Abstract
Passive laser-ranged satellites, launched for geodynamics and geophysics purposes, not only have contributed to significant measurements in space geodesy [1,2], but also provided an outstanding test bench to fundamental physics. Significant examples are represented by the measurement of the Lense-Thirring precession on the combined nodes of the two LAGEOS satellites [3], and by the measurement of the total relativistic precession of the argument of pericenter of LAGEOS II [4,5]. Indeed, the physical characteristics of such satellites and of their orbit, and the availability of high-quality tracking data provided by the International Laser Ranging Service (ILRS) [6], allow for precise tests of gravitational theories. The aim of LARASE (LASer RAnge Satellites Experiment) is to go a step further in the tests of the gravitational interaction in the field of Earth (i.e. in the weak-field and-slow motion (WFSM) limit of general relativity (GR)) by the joint analysis of the orbits of the two LAGEOS satellites and that of LARES [7]. To reach such a goal a key ingredient is to provide high-quality updated models for the perturbing non-gravitational forces acting on such satellites. A large amount of Satellite Laser Ranging (SLR) data of LAGEOS and LAGEOS II has been analyzed using a set of dedicated models for their dynamics, and the related post-fit residuals have been analyzed. A parallel work is ongoing for LARES that, due to its much lower altitude, is subject to larger gravitational and non-gravitational effects; the latter are in part mitigated by its much lower area-to-mass ratio. Recent work on the data analysis of the orbit of such satellites will be presented together with the development of some new refined model to account for the impact of the subtle non-gravitational perturbations.

Introduction
Einstein's theory of GR, after 100 years from its formulation in 1915 [8] and after a huge number of verifications and confirmations during last decades, both from the experimental and observational point of view [9], still represents the best theory we have for the description of the gravitational interaction both at the low and high energy scales and it also represents the pillar of modern cosmology to understand the universe that we observe under a wide number of different techniques. Anyway, despite all these successes, other gravitational theories different from GR have been proposed. Some of these theories, as in the case of metric theories, share with GR the same spacetime structure and the same equations of motion for test particles, but differ in the field equations form. Conversely, other non-metric theories provide more fundamental differences, such as violations of Einstein Equivalence Principle (EEP) [10]. However, the overall validity of GR is not only questioned within alternative theories for the description of the gravitational interaction, but also from quantum theories of physics. Indeed, GR is a classical theory of physics and all the attempts at merging gravitation with the other interactions of nature, in order to encompass all physics in a New Standard Model, have failed. Therefore, it is clear the importance of precisely testing the predictions of GR, as well as those of competing theories, at
all the accessible scales of distances and energies and with a wider as possible use of different techniques. It is precisely in the WFSM of GR that LARASE aims to provide an original contribution in testing and verifying gravitational physics by means of the powerful Satellite Laser Ranging (SLR) technique [6] together with a precise orbit determination (POD) of a dedicated set of passive laser-ranged satellites [11,12]. To obtain a refined POD two fundamental aspects need to be carefully reached: i) high quality for the tracking observations of the satellite, and ii) reliable dynamical models to be included in a software for the orbit determination. Consequently, a major goal of LARASE is to improve the dynamical models of the current best laser-ranged satellites, with a special attention to the subtle, and quite complex to model, non-gravitational perturbations. In the following sections some of the preliminary results that we have recently obtained on the improvements of the models for some of the perturbations acting on the LAGEOS satellites and on LARES will be described and briefly discussed.

**Preliminary results**

The test masses of the LARASE experiment are constituted by the best laser-ranged satellites orbiting the Earth: spherical in shape, fully passive, and with a generally low area/mass ratio in order to minimize the non-gravitational accelerations. Within this family, the two LAGEOS satellites and the more recently launched LARES satellite are the most important to consider, because of the high accuracy of their orbit determination thanks to the very precise SLR technique. LAGEOS (LAser GEOdynamic Satellite) was launched by NASA on May 4, 1976, LAGEOS II was launched by NASA and ASI on October 22, 1992, finally LARES (LAser RElativity Satellite) was launched by ASI on February 13, 2012.

**Spin models** With regard to the non-gravitational perturbations acting on the cited satellites, and in the context of the LARASE activities, we have initially focused our attention on the spin dynamics. Indeed, the rotational dynamics of a satellite represents a very important issue that deeply impacts the goodness of the orbit modelling. Several non-gravitational perturbations depend on the knowledge of the satellite spin vector orientation and rate, among these, the thermal forces are the most important to consider. The rotational dynamics of the two LAGEOS has been deeply investigated in the past by many authors [13-19]. We have deeply reviewed the interactions responsible of the spin evolution of the two LAGEOS and we have removed many of the simplifications at the basis of previous models. The knowledge of the moments of inertia of a satellite plays a significant role in the case of the gravitational torque. Very important is also a reliable model of the magnetic torque, which plays a central role. In particular, we developed a model valid in the so-called rapid-spin approximation — based on the solution of averaged equations for the torques — and a general model based on the full set of Euler equations for the torques. The mathematical formulation adopted accounts for the characteristic periods of the several variables that enter in the spin evolution, namely (i) the rotational period of the satellite, (ii) its period of revolution around the Earth, (iii) the sidereal period of Earth, and (iv) the thermal inertia of the satellite Cube Corner Retro-reflectors (CCRs). For instance, the formulation based on averaged equations is valid when the rotational period of the satellite is small compared to the other characteristic times involved. This work has been also extended to LARES. In Figure 1 the comparison between our spin model (in the so-called rapid-spin approximation) with the available observations is shown for the orientation of LAGEOS II.
Neutral drag The impact of neutral drag on LARES orbit is much stronger with respect to its effect on the orbit of the two LAGEOS satellites. This is due to the much lower orbit of LARES with respect to that of the two LAGEOS (1450 km height vs. 5900 km). Among the different activities that we started on this topic, we mainly focused on i) the comparison of the predictions of the different atmospheric models at the altitudes of the satellites, ii) the estimate of the perturbing accelerations acting on the satellites and iii) the estimate of the disturbing effects on their orbit. In particular, we took advantage of the use of the software SATellite Reentry Analysis Program (SATRAP). Indeed, SATRAP is able to load several different models for the Earth's atmosphere together with the appropriate geomagnetic and solar activities indices while using the following dynamical models for the orbit propagation of the satellite: (i) Earth’s geopotential, (ii) luni-solar perturbations, (iii) solar radiation pressure with eclipses and, of course, (iv) neutral drag. We refer to [23,24] for details. In Figure 2 the comparison among three different models for the atmosphere in the case of the transversal acceleration of LARES is shown.

Figure 1. LAGEOS II spin orientation [degrees] in the J2000 reference frame: right ascension (top) and declination (bottom). The results for the spin evolution, as we obtained from our model (blue line), are compared with the available observations in the literature, see [20-22].

Figure 2. Transversal acceleration [m/s²] of the LARES satellite over a period close to two years since March 10, 2012. The comparison among three different models for the Earth’s atmosphere is shown: the Jacchia–Bowman 2008 model [25], the Empirical Russian GOST-2004 model [26] and the NRLMSISE-2000 model [27]. The eclipses periods undergone by the satellite are shown in gray.
In Table 1 we show the results for the three components of the acceleration of the neutral drag perturbation for LAGEOS, LAGEOS II and LARES. As we can see, despite the smaller value for the area-to-mass ratio, in the case of LARES the accelerations are much larger than those obtained for LAGEOS II. This is of course due to the higher values for the atmosphere density at the height of LARES with respect to the density “felt” by the two LAGEOS, at their much higher altitude. Indeed, the average densities on the analyzed time spans and computed with the NRLMSISE-2000 model are about $6.6\times10^{-18}$ kg/m$^3$, $7.1\times10^{-18}$ kg/m$^3$ and $5.6\times10^{-16}$ kg/m$^3$, respectively for LAGEOS, LAGEOS II and LARES.

Table 1. Average accelerations [m/s$^2$] in the Gauss reference system for the two LAGEOS satellites and LARES. In the case of LAGEOS and LAGEOS II, the average has been computed over a time span of about 4017 days, starting from January 1, 1993. In the case of LARES the average has been computed over a time span of about 764 days, starting from March 10, 2012.

<table>
<thead>
<tr>
<th>Acceleration [m/s$^2$]</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial</td>
<td>$+9.5\times10^{-18}$</td>
<td>$+7.5\times10^{-18}$</td>
<td>$-1.3\times10^{-15}$</td>
</tr>
<tr>
<td>Transversal</td>
<td>$-3.1\times10^{-13}$</td>
<td>$-2.6\times10^{-13}$</td>
<td>$-1.3\times10^{-11}$</td>
</tr>
<tr>
<td>Out-of-plane</td>
<td>$-1.7\times10^{-16}$</td>
<td>$+7.1\times10^{-18}$</td>
<td>$-1.8\times10^{-14}$</td>
</tr>
</tbody>
</table>

Tides Tidal effects must be carefully studied and modelled in space geodesy and in fundamental physics measurements because they influence the orbit of a satellite in three different ways: 1) through kinematic effects, because they produce periodic pulsations of the Earth and, consequently, of the tracking stations; 2) through dynamic effects, which cause a time variation of the geopotential affecting the orbit; 3) through the reference system, because they perturb the Earth rotation, thus perturbing the reference system used in the POD. We have reviewed the effects provoked by both solid and ocean tides on the orbit of the considered satellites. Their effects are particularly important, in the case of relativistic measurements, on the longitude of the ascending node and on the argument of pericenter. The solid tides account for about 90% of the total response to the Moon and Sun tidal disturbing potential and are responsible of the larger effects on the orbit of a satellite. Conversely, the ocean tides are difficult to be modeled because of the greater complexity of the involved phenomena, and are characterized by larger uncertainties with respect to those of solid tides. In Table 2 are shown the results (amplitude and period) we obtained for a few solid zonal tides in the case of the ascending node of the three satellites.

Table 2. Impact of the Earth’s solid zonal tides (with degree $\ell = 2$ and order $m = 0$) on the right ascension of the ascending node of the two LAGEOS satellites and LARES. The periods are in days while the amplitudes are in milliarc seconds [mas]. The positive sign (+) of the period refers to westward tidal waves, while the negative sign (−) refers to eastward ones.

<table>
<thead>
<tr>
<th>Tide</th>
<th>Period [days]</th>
<th>LAGEOS [mas]</th>
<th>LAGEOS II [mas]</th>
<th>LARES [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>055.565</td>
<td>+6798.38</td>
<td>-1080.60</td>
<td>+1977.28</td>
<td>+5352.85</td>
</tr>
<tr>
<td>055.575</td>
<td>+3399.19</td>
<td>-5.23</td>
<td>-9.57</td>
<td>-25.90</td>
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<tr>
<td>056.554</td>
<td>+365.25</td>
<td>+9.97</td>
<td>-18.24</td>
<td>-49.37</td>
</tr>
<tr>
<td>057.555</td>
<td>+182.63</td>
<td>+31.22</td>
<td>-57.12</td>
<td>-154.63</td>
</tr>
</tbody>
</table>
As we can see, in the case of LARES, due to its lower height with respect to that of the two LAGEOS, the perturbation provoked by the tides has a larger impact on the orbit.

**Precise orbit determination (POD)** A POD for a satellite represents an essential prerequisite in order to perform reliable measurements in gravitational physics as well as in space geodesy applications. The orbit analysis strategy we adopted is based on a multi-arc POD, where the overall time span of the analysis is divided into a number of successive short arcs, not causally connected, and then, for each of them, the satellite state vector is estimated along with a set of relevant model parameters. In particular, our preliminary analyses included a preparatory data reduction for the satellites orbit with a tailored setup for the models implemented in the software. Together with the satellites state vector and selected station biases, the radiation coefficient and the corrections to polar motion and length of day have been estimated. Empirical acceleration have been also used at this level. The results of these preliminary analyses are shown in Figure 3, where the EIGEN-GRACE02S model has been used for the background gravitational field.

![Figure 3. Root Means Square (RMS) of the range residuals of LAGEOS (blue), LAGEOS II (red) and LARES (green), the units are in meters. The time is given in Modified Julian Date (MJD). In the case of the two LAGEOS, the starting epoch (MJD 48919) corresponds to October 24, 1992, while, in the case of LARES, the starting epoch (MJD 55975) corresponds to February 18, 2012. The arc length is 14 days for LAGEOS and LAGEOS II, and 7 days for LARES. Notice the higher uncertainty associated with the LARES analysis, showing its currently non-optimal modelling.](image)

As we can see, LAGEOS and LAGEOS II orbits are recovered with a mean error roughly between 1.5 and 1 cm, while LARES orbit has a slightly higher error. This is due to a currently non-optimal modelling for the dynamics of LARES. The decreasing trend that we obtained for the RMS of the range residuals in the case of the two LAGEOS, which approaches the 5 mm (mean) value at the end of the time span, is also in quite good agreement with the results obtained from the data reduction of the orbit of the two LAGEOS satellites performed by the main Analysis Centers of the ILRS network.

**Conclusions**
The goals and preliminary activities of the LARASE program have been briefly introduced and described. Although our final objective is to perform precise relativistic measurements in the weak field of the Earth through the laser measurements of the two LAGEOS satellites and of
LARES, in the meantime a careful re-analysis of the existing dynamical models for such satellites, as well as the development of new and more reliable models, have been started. These activities are important because they provide the basis to perform a POD of the orbit of the laser-ranged satellites and, at the same time, they allow for the estimate of an accurate error budget for the relativistic parameters to which we are interested in. We refer to [28] for further details on the LARASE program.

References: