Relativistic corrections in the European Laser Timing (ELT) experiment
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Introduction
The European Laser Timing (ELT) experiment, which is part of the ESA mission ACES (Atomic Clock Ensemble in Space), aims at enabling picosecond time transfer between ground based clocks and the ultra-stable time scale of the ACES module aboard the International Space Station. To this end, both a classical two-way and an additional one-way optical link shall be established, both of which are based on timing via ultra-short laser pulses. For maximum timing precision, the space based ELT hardware will be equipped with a novel single photon avalanche diode (SPAD), which needs to be gated to reduce the signal-to-noise ratio to an acceptable level. To synchronize pulse transmission dates with the gating of the ELT detector, space and ground clocks shall be referred to a common time scale like UTC.

Hence, the ELT data center is required to compute the relativistic drift of the ACES clock with respect to UTC, as the payload has no access to a sufficiently accurate approximation of UTC, for example through GNSS.

Sagnac delay
The Sagnac delay, which is not a real relativistic effect but often added to them, refers to pulse delay corrections. With the Sagnac delay the movement of the receiver station during the signal time of flight is considered. It is only necessary for a one-way link as it cancels for two-way. The effect can be separated into a first (Fig.7) and a second order (Fig.8) delay, which are defined as follows:

First order delay
\[ D_{Sagnac} = \frac{D_{A1} + D_{A2} + D_{B1} + D_{B2}}{2} \]

Second order delay
\[ D_{Sagnac} = \frac{D_{A1} + D_{A2} + D_{B1} + D_{B2}}{2} \]

Shapiro delay
The Shapiro delay (Fig.9) refers to relativistic pulse delay corrections and describes the curvature of the signal way. It is necessary for a one- and two-way link. The effect is defined as follows:

\[ D_{Shapiro} = \frac{1}{c^2} \left( \frac{D_{A1} + D_{A2} + D_{B1} + D_{B2}}{2} \right) \]

Relativistic effects due to moving clocks and potentials

with
\[ v : \text{Satellite velocity in ECI} \]
\[ c : \text{Speed of light} \]
\[ V : \text{Gravitational potential of earth} \]
\[ \phi_0 : \text{Potential of the geoid} \]
\[ \Delta t : \text{Coordinate time (TT)} \]

Gravitational potential

\[ \Delta t = \frac{V}{c^2} \]

Tidal potentials

\[ \Delta t = \frac{V}{c^2} \]

Relativistic clock corrections

Fig.5: Clock correction for the space station; a) shows the calculated clock correction, b) shows the difference between the calculated and the linear fitted clock correction.

Fig.6: Clock correction for the ground station; a) shows the calculated clock correction, b) shows the difference between the calculated and the linear fitted clock correction.

Summary
Relativistic clock corrections need to be considered due to:

- clocks movements and potentials
- the pulse delay

The corrections due to clocks movements and potentials will be given to SLR stations as a linear coefficient and an offset coefficient. Relating to the simulation the corrections for the pulse delay are for the Shapiro delay around 6.5 ps, the Sagnac delay of first order 110 ns and second order 3 ps. To the proper pulse delay all relativistic delays as well as required non relativistic corrections for instance due to the troposphere and geometry will be added up. Hence, the SLR stations can calculate the offset of the ACES time scale and determine their own offset to UTC.

Literature