Earth Orientation and Relativity Parameters Determined from LLR Data

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Lunar Laser Ranging – in general

- 45 years of observations
- Post-Newtonian model at cm level
- high long-term stability (orbit, reference frames, Earth orientation)
- relativity tests
LLR parameter fit

Analysis
- model based upon
  - Einstein's theory (orbit, rotation, signal propagation, time scales, reference systems, etc.)
  - Consistent application of IERS Conventions
- least-squares adjustment
- parameter estimation (about 200 unknowns, without EOPs) applying various constraints

Results (this talk)
- Pole coordinates $x_p$, $y_p$ and Earth rotation phase $\Delta UT$
- Nutation coefficients for selected periods
- Relativity test: Example $\dot{G}/G$, …
Components of Earth Orientation

Polar motion

Earth rotation phase $\Delta UT$

Precession

nutation

$1 \text{ ms} = 50 \text{ cm at equator}$

DFG Research Unit FOR 584
Pole coordinates \((x_p, y_p)\) from LLR – all sites

Characteristics of the fit

- 15 NPs per night for the time span 4/1984 – 12/2013 (247 n.)
- Simultaneous determination of one component of the pole coordinates and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered
Pole coordinates \((x_p, y_p)\) from LLR – all sites

- **Accuracy**: 1-14 mas
- **Correlation** with coordinates of the observatories up to 10%
Pole coordinates \((x_p, y_p)\) – only APOLLO

Characteristics of the fit

- 5 NPs per night for the time span 6/2006 – 9/2013 (182 n.)
- Simultaneous determination of pole coordinates and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered
Pole coordinates \((x_p, y_p)\) – only APOLLO

Accuracy
0.5 – 40 mas, less after 12/2010

Correlation
with coordinates of the observatories up to 20-40%, with each other 20-40%
Characteristics of the fit

- 5 NP per night for the time span 6/2006 – 9/2013
- Simultaneous determination of $\Delta$UT and all coordinates of the observatories
- Velocities of the observatories fixed to the ITRF values
- a-priori EOP values from IERS C04 series, fixed for the nights that were not considered
**ΔUT from LLR – only APOLLO**

**Accuracy** 0.003 – 0.05 ms
less after 12/2010 due to reduced NP accuracy

**Correlation**
with coordinates of the observatories up to 50%,
with other ΔUT values 20 – 40%  
→ best LLR results can be used to validate VLBI
Nutation determined from only LLR data

**Initials:** precession and nutation according to IAU Resolution 2006 and IERS Conventions 2010

Use of **different realizations** of precession/nutation for ICRS-ITRS transformation

**Fit** of luni-solar nutation coefficients from 44 years of LLR data for nutation periods of 18.6 years, 9.3 years, 1 year, 182.6 days, (13.6 days), i.e. estimation of various $A'$, $A''$, $B'$, $B''$ of

\[
\Delta \psi = \sum_{i=1}^{N} \left( A_i + A_i' t \right) \sin (ARG) + \left( A_i'' + A_i''' t \right) \cos (ARG)
\]
\[
\Delta \varepsilon = \sum_{i=1}^{5} \left( B_i + B_i' t \right) \cos (ARG) + \left( B_i'' + B_i''' t \right) \sin (ARG)
\]
\[
ARG = \sum_{i} N_i F_j \quad \text{N}_j : \text{multiplier, F}_j : \text{Delaunay parameters}
\]
## Results - example

<table>
<thead>
<tr>
<th>Period</th>
<th>MHB2000 [mas]</th>
<th>LLR 1 [mas]</th>
<th>LLR 2 [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18,6 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-17206,42</td>
<td>2.70 ± 0.20</td>
<td>5.21 ± 0.25</td>
</tr>
<tr>
<td>B</td>
<td>9205,23</td>
<td>-0.48 ± 0.10</td>
<td>-1.32 ± 0.11</td>
</tr>
<tr>
<td>A&quot;</td>
<td>3,34</td>
<td>-4.62 ± 0.12</td>
<td>-3.46 ± 0.21</td>
</tr>
<tr>
<td>B&quot;</td>
<td>1,54</td>
<td>-2.29 ± 0.09</td>
<td>-2.19 ± 0.10</td>
</tr>
<tr>
<td>182,6 days</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-1317,09</td>
<td>-2.38 ± 0.08</td>
<td>-1.69 ± 0.11</td>
</tr>
<tr>
<td>B</td>
<td>573,03</td>
<td>0.25 ± 0.05</td>
<td>0.15 ± 0.05</td>
</tr>
<tr>
<td>A&quot;</td>
<td>-1,37</td>
<td>1.80 ± 0.07</td>
<td>1.85 ± 0.09</td>
</tr>
<tr>
<td>B&quot;</td>
<td>-0,46</td>
<td>0.23 ± 0.05</td>
<td>0.22 ± 0.05</td>
</tr>
<tr>
<td>9,3 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>207,46</td>
<td>0.45 ± 0.11</td>
<td>0.85 ± 0.18</td>
</tr>
<tr>
<td>B</td>
<td>-89,75</td>
<td>-0.15 ± 0.07</td>
<td>-0.13 ± 0.08</td>
</tr>
<tr>
<td>A&quot;</td>
<td>-0,07</td>
<td>-1.50 ± 0.12</td>
<td>-0.97 ± 0.20</td>
</tr>
<tr>
<td>B&quot;</td>
<td>-0,03</td>
<td>-0.87 ± 0.08</td>
<td>-1.35 ± 0.09</td>
</tr>
<tr>
<td>365,3 days</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>147,59</td>
<td>-2.91 ± 0.10</td>
<td>-0.51 ± 0.16</td>
</tr>
<tr>
<td>B</td>
<td>7,39</td>
<td>0.55 ± 0.06</td>
<td>0.01 ± 0.07</td>
</tr>
<tr>
<td>A&quot;</td>
<td>1,12</td>
<td>-2.30 ± 0.09</td>
<td>-0.06 ± 0.11</td>
</tr>
<tr>
<td>B&quot;</td>
<td>-0,19</td>
<td>-0.29 ± 0.05</td>
<td>-0.02 ± 0.05</td>
</tr>
</tbody>
</table>

LLR 1: precession according to Fukushima (2003) and Williams (1994)
LLR 2: precession according to P03, Capitaine et al. (2003)
Discussion of Nutation results

Differences in estimated corrections to the MHB2000 nutation model also depend on the implementation of precession/nutation (correlations change …)

Realistic accuracy of nutation coefficients from LLR about 0.1 - 0.3 mas in obliquity and 0.2 - 0.5 mas in longitude

Largest differences in longitude components

Large correlation of 18.6 and 9.3 year periods

Major problems in LLR are the unevenly distributed data (gaps in time series, orbit coverage, only few sites, weather, less accuracy in early years, …)

Future plan: Joined analysis of LLR and VLBI
Variation of the gravitational constant $G$

Use ansatz  \[ G = G_0 + \dot{G} \Delta t + \frac{1}{2} \ddot{G} \Delta t^2 \]

in equations of motion  \[ \ddot{r}_{EM} \approx -\frac{GM_{E+M}}{r_{EM}^2} \] ...

\[ \frac{\dot{G}}{G} = (1.2 \pm 1.5) \times 10^{-13} \text{ yr}^{-1} \]

\[ \frac{\ddot{G}}{G} = (1.4 \pm 3.0) \times 10^{-15} \text{ yr}^{-2} \]

correlation of almost 100% with $k_2 \delta$
Effect of $\dot{G}/G$ perturbation on $r_{EM}$

Here, the effect from the orbit has been reduced by estimating the initial values.
Power spectrum of $\dot{G}/G$ perturbation
Conclusions

- Nutation coefficients from LLR partly well determined (e.g. annual), results affected by the uneven data distribution → combination with VLBI

- Pole coordinates and ΔUT can be obtained with high accuracy for those nights where good data are available

- LLR is a unique tool for studying the Earth-Moon system and testing general relativity, e.g. gravitational constant

\[ \frac{\dot{G}}{G} = (1.2 \pm 1.5) \times 10^{-13} \text{ yr}^{-1} \]

\[ \frac{\ddot{G}}{G} = (1.4 \pm 3.0) \times 10^{-15} \text{ yr}^{-2} \]

- Good results are only possible because of fantastic long-term lunar tracking by observatories (> 45 years of data). Thanks!
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