Update of the IfE LLR analysis model and new fit of relativistic parameters
LLR analysis at IfE – program LUNAR

- Goes back to mid of 1980ies (FESG, Munich)
- 2 main development phases: ~1985-2001 (Munich), since 2006 at IfE

- LLR-only analysis
  - ephemeris computation based on initial values from DE421 (Sun, planets, largest asteroids)
  - no further planetary data (optical, radar,…) included
  - ephemeris model: EIH equations of motion for all bodies as point masses
  - Earth orientation: IERS conventions
  - lunar orientation: Euler equations integrated together with translation including relativistic corrections (geodetic precession, Lense-Thirring)
  - additional forces due to inhomogeneous gravity fields, tides

- Combined analysis of solar system data and LLR planned in future (project in research unit FOR 1503)
Program updates at IfE

- Data reduction compared with IMCCE, good agreement
- Results show some room for improvements → updates in ephemeris computation needed
  - many modeling parts still from first development phase
  - simplifications (due to computation time reasons and accuracy requirements)
  - slight inconsistencies in force model (‘interactions‘) resolved
  - 3 parts: Earth-related, additional gravitational effects, Moon-related
Program updates at IfE – Earth related

- Earth tides – tidal acceleration
  - former model
    - single lag angle of time delayed tides
    - only Moon as tide generating body
    - effect on lunar translation
  - new model (according to DE430 ephemeris)
    - degree 2 tidal potential (~98% of tidal effect)
    - arbitrary tide generating body possible
    - 5 tidal time delays (2 estimated, 3 fix at DE-values)
    - effect on Moon:
      - via change in Earth‘s degree 2 potential coefficients on lunar translation and rotation
      - via acceleration on lunar translation
Program updates at IfE – Earth related

- Earth tides – tidal acceleration
  - some **results**
    - Sun + Moon as tide generating body
    - tides from Jupiter, Venus $\rightarrow$ <0.1 mm in $r_{EM}$ in 45 years
    - estimated time delays
      - diurnal $\tau_{21}$=575 s (DE430: 636 s)
      - semi-diurnal $\tau_{22}$=226 s (DE430: 219 s)

- Secular trend in $C_{20}$
  - several models tested (linear, quadratic)
  - best result in LLR analysis with model from IERS Conventions 2010 with
    \[ \dot{C}_{20} = 2.6 \times 10^{-11} \text{ yr}^{-1} \]

Roy and Peltier (2011)
Program updates at IfE – additional gravitational effects

- Interaction of Sun/planets with Moon
  - former model – interaction of
    • planets with point-mass Moon
    • Sun with lunar degree 2
  - new model
    • planets with lunar degree 2
    • Sun with lunar degree 2+3
  - main effect on lunar rotation, e.g.
    • ~ 19 mm on surface from Venus
    • ~ 4 mm on surface from Jupiter
      (from ephemeris with equal initial conditions)
  - in analysis: residuals decrease ~0.1 mm on average
    → small effect but maybe needed in future
Program updates at IfE – additional gravitational effects

- Figure-figure interaction between Earth and Moon
  - former model
    - simplified version of degree 2 – degree 2 coupling
    - effect on lunar rotation considered
  - **new model**
    - coupling up to any degree/order of the gravitational field of Earth and Moon possible
      (Ilk, 1983)
    - effect on translation and rotation
  - **results**
    - improvement due to complete degree 2-degree 2 coupling
    - Earth degree 2 – Moon degree 3
      → some mm on surface
Program updates at IfE – Moon related

- **Rotation of deformable Moon**
  - former model
    - tidally deformed tensor of inertia only in rotation
    - no changes of potential coefficients (interaction with Earth, additional effect on translation and rotation)
    - no consistent core-implementation
  - **new model** (according to 2-layer model of DE430)
    - basis tide-free tensor of inertia, elements from
      - $C_{20}$ (GRAIL)
      - $C_{22}$, dynamical $\beta$ (estimated in analysis)
      - dynamical $\gamma$ derived
    - core moments with DE-fixed values for inertia-ratio $C_{\text{core}}/C_{\text{Moon}}$
      and core flattening
Program updates at IfE – Moon related

- tidal + spin deformations on mantle tensor of inertia
  - Love number $k_2$ fixed on GRAIL-value
  - tidal deformation due to Earth (much larger than from Sun)
  - 1 time delay used
  → equations for rotation complete
  → coupled differential equations for core + mantle
  → coupling constant, initial rotation vector of core estimated

- degree-2 changes in potential coefficients from deformed mantle tensor
  → enters in computation of external forces (translation/rotation)
Program updates at IfE – Moon related

- Effect of new rotation-modelling on post-fit residuals, wrms (without core)

- Effect including core (which leads to improved libration modelling) on 2-way residuals
Results - wrms

- 1-way wrms, comparison with former model

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- Further investigations
  - effects in longitude libration visible → empirical correction as in DE430
  - core flattening not yet estimated → possible effect on librations
  - tidal and lunar rotation modelling → room for improvement
## Results – coefficients lunisolar nutation

<table>
<thead>
<tr>
<th>Periode</th>
<th>MHB2000 [mas]</th>
<th>old model</th>
<th>new model (two test cases)</th>
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<tr>
<td>18.6 a</td>
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<td></td>
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<td>A</td>
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<tr>
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<td>-0.06 ± 0.03</td>
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<td>1.22</td>
<td>-0.01 ± 0.06</td>
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<tr>
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<tr>
<td>B''</td>
<td>0.14</td>
<td>-2.54</td>
<td>-0.21 ± 0.11</td>
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</table>

Less accurate, leads to larger discrepancies in other coefficients.
Results – relativistic parameters

- **Temporal variation of gravitational constant**
  
  - modelled as \( G(t) = G_0 \left(1 + \frac{\dot{G}}{G_0} \Delta t + \frac{1}{2} \frac{\ddot{G}}{G_0} \Delta t^2 \right) \)
  
  - moderate to strong correlations with
    - lunar core rotation vector (fixed in solution)
    - some station coordinates (constrained a little bit)
  
  - as single parameters:
    \[ \frac{\dot{G}}{G_0} = (0.7 \pm 0.8) \times 10^{-13} \text{ yr}^{-1} \]
    \[ \frac{\ddot{G}}{G_0} = (1.6 \pm 2.0) \times 10^{-15} \text{ yr}^{-2} \]
  
  - estimated together:
    \[ \frac{\dot{G}}{G_0} = (0.8 \pm 1.1) \times 10^{-13} \text{ yr}^{-1} \]
    \[ \frac{\ddot{G}}{G_0} = (-0.3 \pm 2.4) \times 10^{-15} \text{ yr}^{-2} \]
Results – relativistic parameters

- **Equivalence principle**
  - estimating ratio $\Delta \left( \frac{m_g}{m_i} \right)_{\text{EM}}$
    - partials: computed numerically by introducing additional acceleration of Moon towards Sun
      - $\Delta \left( \frac{m_g}{m_i} \right)_{\text{EM}} = (-3.0 \pm 6.6) \times 10^{-14}$
  - Estimating Nordtvedt parameter $\eta$
    - partials: analytical from synodic range variation: $13.1m \cdot \cos(D) \cdot \eta$
      - $\eta = (-0.2 \pm 1.2) \times 10^{-4}$
  - into direction of galactic center (e.g. due to dark matter)
    - amplitude: $A_{gc} = 3.0 \pm 3.3 \text{ mm}$
    - additional acceleration $a_{gc} = (-1.1 \pm 1.2) \times 10^{-6}$
      - in parts of $1.9 \times 10^{-8} \text{ cm/s}^2$
Results – relativistic parameters

- **PPN parameters** $\beta, \gamma$
  - included in EIH-equations of motion, partials numerically
  - correlated with station coordinates (constrained)
  - show also correlations with $z$-coordinate of lunar initial orbit values

\[
\beta - 1 = (0.9 \pm 1.0) \times 10^{-4}
\]

\[
\gamma - 1 = (-1.2 \pm 1.6) \times 10^{-4}
\]  (not as accurate as Cassini-result)

- $\beta$ from combination of PPN-parameters $0.25(\gamma+\eta+3)$ and Cassini-$\gamma$

\[
\beta - 1 = (0.03 \pm 6.1) \times 10^{-5}
\]

- **PPN preferred frame** $\alpha_1, \alpha_2$ w.r.t. cosmic microwave background

\[
\alpha_1 = (-1.1 \pm 2.0) \times 10^{-5}
\]

\[
\alpha_2 = (-0.6 \pm 0.9) \times 10^{-5}
\]  (not as accurate as test with Sun‘s spin)
Results – relativistic parameters

- **Geodetic precession** of lunar orbit
  - introducing GP a second time as additional acceleration
  - factor h gives relative deviation in from Einstein's theory (~1.9 as/cy)
  - strong correlation with
    - lunar core rotation vector (fixed)
    - z-component of lunar initial velocity (fixed)
  
  \[ h = (-0.6 \pm 2.0) \times 10^{-3} \]

- **Yukawa-term** (1/r²-test), acceleration due to

  \[ V_{EM} = -\frac{GM_E M_M}{r} \left(1 + \alpha e^{-r/\lambda}\right) \]

  - interacting range \( \lambda = 380000 \text{ km} \)
  - coupling constant \( \alpha \) estimated
  - correlations and fixed values like GP
  - \( \alpha = (-4.0 \pm 5.0) \times 10^{-12} \)
Summary

- IfE-LLR ephemeris model updated
  - tidal acceleration, secular trend in Earth‘s $C_{20}$
  - additional gravitational interactions planets-Moon, Earth-Moon
  - lunar rotation as 2-layer core/mantle model
  - effect of lunar deformation in all lunar equations of motion (translation plus rotation)

- Smaller residuals and more accurate parameter estimation
  - increased accuracy in relativistic parameters (strong limits for validity of equivalence principle and gravitational constant)
  - no deviation from Einstein‘s theory of gravity up to now