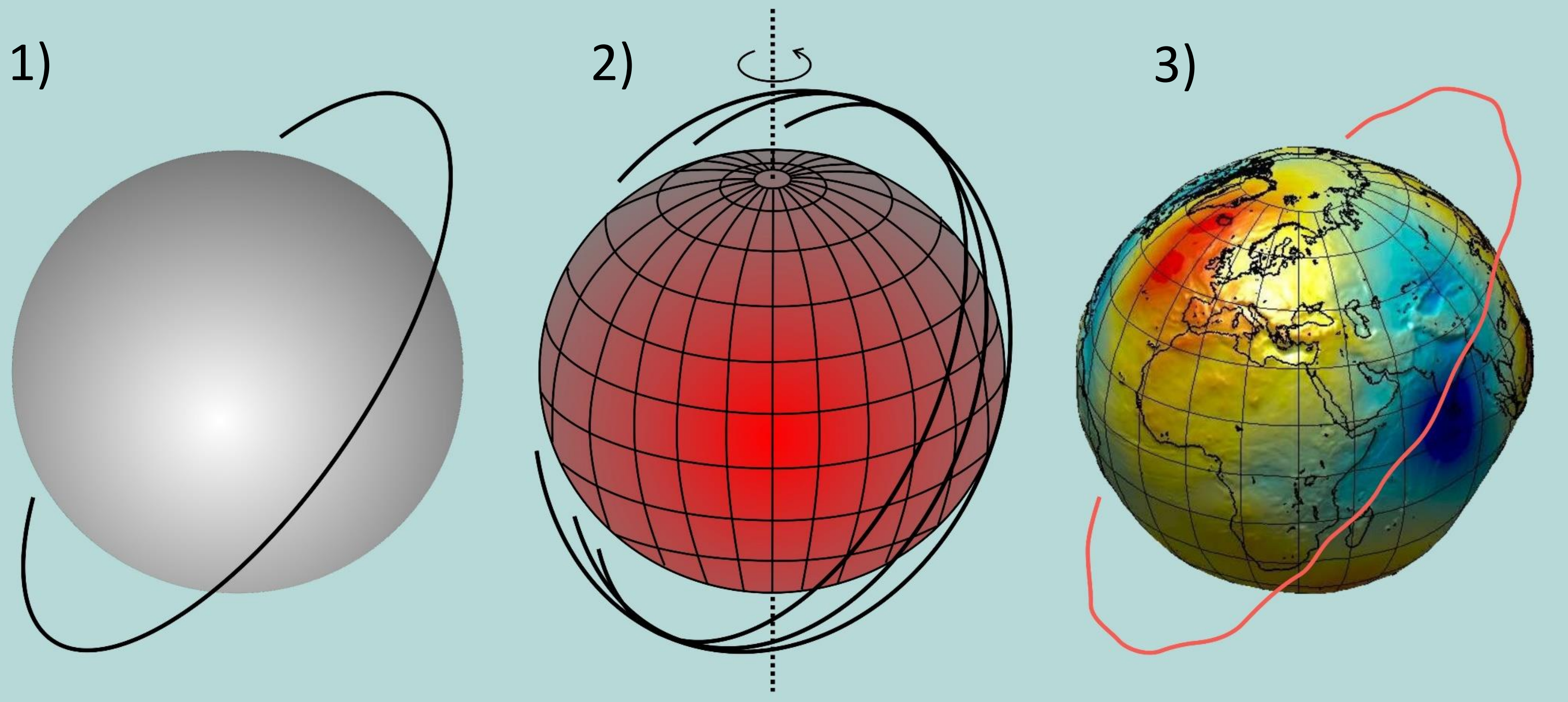
The solution: Gravity field determination from space by using satellites

→ **Basic principle: Evaluation of satellite orbit pertubations**

1) Spherical central body – Elliptical satellite orbit with fixed orientation

2) Rotational ellipsoid as central body – The orbit plane of the Kepler ellipse is rotating around the central body (node rotation)

3) The real Earth as central body – Node rotation with further curvatores



Source: R. Rummel / TU Munich

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The solution: Gravity field determination from space by using satellite

Basic principle: Evaluation of satellite orbit pertubations

– Applied since the launch of the first satellites (1957)
Photo-optical satellite tracking **= Azimuth/Elevation angle measurements** w.r.t. the starry sky

– Improved accuracy by Laser tracking systems since ~ 1980
= Satellite Laser Ranging (SLR)

– Important „Quantum jump" in the accuracy has been achieved by **continuous orbit tracking** using GNSS, since the launches of CHAMP (2000 - 2010) and GRACE (seit 2002)
= Position measurements (in principle), a few second sampling

Gravity field determination using satellites (by evaluation of orbit pertubations) means solving the equation of motion on the basis of precise orbit determination:

$$\ddot{\vec{r}} = \vec{F}(t, \vec{r}, \dot{\vec{r}}) / m = \vec{f}_g + \vec{f}_{ng} + \vec{f}_{emp}$$

where f_g = Gravitational forces (contains the spher. harm. coefficients)

f_{ng} = Non-gravitational forces

f_{emp} = Unknown residual forces

But it's practically not possible to solve this equation analytically.

Common numeric principles for solving the equation of motion:

- Numerical integration (Dynamic approach)
- Kinematic orbit determination and taking the obtained positions/velocities as observations for specific algorithms like:
  - Energy balance approach
  - Reduced dynamic approach
  - Short arc approach

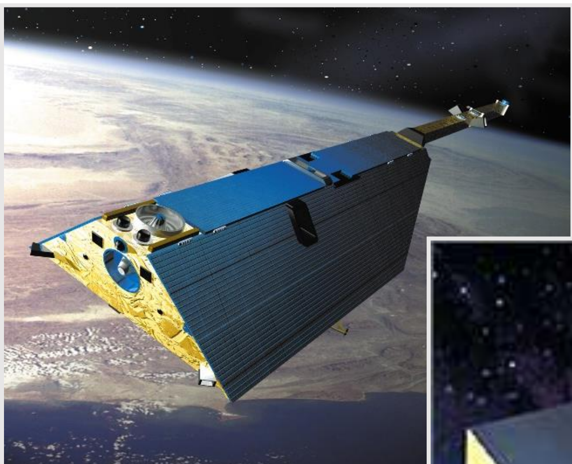
Estimation of the spher. harm. coefficients:

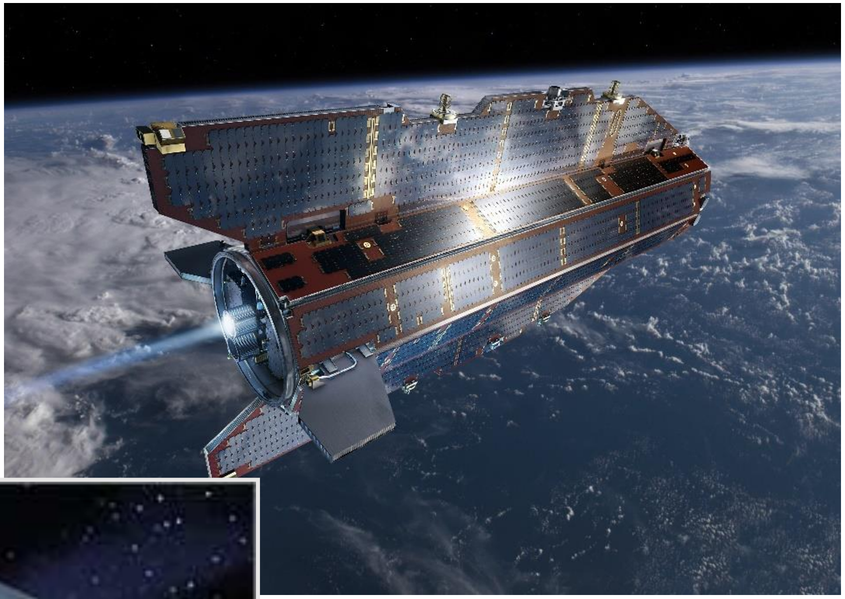
- Obtained from least squares adjustment
- Based on the functional dependency of the satellite orbit from the spher. harm. coefficients
- Includes estimation of empirical parameters to consider the unknown residual forces

Recent gravity satellite missions and some results

Famous recent gravity satellite missions:

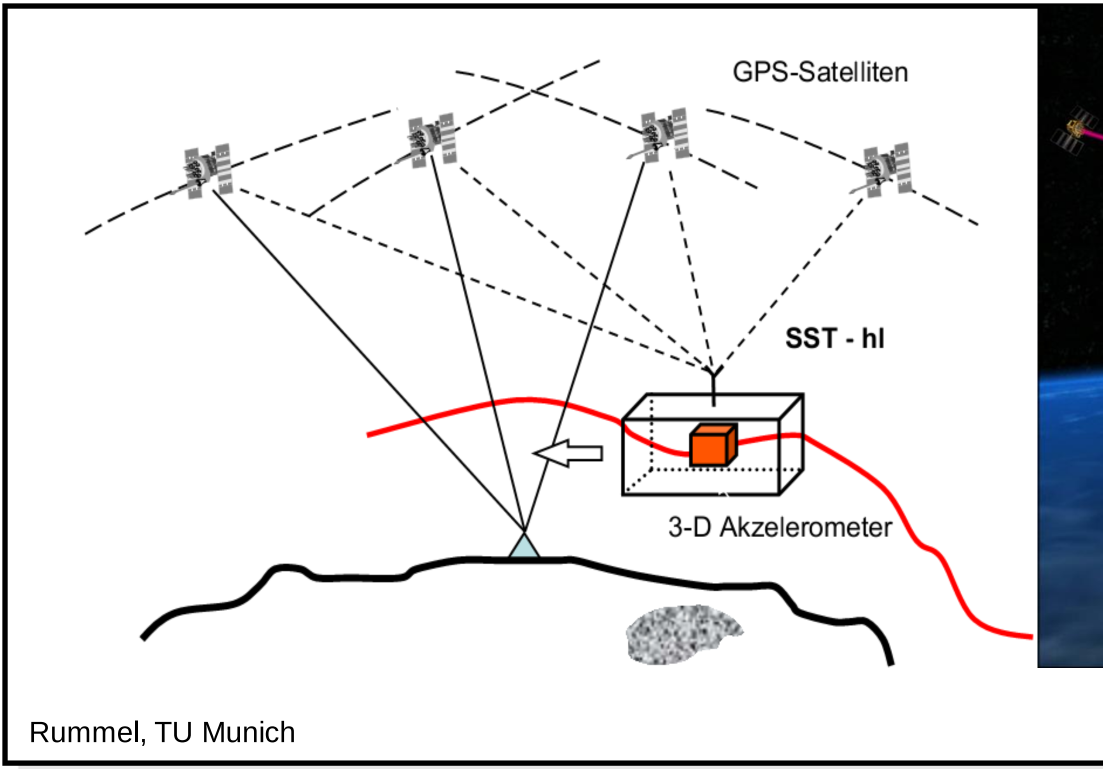
- CHAMP (2000 – 2010)
- GRACE (since 2002)
- GOCE (2009 – 2013)

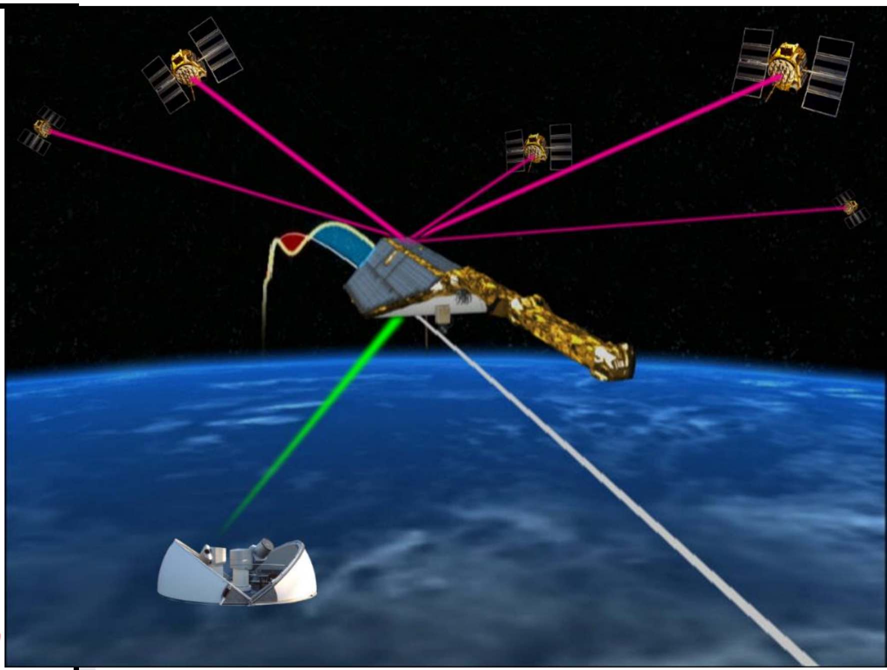




The Satellite Mission CHAMP

(CHAMP = CHAllenging Minisatellite Payload)

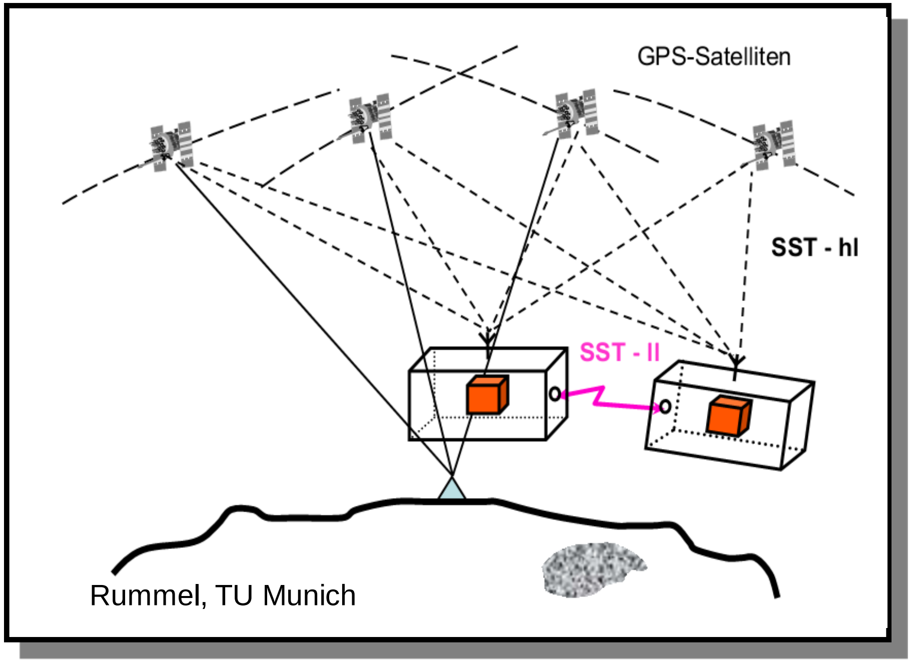


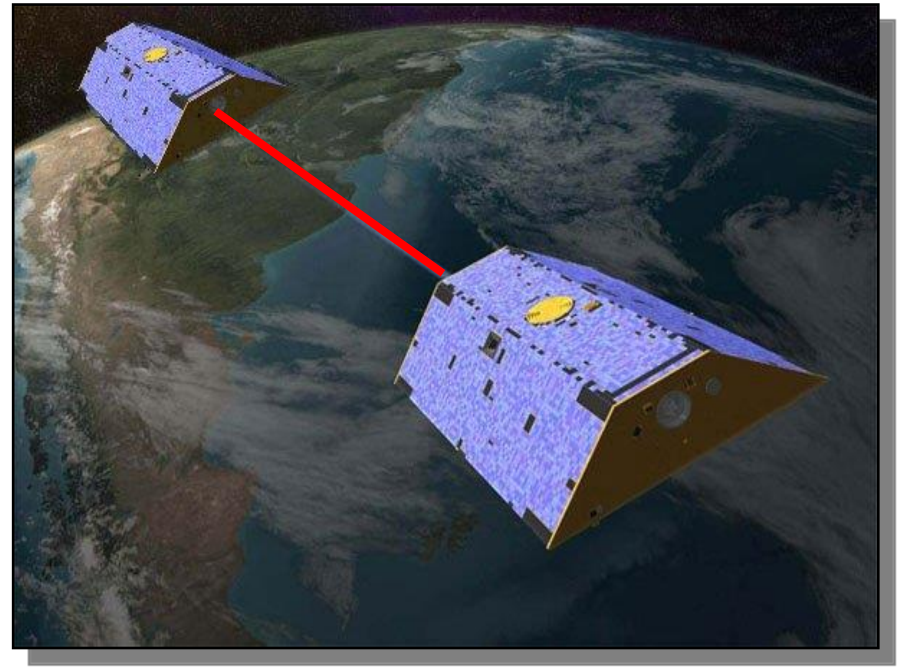


- Continous GPS-measurements to the high-altitude GPS satellites (GPS-SST)
- Measuring of the non-gravitative forces by a 3D accelerometer
- Additional „classical“ SLR tracking from ground
- Allows for much more precise determination of the fine structure of the gravity field than from ground tracking only
- 2000 - 2010

The Satellite Mission GRACE

(GRACE = Gravity Recovery And Climate Experiment)





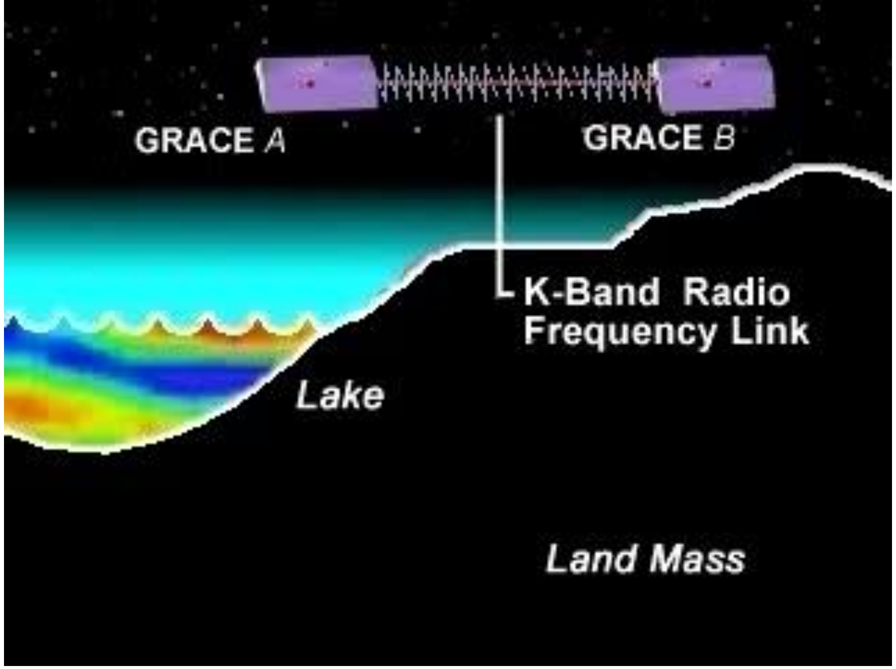
- Two CHAMP-type satellites (launched in 2002) in an altitude of about 500 km are flying one after another in a distance of about 200 km

  In addition to GNSS and SLR tracking:
- Ultra precise relative range and range-rate measurements between the satellites (Microwave Ranging System of a few Micrometer accuracy)
  - Measuring of **differential orbit pertubations** between the satellites
  - Higher accuracy in gravity field determination than with single satellites like CHAMP

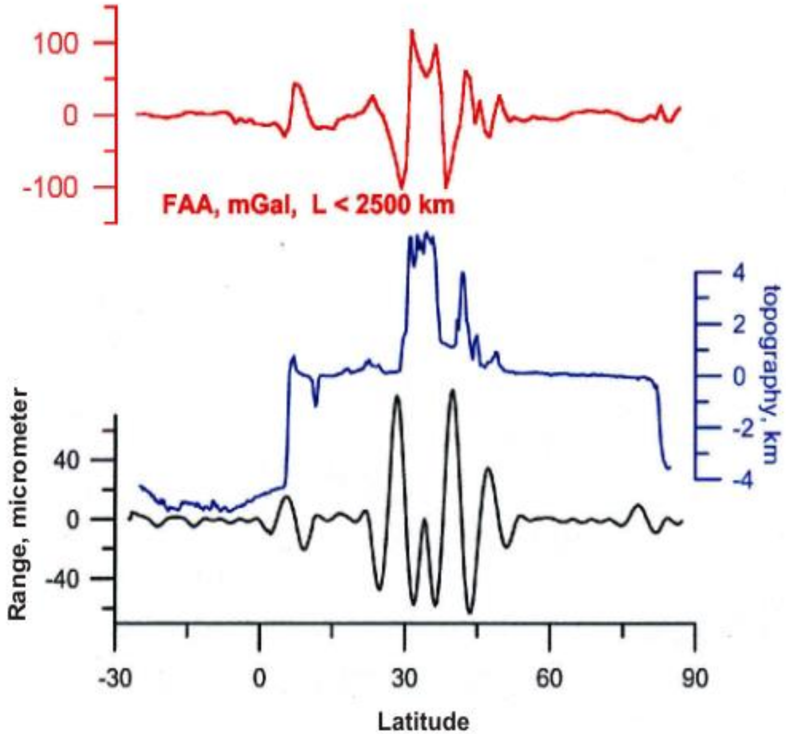
→ **GRACE is sensitive for large scale temporal variations in the gravity field and enables monitoring of mass redistributions on the Earth surface (glacier melting, ground water storage variations …)**

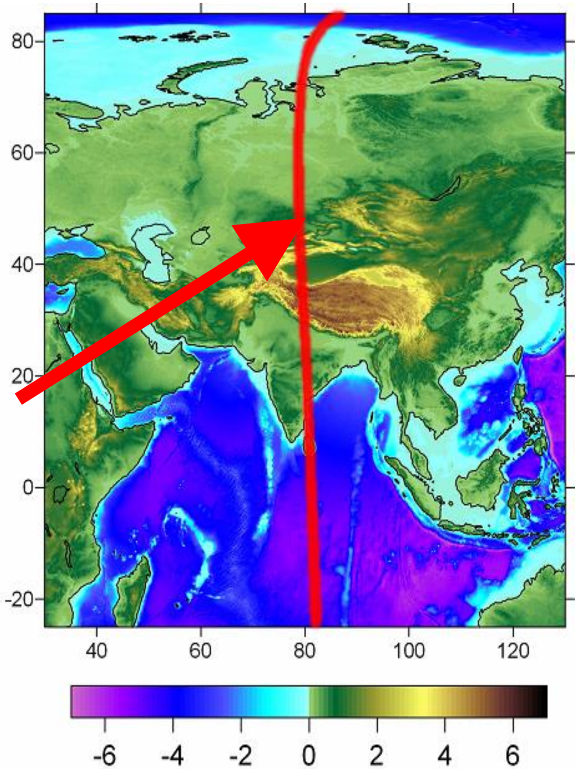
GRACE
Inter-Satellite Ranging

accuracy = few µm
(few tenth the diameter of
human hairs)



Example 3. May 2003





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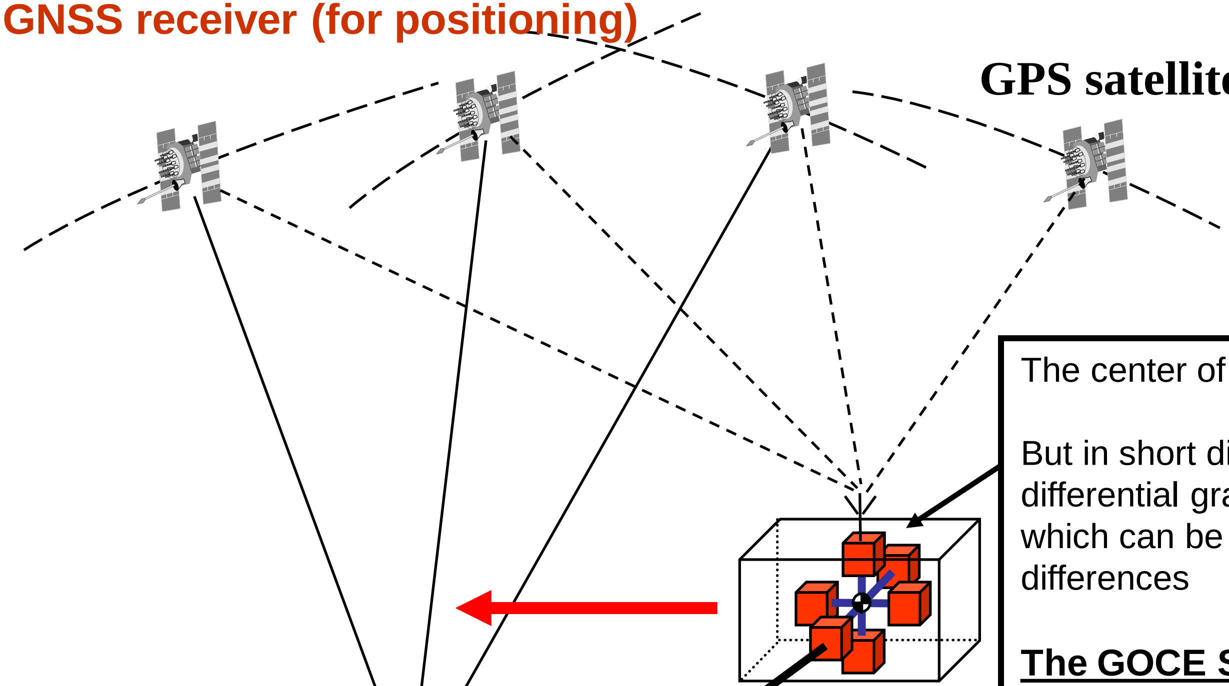
The Satellite Mission GOCE

(GOCE = Gravity field and steady-state Ocean Circulation Explorer)

GOCE instrumentation:

- Three-axial satellite gradiometer
- GNSS receiver (for positioning)



The center of mass of the satellite is weightless.

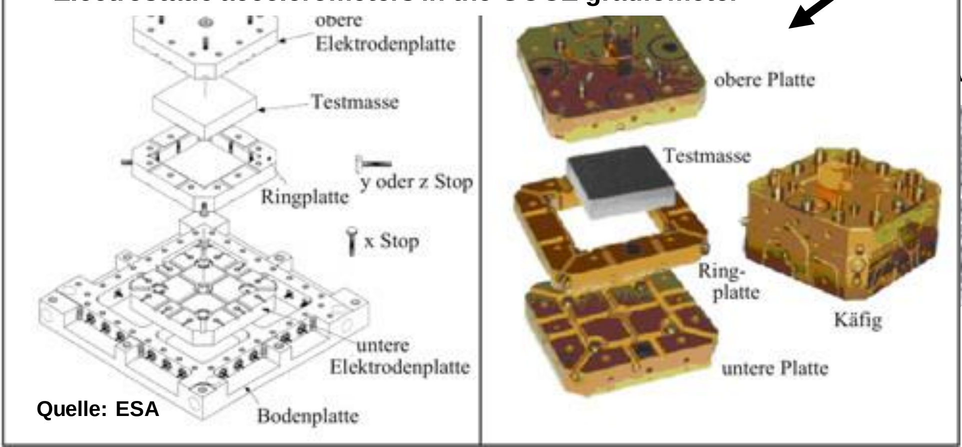
But in short distances to the center of mass differential gravity forces (gravity gradients) occur, which can be measured as acceleration differences

The GOCE Satellite Gradiometer:

3 pairs of accelerometers
(1 pair per spatial direction, distance 0.5 m, measurement accuracy ~ 10^{-12} ms^{-2} = 100 x more sensible as in previous satellite missions)

An accelerometer sensor contains a 4cm x 4cm x 1cm large free-floating Platinum-Rhodium prove mass which is kept in an electrostatically balanced state.

Electrostatic accelerometers in the GOCE gradiometer



Quelle: ESA

The GOCE measurement principle:

- The acceleration differences along each axis can be taken as second derivatives of the gravitational potential (gravity gradient) in the respective direction

- This measurement principle means the direct measurement of a functional of the Earth gravity field (instead of the indirect gravity measurements via evaluation of orbit pertubations)

Remark:

- From theoretical point of view: A free-floating accelerometer proofmass outside the satellite‘s center of mass can be seen as a small satellite flying in a slightly deviating orbit

→ Satellite gravity gradiometry is nothing else than measuring differential orbit pertubations (but on a different scale as GRACE)

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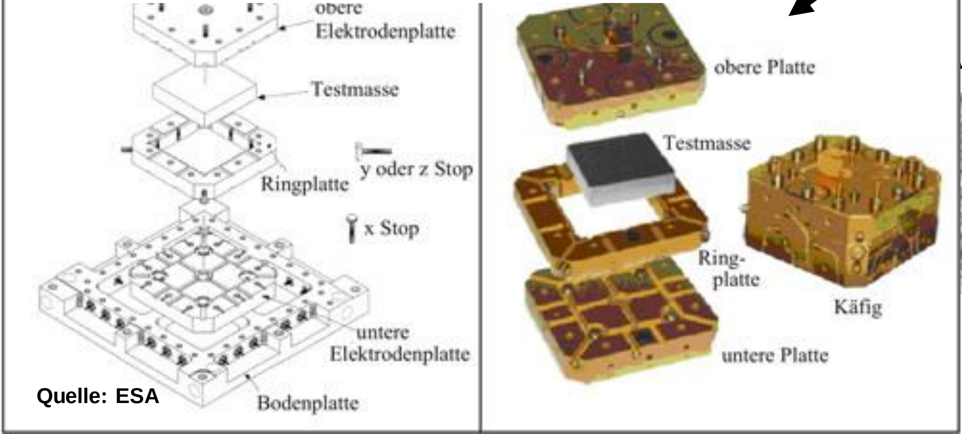
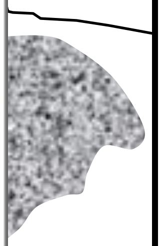
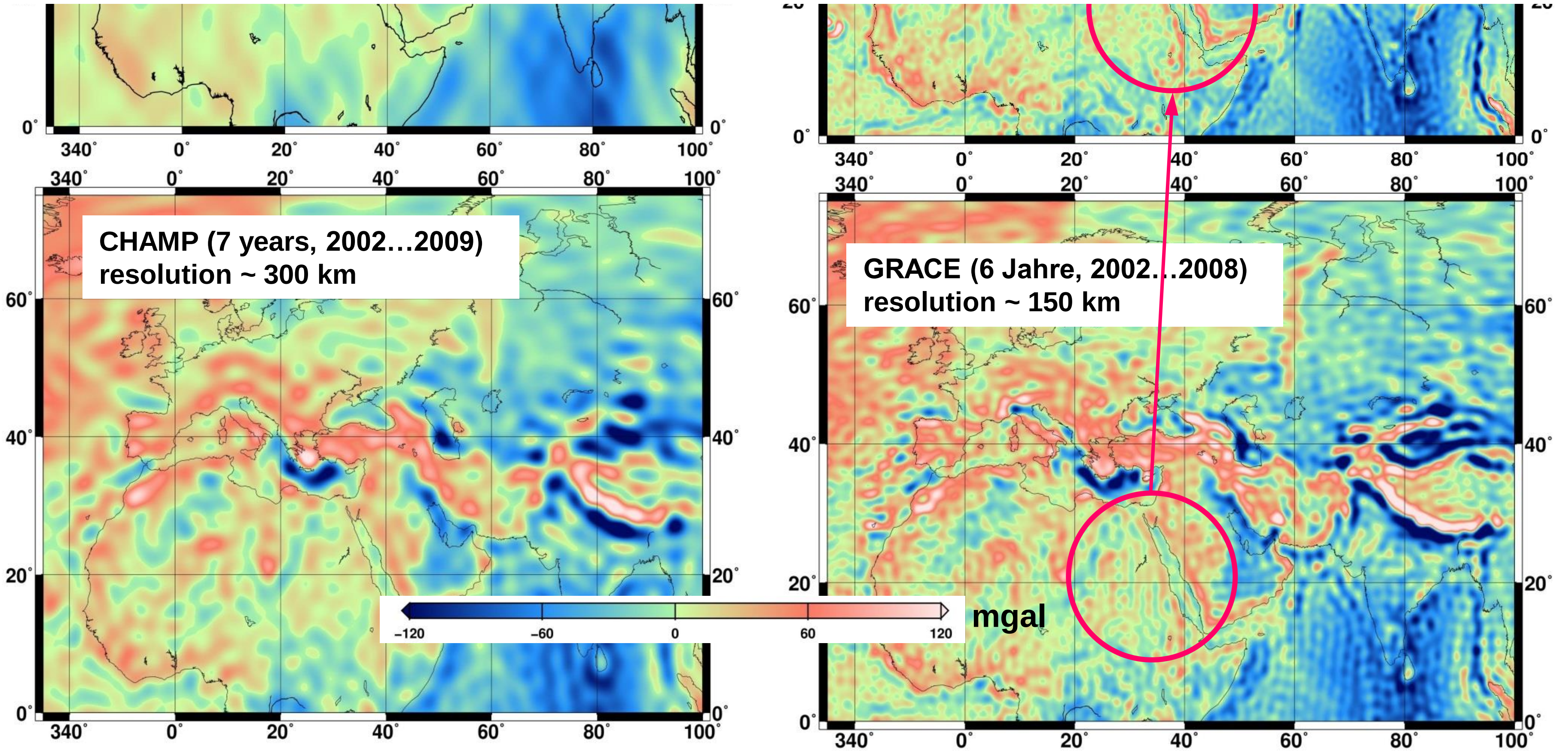


Quelle: ESA



One example from the results obtained with CHAMP, GRACE and GOCE



List of recent Satellite-only gravity field models

from: http://icgem.gfz-potsdam.de

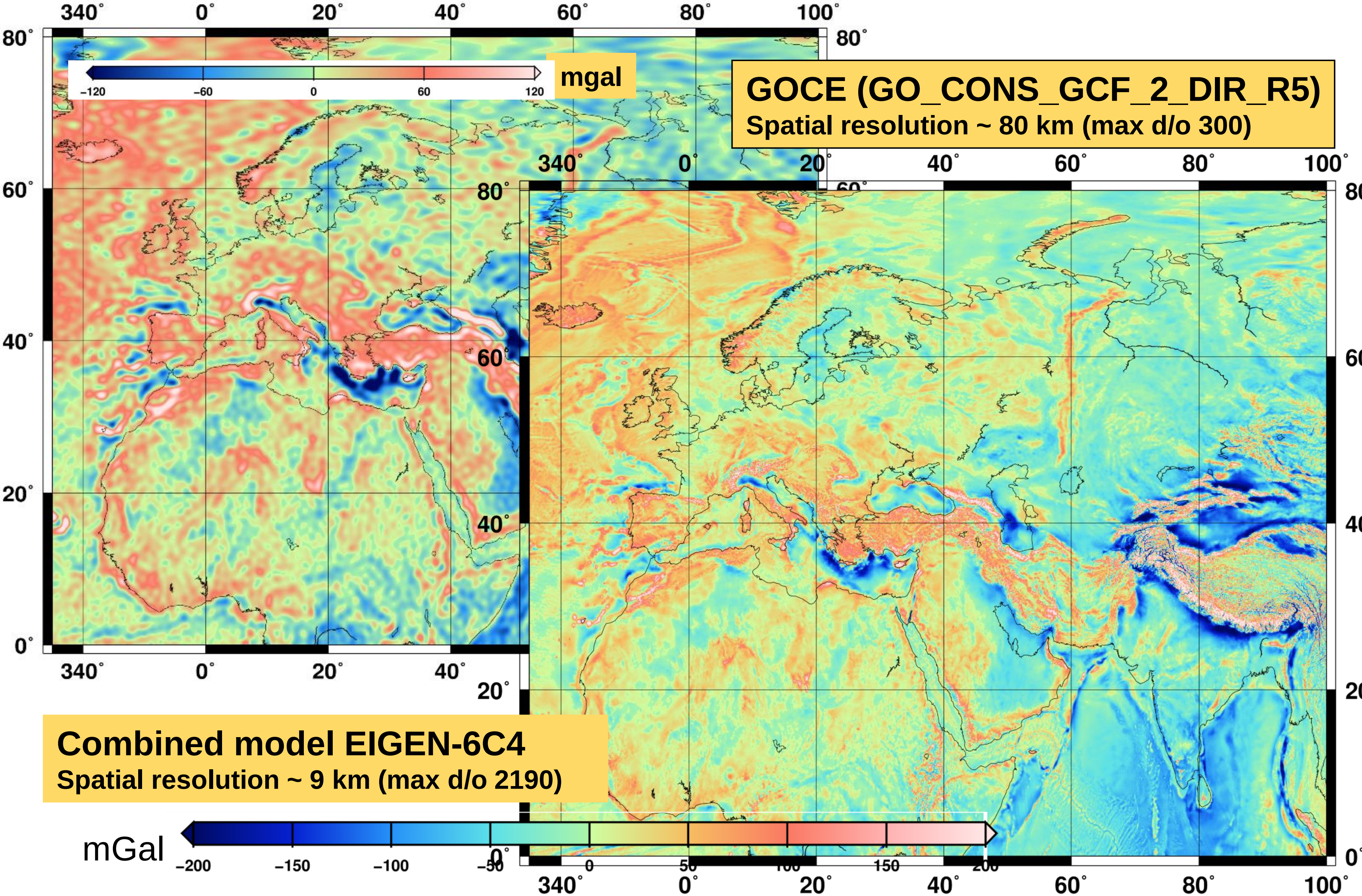
Nr ▲	Model ⇅	Year ⇅	Degree ⇅	Data ⇅	Reference ⇅
157	ITU_GRACE16	2016	180	S(Grace)	Akyilmaz et al, 2016b
156	ITU_GGC16	2016	280	S(Grace,Goce)	Akyilmaz et al, 2016a
155	EIGEN-6S4	2016	300	S(Goce,Grace,Lageos)	Förste et al, 2016
151	GGM05G	2015	240	S(Grace,Goce)	Bettadpur et al, 2015
150	GOCO05s	2015	280	S(see model)	Mayer-Gürr, et al. 2015
149	GO_CONS_GCF_2_SPW_R4	2014	280	S(Goce)	Gatti et al, 2014
147	ITSG-Grace2014s	2014	200	S(Grace)	Mayer-Gürr et al, 2014
146	ITSG-Grace2014k	2014	200	S(Grace)	Mayer-Gürr et al, 2014
145	GO_CONS_GCF_2_TIM_R5	2014	280	S(Goce)	Brockmann et al, 2014
144	GO_CONS_GCF_2_DIR_R5	2014	300	S(Goce,Grace,Lageos)	Bruinsma et al, 2013
143	JYY_GOCE04S	2014	230	S(Goce)	Yi et al, 2013
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
134	ITG-Goce02	2013	240	S(Goce)	Schall et al, 2014

<u>Purpose of Satellite-only gravity field models:</u>

- Precise orbit computation for various Earth observation satellites (e.g. SAR-Interferometry, Radar Altimetry and other remote sensing techniques)
- Oceanography (e.g. ocean currents)
- Regional gravity field modelling together with terrestrial data
- ....

Further enhancement of the spatial resolution

by combination of satellite data with ground gravity data (Altimetry, Airborne and terrestrial gravimetry



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Nr ▲	Model ⇅	Year ⇅	Degree ⇅	Data ⇅	Reference ⇅
154	GOCO05c	2016	720	S,G,A (see model)	Pail, et al. 2016
153	GGM05C	2016	360	S(Grace,Goce),G,A	Ries et al, 2016
152	GECO	2015	2190	S(Goce),EGM2008	Gilardoni et al, 2015
148	EIGEN-6C4	2014	2190	S(Goce,Grace,Lageos),G,A	Förste et al, 2015
104	EGM2008	2008	2190	S(Grace),G,A	Pavlis et al, 2008

- The development of EGM2008 set a benchmark in high resolution gravity field modelling
- All other models are improvements based on EGM2008 by combination of new satellite data (incl. Altimetry) with the up to now unmatched continental gravity data from EGM2008

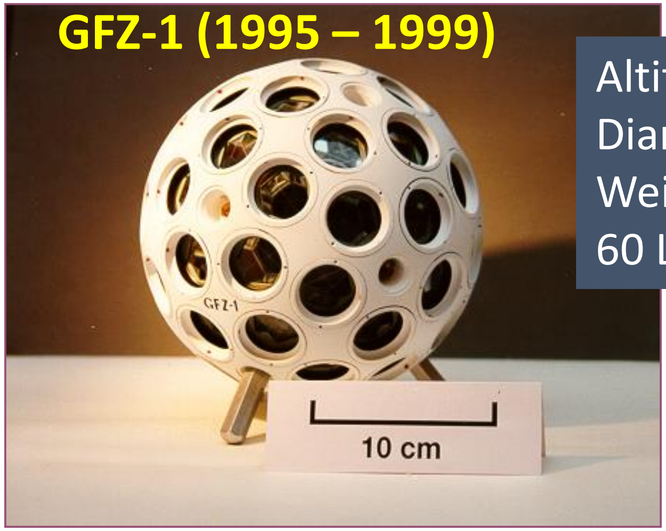
Purpose of combined gravity field models:

- Regional and global gravity field modelling (e.g. height system(s))
- Geophysical modelling
- Oceanography (e.g. ocean currents)
- …

Applications of SLR data in the context of satellite based gravity field determination

Applications of SLR data in the context of satellite based gravity field determination:

1) Direct use for gravity field determination (long wavelengths part incl. temporal variation)
2) Validation of gravity field models via orbit computation tests using SLR tracking data
3) Validation of GNSS-based orbits for satellites which are used for gravity field determination



Altitude: 387 - 380 km
Diameter: 22 cm
Weight: 21 kg
60 Laser-Retroreflektors

Altitude: 5858 – 5958 km
Diameter: ~ 60 cm
Weight: 411 kg
426 Laser-Retroreflektors

Validation of GNSS-based orbits for satellites which are used for gravity field determination

– The orbits of Low Earth Orbit satellites (like GRACE, CHAMP and GOCE) are computed from GNSS measurements to the high-altitude GNSS satellites

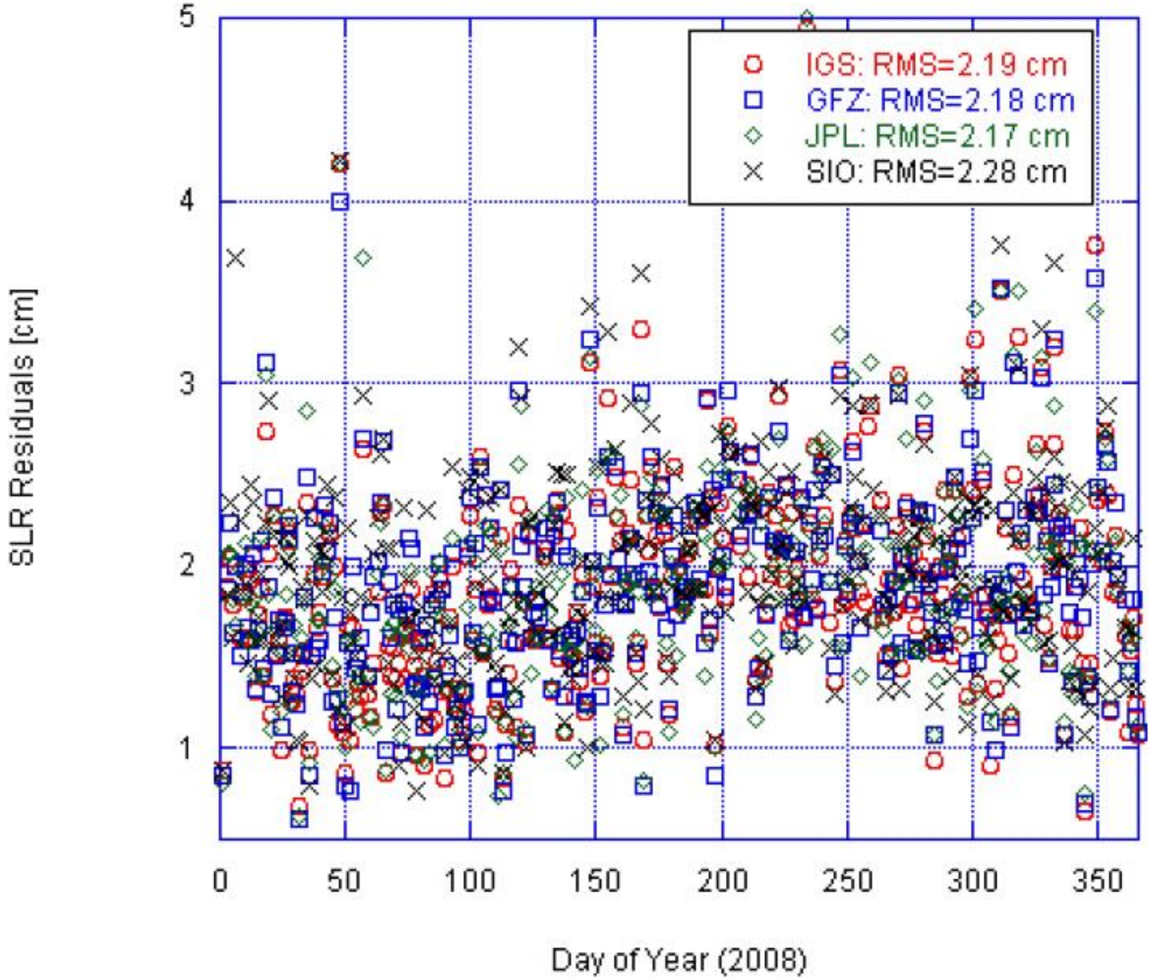
– Precise orbits and clock errors of the GNSS satellites are needed

– The accuracy of the GNSS satellite orbits and clock errors has an impact on the accuracy of the LEO orbits and thus on the gravity field determination

☞ SLR observations to satellites like GRACE can be used for independent quality checks of LEO satellite orbits

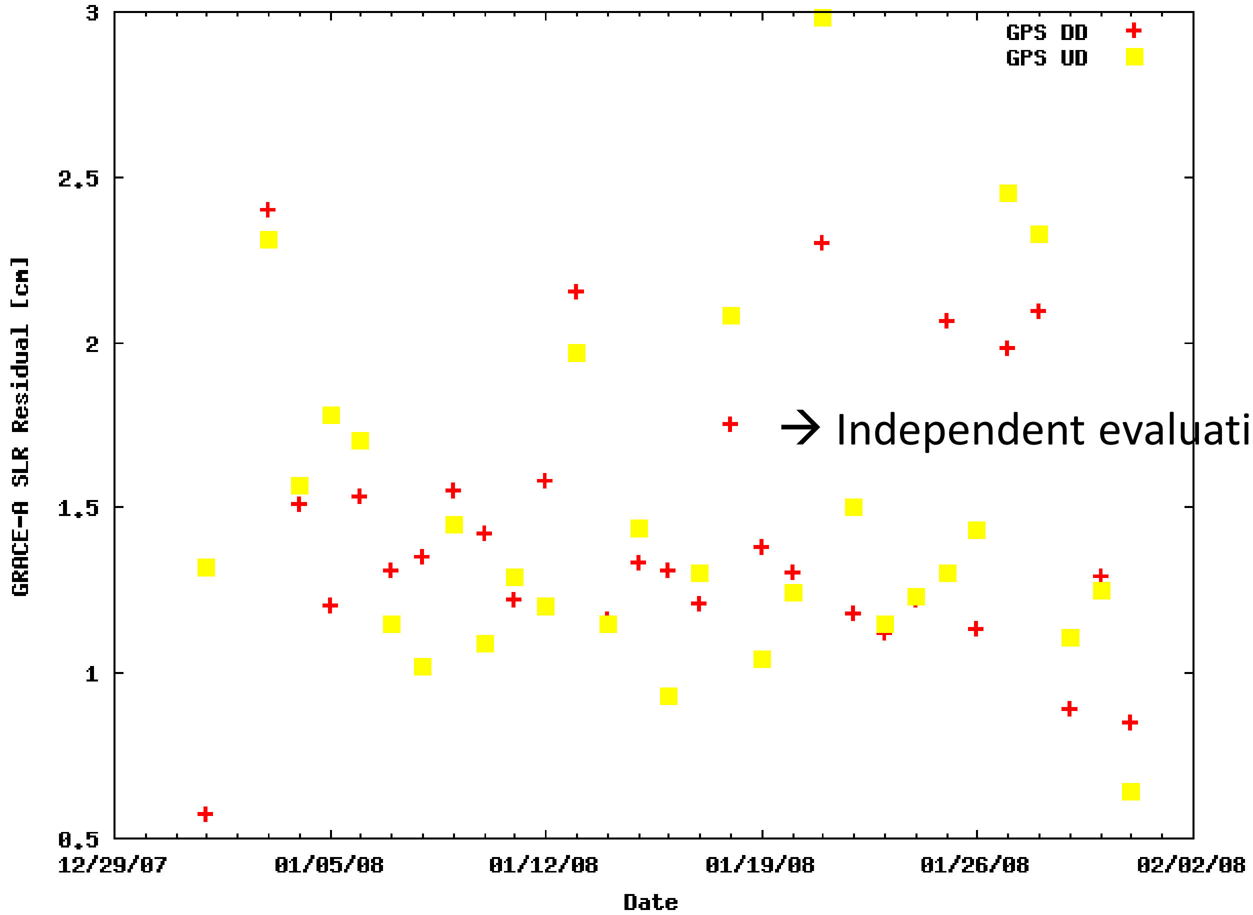
Daily SLR residuals to GRACE-A orbits for 2008 based on GNSS-orbits and clocks from different IGS analysis centers (status 2009)

→ Independent evaluation of GPS-orbits



From: Z. Kang, B. Tapley, S. Bettadpur, H. Save (2009), Quality of GRACE Orbits Using the Reprocessed IGS Products, presented at AGU Fall Meeting 2009

Daily SLR residuals to GRACE-A orbits for 2008 based on GPS DD and UD observations using IGS final orbits and clocks (status 2016)



→ Independent evaluation of GPS-orbits

GRACE-A & B SLR Residual RMS [mm]

Case name	GRACE-A	GRACE-B
GPS DD	14.4	12.5
GPS UD	14.6	13.1

Improvement in orbit quality w.r.t. the previous results from 2009

Courtesy of Z. Kang, S. Bettadpur, Center for Space Research, Austin/TX, 2016

Validation of gravity field models via orbit computation test using SLR tracking data

- Dynamic orbit computation and fit to SLR data by adjustment of orbital elements (and some empirical accelerations)
- Selected Example: Comparison of the following GOCE-containing Gravity Field models by using STARLETTE and LAGEOS orbits :

Model	Max d/o	Reference	Included data
GO_CONS_GCF_2_DIR_R3	240	Bruinsma et al. 2010	GOCE + GRACE + LAGEOS-1/2
GO_CONS_GCF_2_DIR_R4	260	Bruinsma et al. 2013	GOCE + GRACE + LAGEOS-1/2
GO_CONS_GCF_2_TIM_R3	250	Pail et al. 2011	GOCE-only
GO_CONS_GCF_2_TIM_R4	250	Pail et al. 2011	GOCE-only
GOCO03S	250	Mayer-Gürr et al. 2012	GOCE+GRACE+LAGEOS-1/2, Starlette, Stella, Ajisai

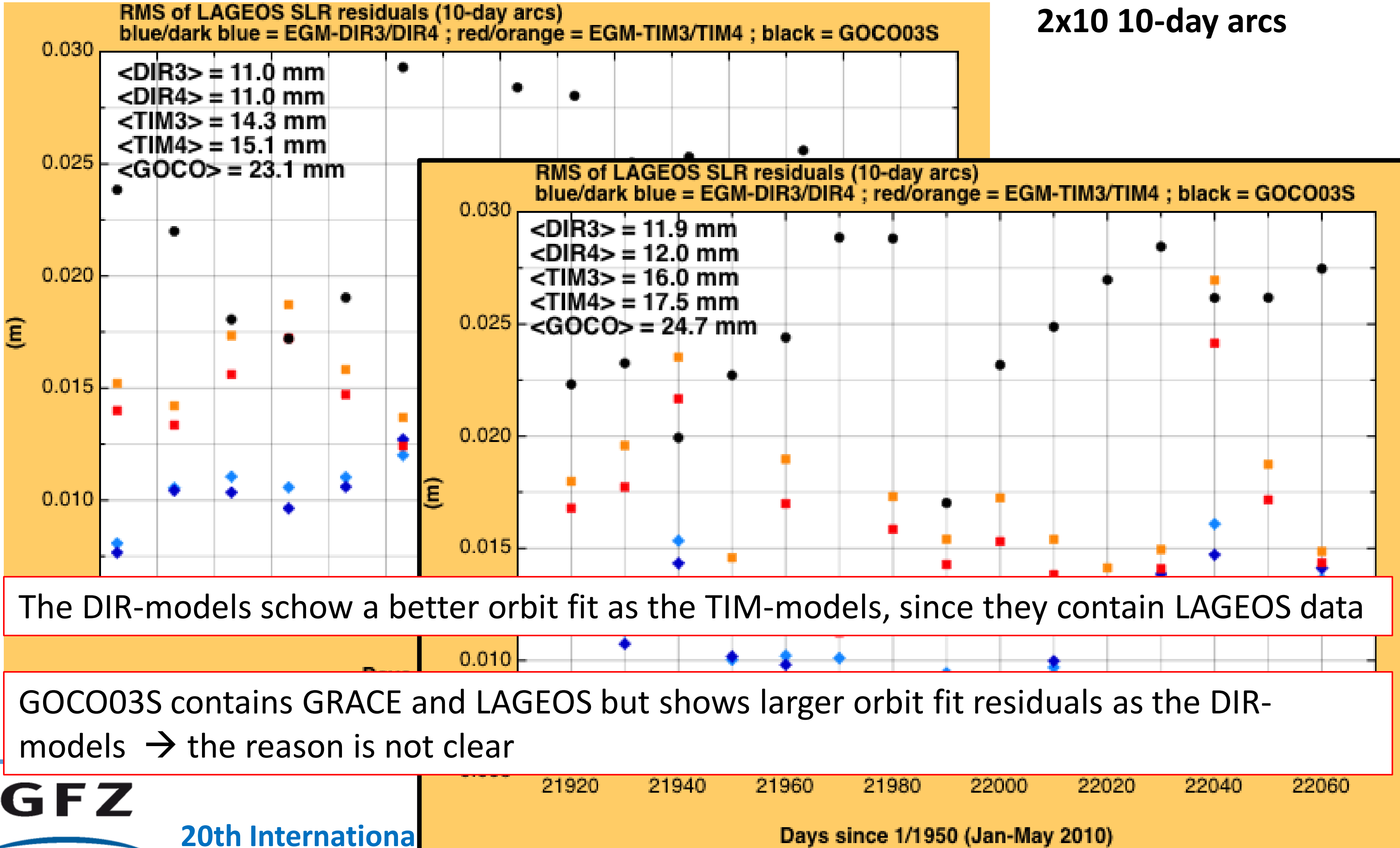
References:

- Bruinsma S.L., Marty J.C., Balmino G., Biancale R., Foerste C., Abrikosov O. and Neumayer H, 2010, GOCE Gravity Field Recovery by Means of the Direct Numerical Method, presented at the ESA Living Planet Symposium, 27th June - 2nd July 2010, Bergen, Norway; See also: earth.esa.int/GOCE
- Bruinsma, S., Foerste, C., Abrikosov, O., Marty, J.-C., Rio, M.-H., Mulet, S., Bonvalot, S. (2013): The new ESA satellite-only gravity field model via the direct approach, Geophysical Research Letters, 40, 14, p. 3607-3612. doi.org/10.1002/grl.50716
- Mayer-Gürr T., et al. (2012): The new combined satellite only model GOCO03s. Presentation at GGHS 2012, Venice, October 2012
- Pail R., Bruinsma S., Migliaccio F., Foerste C., Goiginger H., Schuh W.-D, Hoeck E, Reguzzoni M., Brockmann J.M, Abrikosov O., Veicherts M., Fecher T., Mayrhofer R., Krasbutter I., Sanso F. & Tscherning C.C. (2011) First GOCE gravity field models derived by three different approaches. Jounal of Geodesy, 81:11, doi: 10.1007/s00190-011-0467-x.

Validation of satellite-only gravity field models: LAGEOS orbit fit using GOCE-models (2000&2010)

from Bruinsma et al. 2013, presented at EGU2013

2x10 10-day arcs



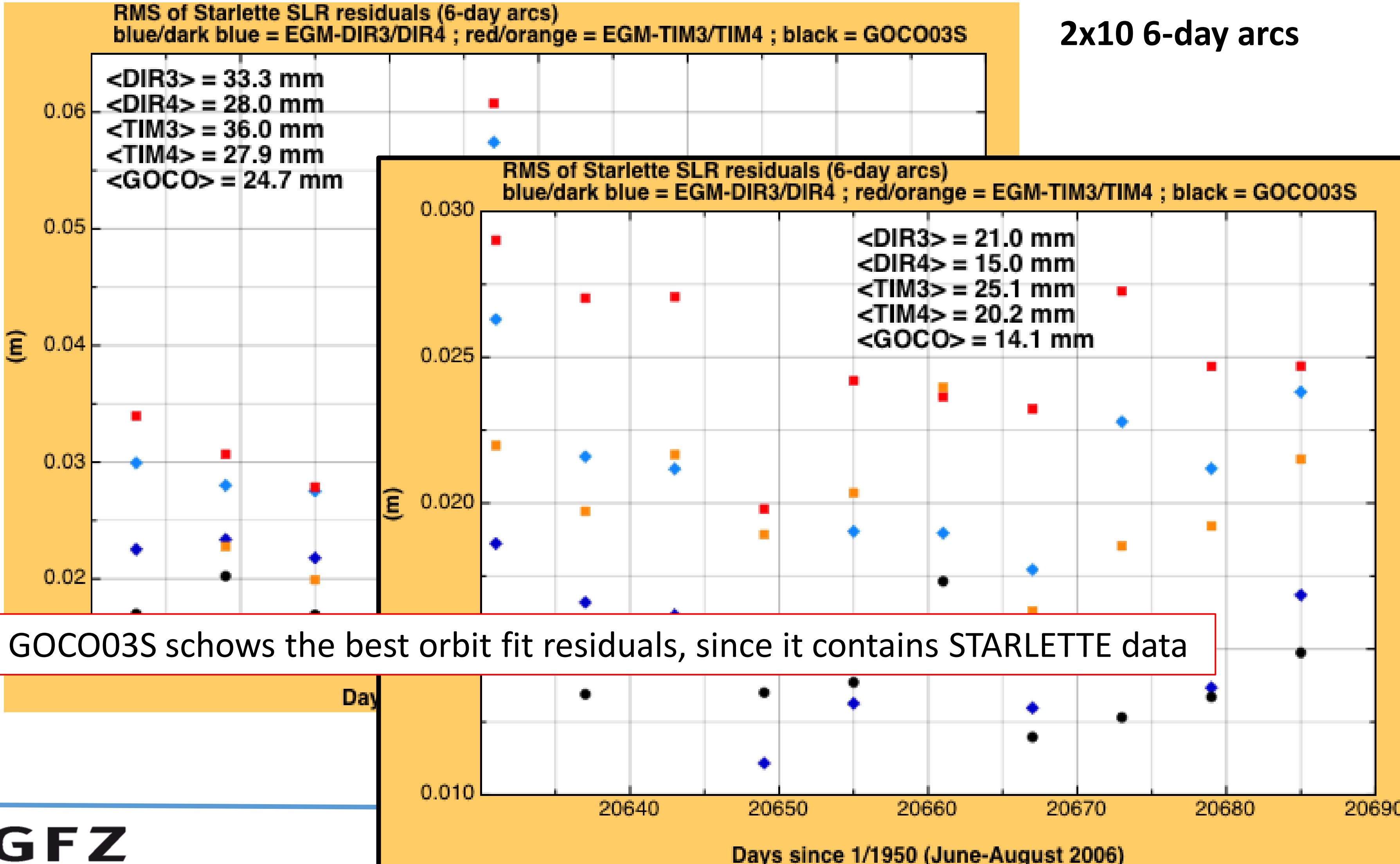
The DIR-models schow a better orbit fit as the TIM-models, since they contain LAGEOS data

GOCO03S contains GRACE and LAGEOS but shows larger orbit fit residuals as the DIR-models → the reason is not clear

Validation of satellite-only gravity field models: Starlette orbit fit using GOCE-models (2000&2006)

from Bruinsma et al. 2013, presented at EGU2013

2x10 6-day arcs



GOCO03S schows the best orbit fit residuals, since it contains STARLETTE data

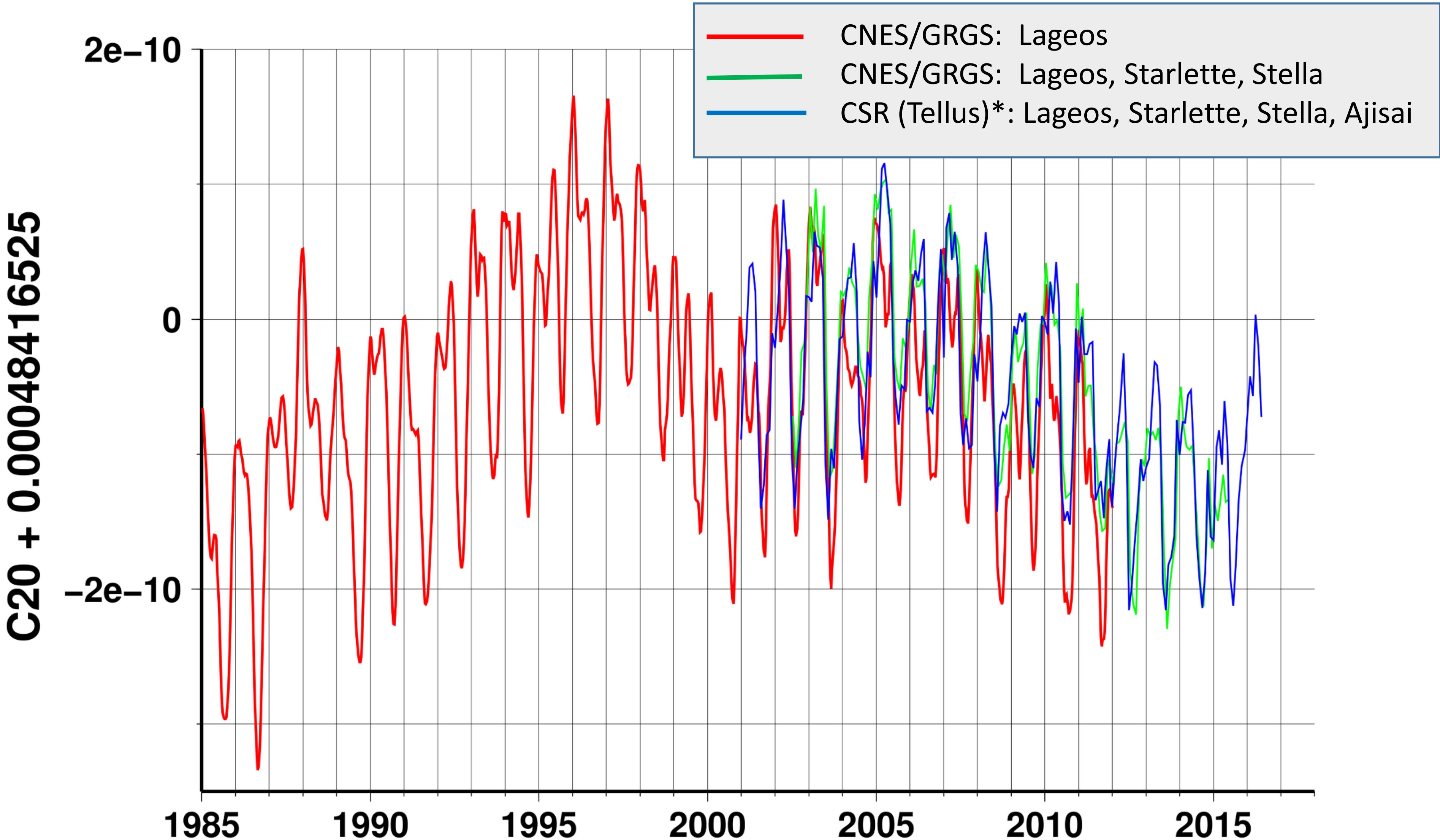
On the use of SLR data for gravity field determination

- SLR allows for estimation of low-degree/low order spherical harmonic coefficients (incl. temporal variations of a few of them):
  - C_{00} and GM (Geocentric Gravitational Constant)
  - C_{10}, C_{11}, S_{11} (coordinates of centre of mass)
  - C_{20} (~ flattening of the Earth)
  - C_{21}, S_{21} (mean pole position, i.e. principal axis of inertia)
  - Others, partially up to degree/order ~ 100

- For present-day gravity field determination, mainly SLR data from the famous „cannonball" satellites are used (LAGEOS-1/-2, STARLETTE, STELLA etc.)

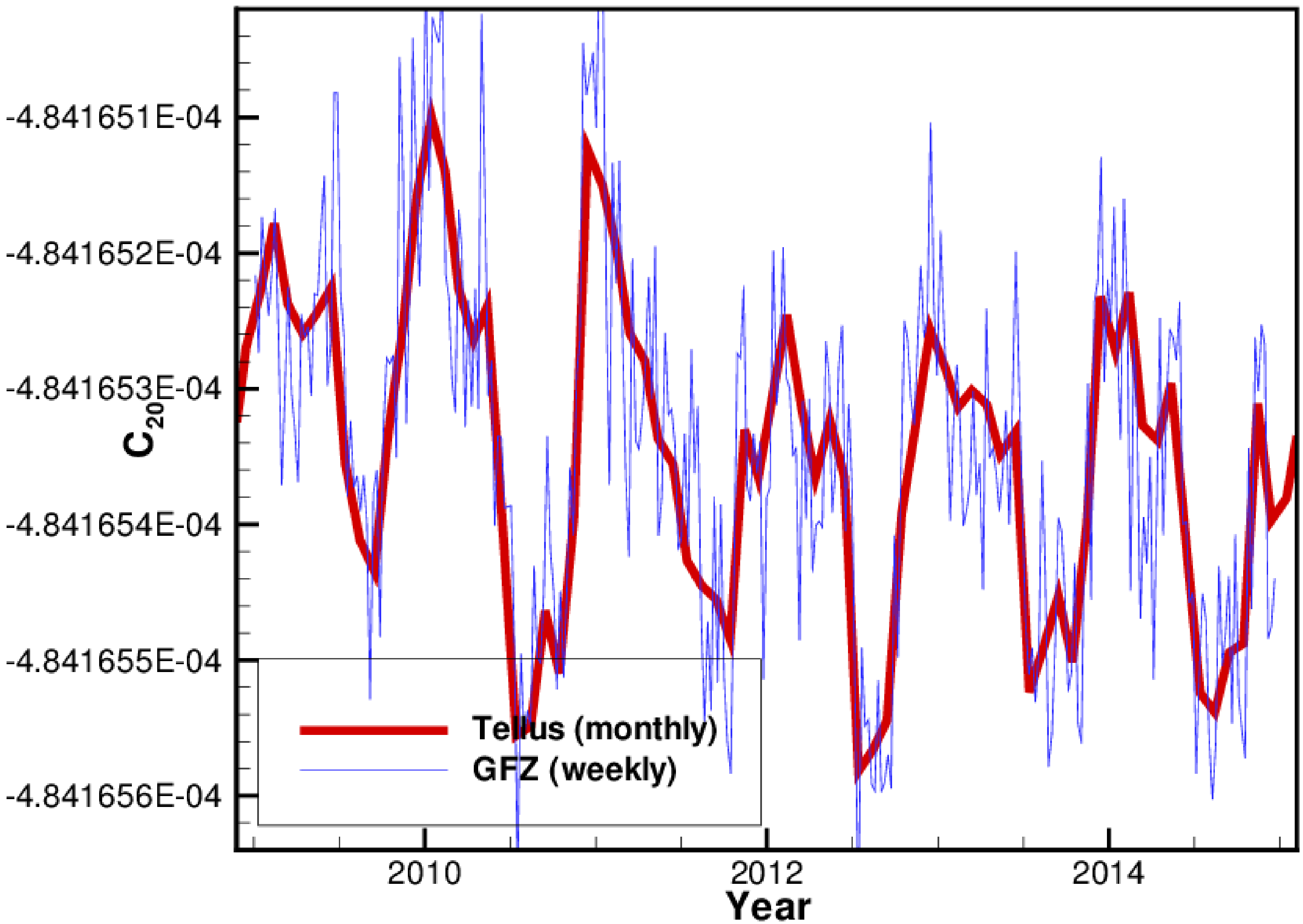
In the following I focus on recent examples from the determination of C_{20}

Monthly variation of C20 from SLR data

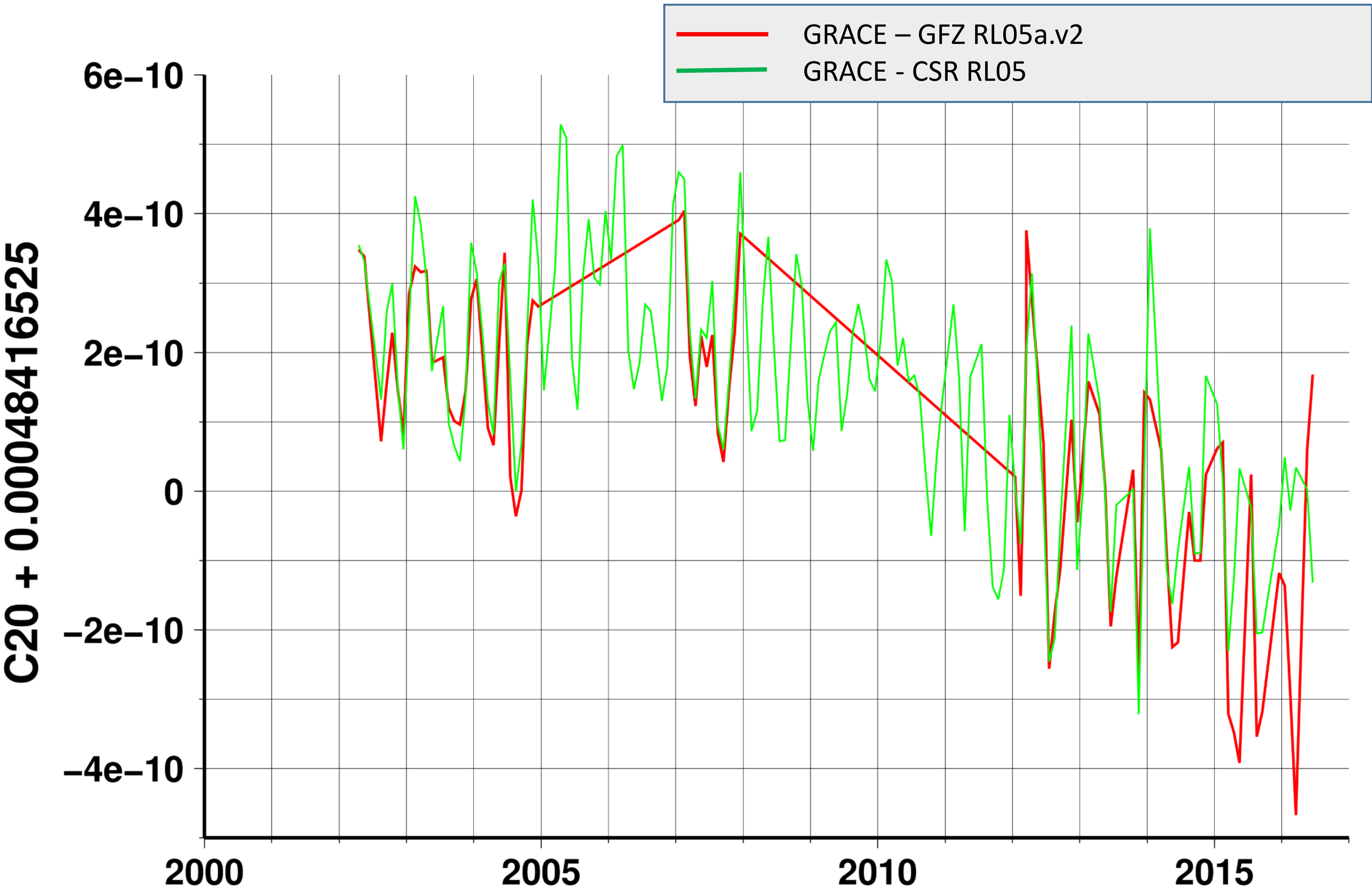


* Cheng, M., J. C. Ries, and B. D. Tapley (2011), Variations of the Earth's figure axis from satellite laser ranging and GRACE, J. Geophys. Res., 116, B01409, doi:10.1029/2010JB000850

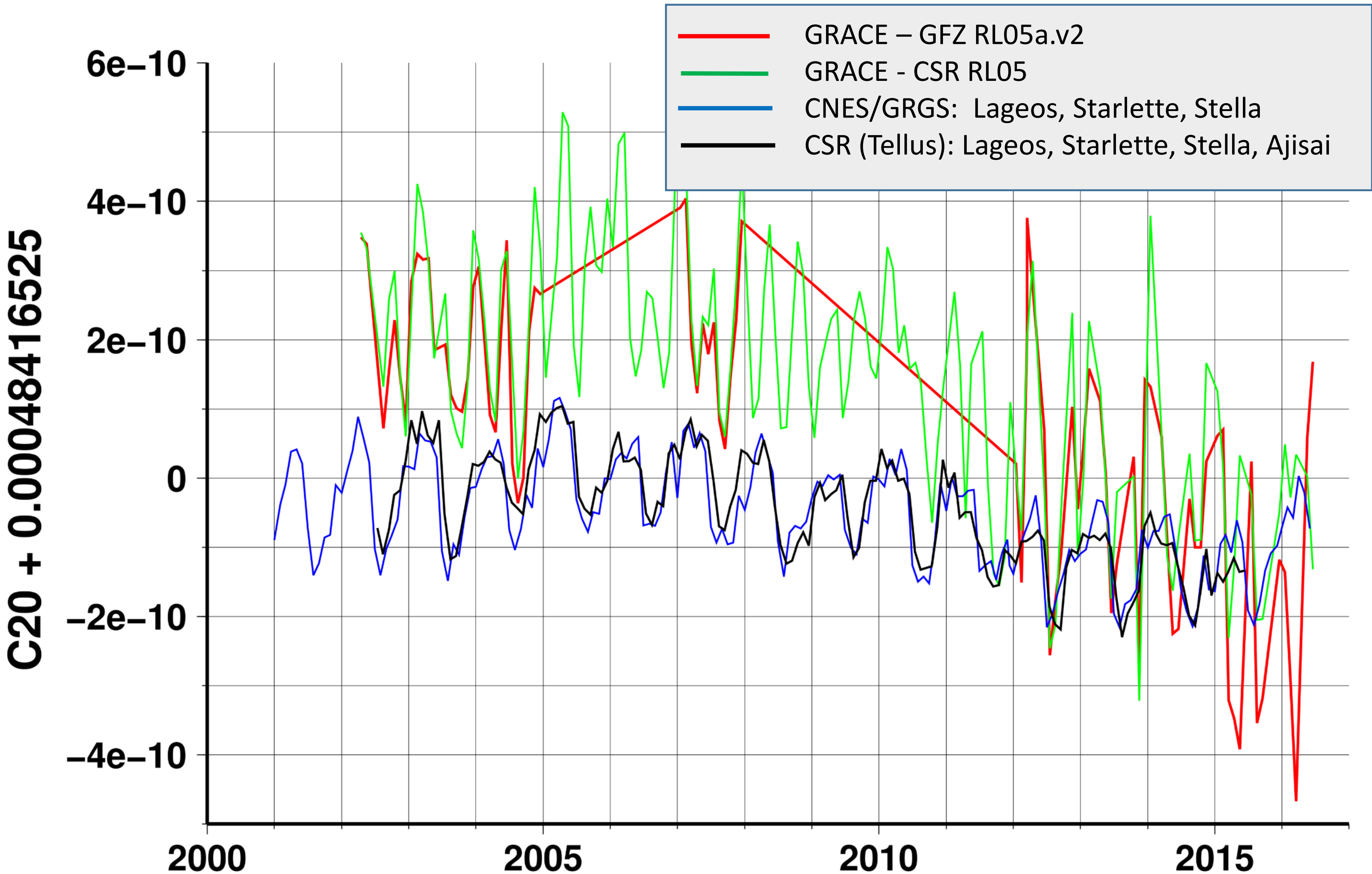
Weekly variation of C_{20} from SLR data



C_{20} estimation from GRACE

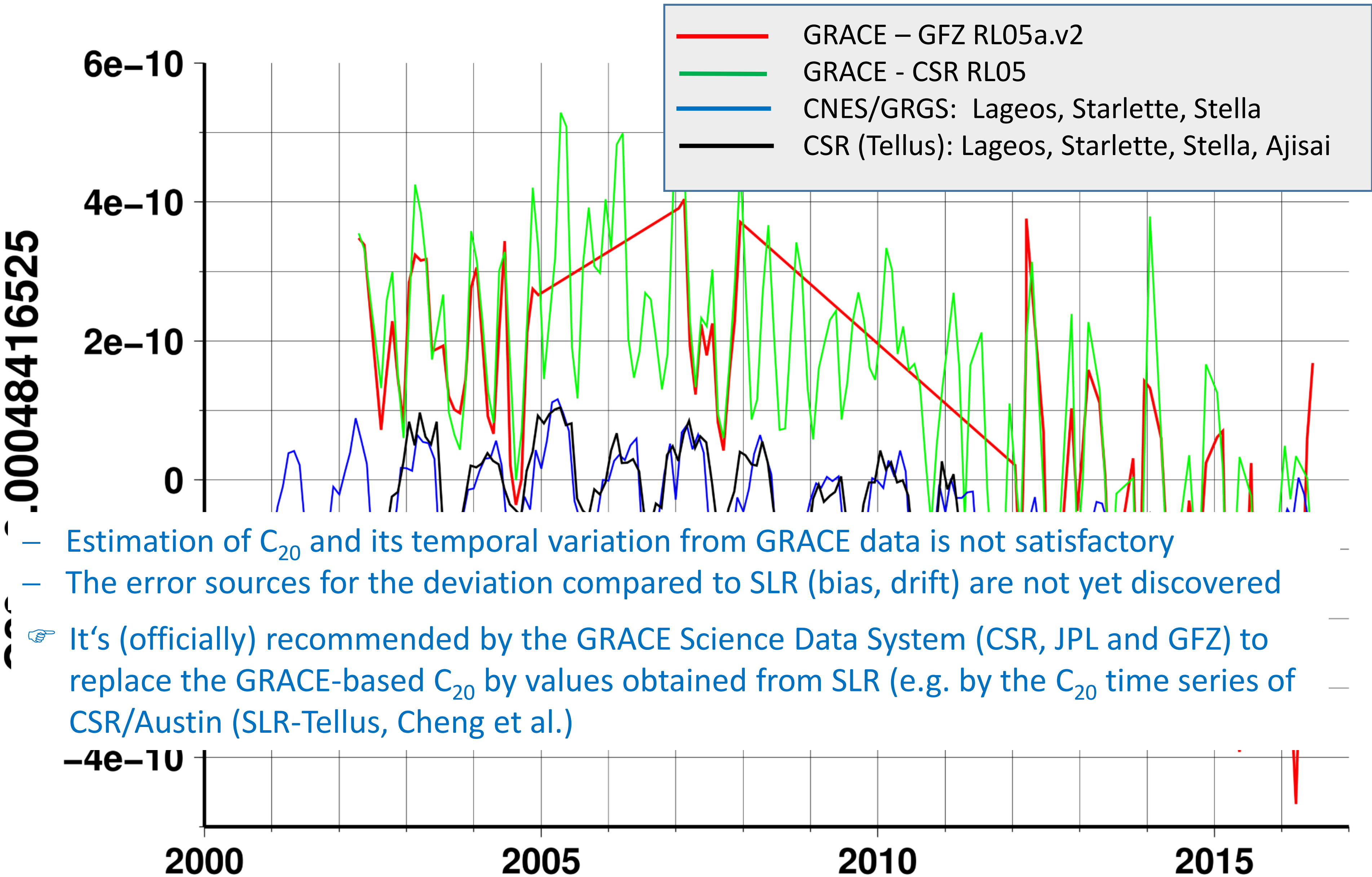


C_{20} estimation from GRACE



GROUPE DE RECHERCHE DE GEODESIE SPATIALE

C_{20} estimation from GRACE



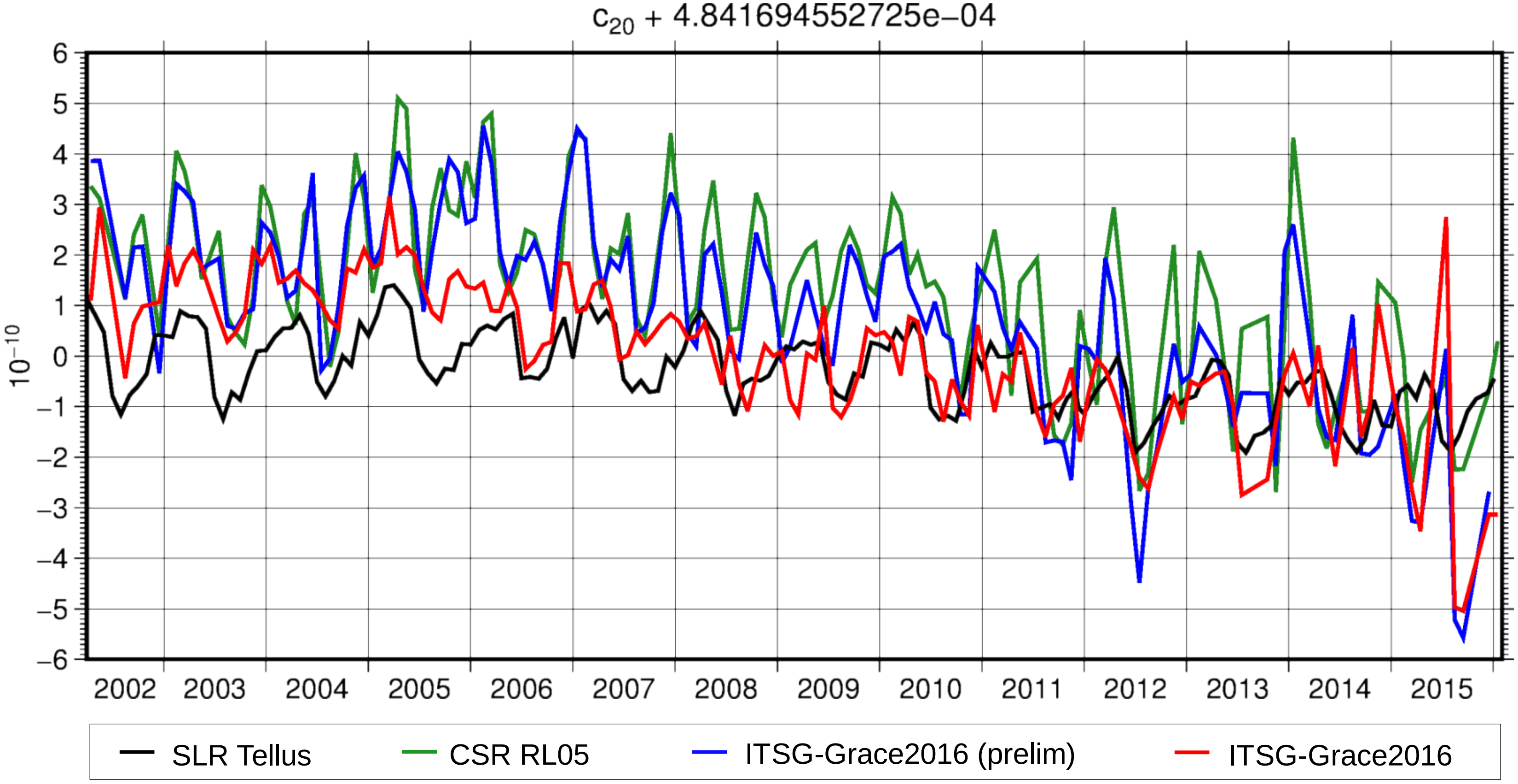
- Estimation of C_{20} and its temporal variation from GRACE data is not satisfactory
- The error sources for the deviation compared to SLR (bias, drift) are not yet discovered

☞ It's (officially) recommended by the GRACE Science Data System (CSR, JPL and GFZ) to replace the GRACE-based C_{20} by values obtained from SLR (e.g. by the C_{20} time series of CSR/Austin (SLR-Tellus, Cheng et al.)

GRACE accelerometer parametrization has an impact on the C_{20} time series:

Considering possible misalignments of the Accelerometers onboard GRACE:
→ Estimation of a full accelerometer scale factor matrix

Results: Offset w.r.t SLR is reduced (2008-2014)
Differences increase at the beginning and end of GRACE time-series



Taken from: B. Klinger & T. Mayer-Gürr (2016), The role of accelerometer data calibration within the ITSG-Grace2016 release: impact on C20 coefficients, presented at the GRACE Science Team Meeting 2016, Potsdam

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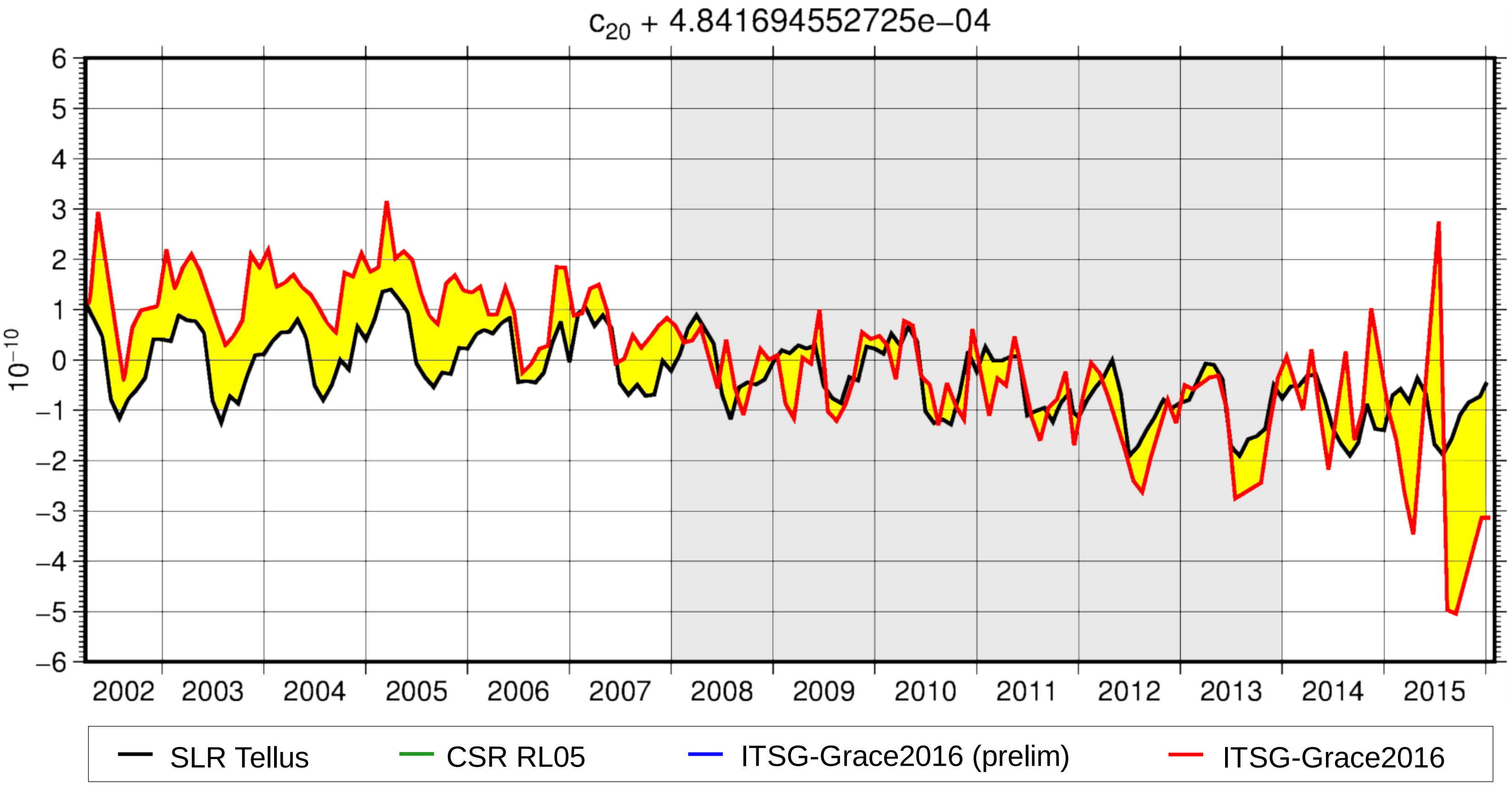

Taken from: B. Klinger & T. Mayer-Gürr (2016), The role of accelerometer data calibration within the ITSG-Grace2016 release: impact on C20 coefficients, presented at the GRACE Science Team Meeting 2016, Potsdam

TU Graz ifG

Conclusion

SLR tracking is directly and indirectly indispensible for global gravity field determination.

Please keep the SLR station network always running!