Can Planetary SLR Measure the Expansion of the Solar System?

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Abstract
It has been suggested that the mass (M) of the sun and the gravitational constant (G) may both be changing which would affect the scale of the solar system. The sun is continually converting hydrogen to helium and the resulting loss of mass is estimated to be or order \(10^{-13}/\text{yr}\). Further, it is suggested that G might also be decreasing or increasing, possibly as a result of the expansion of the universe, at \(-10^{-13}/\text{yr}\). These rates of change of the solar GM will be having a significant effect on the orbital motions of all the masses in the solar system at levels we can now measure. Although these changes are estimated to be decimeters/year, they represent the best tool to measure the changes in solar system. Further, laser ranging to spacecraft in Mars (Abshire et al. 2006) and lunar orbits (Zuber et al. 2010), and to a spacecraft en route to Mercury (Smith et al. 2006) have demonstrated the technology over planetary distances. A suggested rate of change of GM of \(10^{-12}/\text{yr}\) would change the positions and orbits of the planets by hundreds of thousands of kilometers over a billion years interfering with the resonant relationships of many planets and their satellites. The detection of a change in GM is most easily accomplished by measuring the change in the distances between planets (Smith & Zuber, 2016). Measurements between landers on widely separated planets might appear the easiest but for gaseous planets, such as Jupiter, it would need to be to a spacecraft in orbit. The measurements, technically feasible, would be difficult but the knowledge of whether the size of the solar system is changing would be a significant accomplishment with possibly important implications for the solar system’s past and future.

Introduction
Although never observed or proven, it has been suggested the gravitational constant, G, could change over time. One of the earliest measurements was by van Flandern (1981) who estimated the change in G from lunar occultation data, lunar laser ranging and planetary radar as:

\[
\frac{G_{\text{dot}}}{G} = (-6.4 \pm 2.2) \times 10^{-11} \text{yr}^{-1}
\]

A recent lunar laser ranging data analysis, Hofmann et al., (2010) yielded:

\[
\frac{G_{\text{dot}}}{G} = (-0.7 \pm 3.8) \times 10^{-13} \text{yr}^{-1}
\]

and other recent measurements from lunar laser ranging suggest small and possibly positive values (Muller, J., and Biskupek, L. 2007; Turshev, S.G. and Williams, J.G, 2007). In addition, using the timing of binary pulsars over a period of 21 years Zhu et al., (2015) arrived at the value:

\[
\frac{G_{\text{dot}}}{G} = (-0.6 \pm 1.1) \times 10^{-12} \text{yr}^{-1}
\]

But in all cases the results are consistent with a G-dot of zero.

A change in G affects everything in the universe, but within the solar system we are also governed by the distribution of mass, particularly the mass of the Sun, which is changing as the
sun ages. The electromagnetic radiation emitted by the sun is the result of the conversion of hydrogen to helium and in the process energy is released and the mass of the sun is reduced. The estimated change in solar mass ($M$) from the conversion of H to He is:

$$\left(\frac{\dot{M}}{M}\right) = -0.07 \times 10^{-12} \text{ y}^{-1}$$

In addition, the sun loses mass through the solar wind, composed mainly of protons. The flux of particles in the solar wind at the Earth provides a measure of the mass loss by the sun, and is estimated to be:

$$\left(\frac{\dot{M}}{M}\right) = -0.02 \times 10^{-12} \text{ y}^{-1},$$

leading to a total change in mass of:

$$\left(\frac{\dot{M}}{M}\right) = \sim -0.1 \times 10^{-12} \text{ y}^{-1}$$

Over the remaining life of the sun this rate of mass loss is equal to $<0.1\%$.

**Effect on Planetary Motions**

Although we do not have a definite value for the change in the solar GM we can at least estimate the effect on the orbits of the planets, based upon a nominal value, and how observation of their positions would be affected.

If we assume the present understanding and knowledge of the positions of the planets and that there is a change in the $GM$ of the sun of $-1 \times 10^{-12}$ per year then Figure 1 indicates the approximate change on the positions of the terrestrial planets. We have chosen a negative change in GM since we are more confident in the decrease in the sun’s mass than we are about the change in G, although the former is estimated to be an order of magnitude smaller.

![Figure 1](image1.png)

**Figure 1. Changes in the orbits of the terrestrial planets during the year 2008 due to a fractional change of $10^{-12}$/yr in GM of the sun. The calculations are approximate.**
The effect of a change in GM, although primarily a scale factor in distance, also scales the velocity and since angular momentum is conserved the resulting motions are not necessarily intuitive. As an example, scaling of the Earth’s distance provides a radius change of Earth’s orbital radius of ~15cm/year and a scaling of it’s velocity indicates a positional change of ~1m/year. Over time these rates become observable in the positions of the planets.

Figure 1 shows that the approximate changes in the radial and along track positions of the terrestrial planets change after 1 year. In general, the along track position due to the change in GM increases with time and enables the perturbation to become of measureable magnitude after a few years. However, since observations are currently limited to measurements from Earth to the planets it is the changes with respect Earth that are important. Figure 2 shows the approximate change during 1 year between Earth and Mercury, Earth and Venus and Earth and Mars. Figure 2 also shows the approximate change between Venus and Mars.

![Figure 2](image)

**Figure 2. Changes in the distances between the terrestrial planets during the year 2008 due to a fractional change in the GM of the sun of \(10^{-12}/yr\). The calculations are approximate.**

The largest changes are seen in the Earth to Mercury distance due largely to the more rapid orbital motion of Mercury and hence larger perturbation. The pattern in the observed range is approximately the synodic period of Earth and Mercury and after a year the amplitude has grown to over 50 cm.

**How can we measure the expansion?**

The most accurate approach is the high precision monitoring of the distances between the centers of mass of the several planets over an extended period of time thus benefiting from the accumulative effect. So far, all planetary-scale distance measurements have been Earth-centric, i.e. they are one way from Earth (with the exception of lunar laser ranging, a few LOLA laser altimeter measurements from LRO at the Moon, and the laser two-way measurement to the MESSENGER spacecraft by laser altimeter (MLA) instrument and the ground station at Greenbelt, MD). The effect of a change in the Sun’s GM on measurements to the Moon,
although easier and more accurate than measurements to the planets, is much smaller on the Earth-Moon distance, and hence more difficult to extract.

Figure 2 indicates that over a relatively short period of a few years a change in the solar GM of $10^{-13}$/year. If measurements are made between 3 planets, such as Earth, Mars and Venus, any signal would be present in each line with its unique periodic signature, but also in the overall expansion of the “triangle” formed by the planets.

The measurements are probably best made between spacecraft orbiting the planets rather than from landers (planet’s rotation, power constraints of landers, etc) but a simulation of a specific planetary configuration would indicate whether an orbiter or a lander is better for locating and monitoring the position of the center of the planet. Landers, of course, are not options for the gas giants, although their natural satellites might be.

SLR and LLR measurements are routinely made at the centimeter level but it is not clear that such accuracy is necessary between the planets. A 10 cm capability might be adequate, considering the magnitude of the effect over large distances, and the effect accumulative of time.

Conclusions
The change in scale of the solar system by a change in the solar GM is small but measurable with today’s technology. Experiments on MGS, MESSENGER, and LRO have provided evidence that we know how to make accurate measurements over planetary distances. The detection of a change in the solar mass and/or the value of G (different from zero) could have significant impact on our understanding of the evolution of the solar system and, probably, on the interior working of our Sun. If there is the possibility of measuring a change in the solar $GM$, shouldn’t the SLR community be the ones to do it?

References


Turyshev, S.G. and Williams, J.G., Space-Based Tests of Gravity with Laser Ranging, Int. J.
