Lunar Dynamical Modeling with Improved IR Lunar Laser Ranging data

V. Viswanathan [1,2], A. Fienga [1], H. Manche [2], C. Courde [1], A. Belli [1,3], J. M. Torre [1], P. Exertier [1], J. Laskar [2]

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(1) Observatoire de la Côte d’Azur, CNRS-Géoazur, OCA
(2) Observatoire de Paris, CNRS-IMCCE, PSL
(3) UTINAM/Université de Franche Comté
Overview

1. New Dataset
   - IR (1064nm) Lunar Laser Ranging at OCA
     • advantages
     • impact on tests of general relativity

2. Improved dynamical model
   - INPOP Planetary and Lunar Ephemeris
     • description
     • latest residuals comparison with DE430 ephemeris
     • preliminary estimates

3. Conclusion + Future work
   - Multitechnique at MeO-OCA
     • SLR + LLR
     • Hydrology loading
**Improved IR LLR data**

- **LASER**: Infrared wavelength (1064nm)
- **Advantages**: 
  - Better atmospheric transmission
  - Observations round the clock (high SNR)
  - Diversification of observed reflectors

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**Diversity of Observations**

- **Green**:
  - A11: 6%
  - L1: 16%
  - L2: 1%
  - A14: 12%
  - A15: 64%

- **IR**:
  - L1: 15%
  - L2: 17%
  - A11: 17%
  - A14: 8%
  - A15: 48%
**Improved IR LLR data**

- **LASER**: Infrared wavelength (1064 nm)
- **Advantages**\(^7\):
  - Better atmospheric transmission
  - Observations round the clock (high SNR)
  - Diversification of observed reflectors
  - Observations during new and full moon

Maximum sensitivity for tests of EP: \(\cos(D)\)
**Improved IR LLR data**

- **LASER**: Infrared wavelength (1064nm)
- **Advantages[^7]**:
  - Better atmospheric transmission
  - Observations round the clock (high SNR)
  - Diversification of observed reflectors
  - Observations during new and full moon
  - Dense observations

**Graphs**

1. **1 year IR (2015-2016) @ Calern**: # 907
2. **1 year IR (2015-2016) @ Calern**: # 168
Lunar Dynamical model
INPOP
Intégrateur Numérique Planétaire de l'Observatoire de Paris

• Uses Numerical integration of the (Einstein-Imfeld-Hoffmann, c⁻⁴ PPN approximation) equations of motion
• Adams-Cowell integrator in extended precision
• 8 planets + Pluto + Moon + asteroids (point-mass, ring), GR, J₂sun, Earth rotation (Euler angles)

\[ \ddot{x}_{\text{Planet}} = \sum_{A \neq B} \frac{\mu_B}{\|r_{AB}\|^3} \frac{r_{AB}}{\|r_{AB}\|^3} + \ddot{x}_{GR}(\beta, \gamma, c^{-4}) + \ddot{x}_{\text{AST},300} + \ddot{x}_{J_2} \]

• Moon: orbit and librations
• Simultaneous numerical integration TT-TDB, TCG-TCB
• Fit to observations in ICRS over 1 cy (1914-2016)
• GAIA ESA planetary ephemerides
• Asteroid physics, Tests of gravity, solar physics
INPOP15b Lunar dynamical model description:

Simplified, differentiated 2 layer model

a. Solid mantle
b. Liquid core:
   - Axial symmetry (C22 Core = 0)
   - Non-differential rotation
   - Shape constrained by core-mantle boundary

Perturbations on lunar orbit:

a. Interaction - Moon’s figure and point masses:
   - Earth, Sun, Venus, Jupiter
b. Interaction - Earth’s figure and point masses:
   - Moon, Sun, Venus, Jupiter
c. Interaction - distorted part of Earth and Moon:
   (acceleration + 5 time delays)
   - Distortions:
     o Solid tides raised by Moon and Sun
     o Deformation due to spin
   - Force exerted on the Moon

Interactions at Lunar Core-Mantle Boundary (CMB):

✓ Dissipation: Viscous drag of core fluid flowing past boundary
   - Torque on the mantle due to coupling (no topography at CMB)
LLR Reduction Model
Calern : A Multi-Technique Station

- GINS Reduction Model
- Calern => SLR + LLR
- GINS allows SLR + LLR processing
- Calibration of SLR/LLR reduction procedure with LAGEOS
- Under study : Hydrology loading and horizontal gradients in the troposphere

What is GINS ?
(Géodésie par Intégrations Numériques Simultanées)
- Precise Orbit determination applied to space geodesy
- Developed and maintained by OCA-GRGS-CNES
- Time of flight (photon) to Residuals
- Planetary and lunar ephemeris (libration angles)
- Earth orientation (IERS C04 / JPL KEOF)
- Tides and loading
- Tropospheric delay
- Crustal deformation (Love & Shida numbers)
- Relativistic effects
- Under study : Hydrology loading
New solution : INPOP15b

- INPOP dynamical modeling
- fitted over LLR observations 1969-2016
- inclusion of IR LLR dataset
- GINS LLR reduction model
- Model differences between DE430 and INPOP

<table>
<thead>
<tr>
<th>Shape of Moon</th>
<th>DE430(^{[1][2]})</th>
<th>INPOP13c(^{[3]})</th>
<th>INPOP15b*</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20 (\beta = (C-A)/B) (\gamma = (B-A)/C) C22 (derived) C/MR(^2) (derived)</td>
<td>C20 C22 C/MR(^2)</td>
<td>C20 C22 C/MR(^2)</td>
<td></td>
</tr>
<tr>
<td>Shape of fluid core</td>
<td>CMB flattening (f)</td>
<td>-</td>
<td>C20 Core C/MR(^2) Core</td>
</tr>
<tr>
<td>Fluid Moment ratio</td>
<td>fixed</td>
<td>-</td>
<td>derived from: C/MR(^2), (f) and C20 Core</td>
</tr>
<tr>
<td>Symmetry of core</td>
<td>Axisymmetric</td>
<td>-</td>
<td>Axisymmetric</td>
</tr>
<tr>
<td>Additional longitude libration ((\Delta \tau))</td>
<td>A1 A2 A3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lunar Gravity field</td>
<td>GRAIL660b(^{[5]})</td>
<td>LP150(^{[4]})</td>
<td>GRAIL660b(^{[5]})</td>
</tr>
</tbody>
</table>

Post-fit residual comparison: INPOP15b vs DE430 (5 sigma filtered)

INPOP: 0.0006803 +/- 0.04481 m #: 11038 / 11158

DE430: 0.02751 +/- 0.04848 m #: 11018 / 11133

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Post-fit residual comparison: INPOP15b vs DE430 (5 sigma filtered)

INPOP: 0.00121 +/- 0.01992 m #: 2352 / 2352

DE430: 0.03324 +/- 0.02239 m #: 2278 / 2278
Post-fit residual comparison: INPOP15b vs DE430 (5 sigma filtered)

INPOP: 0.006975 +/- 0.1662 m #: 7098 / 7161

DE430: -0.06851 +/- 0.2027 m #: 6905 / 6956

3 McDonald stations
Reflector-wise $\sigma(m)$ after 5$\sigma$ filter

<table>
<thead>
<tr>
<th>Reflector</th>
<th>INPOP15b*</th>
<th>DE430</th>
</tr>
</thead>
<tbody>
<tr>
<td>A15</td>
<td>0.015</td>
<td>0.016</td>
</tr>
<tr>
<td>A14</td>
<td>0.017</td>
<td>0.018</td>
</tr>
<tr>
<td>A11</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>L1</td>
<td>0.017</td>
<td>0.020</td>
</tr>
<tr>
<td>L2</td>
<td>0.018</td>
<td>0.070</td>
</tr>
</tbody>
</table>

DE430 L2 signature outside the DE fitted data interval
Preliminary estimates with formal uncertainties from INPOP15b WLS fit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>INPOP15b</th>
<th>DE430[1][2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius Moon km</td>
<td>1.738E+03</td>
<td>1.738E+03</td>
</tr>
<tr>
<td>EMRAT</td>
<td>81.3005718</td>
<td>81.3005691±0.0000024</td>
</tr>
<tr>
<td>GM EMB</td>
<td>8.99701141E-10</td>
<td>8.99701139E-10</td>
</tr>
<tr>
<td>k2 Moon</td>
<td>2.545E-02 ± 2E-05</td>
<td>2.4059E-02</td>
</tr>
<tr>
<td>h2 Moon</td>
<td>4.315E-02 ± 9.9E-05</td>
<td>4.76E-02 ± 6.4E-03</td>
</tr>
<tr>
<td>l2 Moon</td>
<td>1.070E-02</td>
<td>1.070E-02</td>
</tr>
<tr>
<td>C/MR2 Moon</td>
<td>3.9313E-01 ± 1.331E-06</td>
<td>3.93142E-01</td>
</tr>
<tr>
<td>Gravity field coefficients</td>
<td>GRAIL 660b (BVLS 2 x sig)</td>
<td>GRAIL 660b</td>
</tr>
<tr>
<td>C(2,0) Core</td>
<td>-4.74E-08 ± 3.052E-10</td>
<td>-6.78E-08 (computed)</td>
</tr>
<tr>
<td>C/MR2 Core</td>
<td>2.75E-04</td>
<td>2.75E-04 (computed)</td>
</tr>
<tr>
<td>K CMB</td>
<td>6.20E-09 ± 1.167E-11</td>
<td>6.43E-09</td>
</tr>
<tr>
<td>Angular velocities</td>
<td>6.241E-03 ± 2.544E-06</td>
<td>-2.41999E–03</td>
</tr>
<tr>
<td></td>
<td>-5.136E-04 ± 1.232E-06</td>
<td>4.110195E-01</td>
</tr>
<tr>
<td></td>
<td>-1.89E-04 ± 5.090E-06</td>
<td>-4.630947E-01</td>
</tr>
<tr>
<td>Cf/C ratio</td>
<td>7.0E-04</td>
<td>7.0E-04</td>
</tr>
</tbody>
</table>

Current assumptions

- Axial symmetry of liquid core
- Non-differential rotation
- Shape constrained by CMB
- Only viscous drag at CMB
- No topography at CMB

References:


*bold: fixed parameters

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Conclusion and Future Work

- **Reduction model**
  - Hydrology loading
    - EOST/IPGS Loading service
  - Tropospheric delay
    - Horizontal gradients

- **Multi-technique**
  - Calern: SLR, LLR, GPS

- **Normal point computation algorithm**
  - Semi-train accumulation

![Graphs showing residuals vs time (year) for INPOP and DE430 models with NPTs: 22314 and 21982 respectively.](image)
Thank you for your attention

Questions?
viswanat@geoazur.unice.fr

OCA LLR Distribution:
http://www.geoazur.fr/astrogeo/?href=observations/donnees/lune/