The Error Analysis of SHAO Terrestrial Reference Frame and EOPs

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18th International Workshop on Laser Ranging
11-15 November 2013, Fujiyoshida, Japan
Outline

1、Introduction
2、Methods
3、Data processing and results
4、Error analysis
5、Conclusion and future plan
1. **Introduction**

- **Terrestrial Reference Frame (TRF)**
- **Earth Orientation Parameters (EOP)**
- **Celestial Reference Frame (CRF)**

- They are fundamental parameters connecting the high precision TRF and CRF.
- Including precession, nutation and ERP. Precession and nutation show the motion of the earth rotation axis in space and can be obtained by models such as IAU2006 precession model and IAU2002 nutation model.
- $Px, Py, UT1-UTC$ and LOD: difficult to model in theory and need real observation to obtain them.

The most authoritative TRF and EOP are provided based on VLBI/SLR/GNSS/DORIS by IERS such as ITRF2008 and IERS C04. Other institutions from USA/EUROPE/Japan also do some study in some degree.
1、Introduction

High accuracy EOP and TRF are necessarily needed for space navigation of satellites and almost all space orbiters. They need EOP to complete the TRF transformation for the orbit determination.

EOPs are also the basic and main observation and scientific data for several scientific fields such as Astro-Geodynamics, geophysics, geodesy and so on. They are basic information for studying the rotation of the Earth, facial motions of the Earth, core-mantel couple and earthquake. The improved EOP monitoring will be helpful to these fields.
Because EOPs and TRF are base of many scientific fields such as sea level rise, crustal deformation and movement mechanisms of the Earth different spheres, high precise geodesy and so on the establishment and maintain of high accuracy TRF and determination of EOP have been hot spots of international scientific research. Some research needs mm TRF and EOPs.

Future ITRF of GGOS requires the origin accuracy at 1mm and its stability at 1mm/yr. There is still some distance for these requirements

Therefore we have necessary to analyze the errors and improve the accuracy of our TRF and EOP.
2. Methods

ITRF2000 → ITRF2005 → ITRF2008 (ITRF 2013)

IERS

- EOP Bulletin A/B, C01/C04
  (Polar Motion, LOD, UT1-UTC, Nutation, precession)

- TRF+EOP simultaneous estimation
- Combine at the normal equation level or observational level

720 sites
## 2. Methods

### Input data

<table>
<thead>
<tr>
<th>Technique</th>
<th>International Service/Combination Center</th>
<th>Analysis Center</th>
<th>Data Span</th>
<th>Resolution</th>
<th>Type</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLR</td>
<td>ILRS/ASI DGFI</td>
<td>ASI,DGFI,GFZ,JCET, NSGF,etc.</td>
<td>1983.0-1992 1993.0-2011.5</td>
<td>14DAYS 1 WEEK</td>
<td>STA+EOP</td>
<td>Loose constraints</td>
</tr>
<tr>
<td>GPS</td>
<td>IGS/NRCan</td>
<td>CODE,ESOC,GFZ,JP L,NOAA,NRCan,etc.</td>
<td>1997.0-2012</td>
<td>1 WEEK</td>
<td>STA+EOP</td>
<td>Minimum constraints</td>
</tr>
<tr>
<td>VLBI</td>
<td>IVS/GIUB</td>
<td>BKG,DGFI,GSFC,SH AO,etc.</td>
<td>1980-2013</td>
<td>Session-wise</td>
<td>Normal equation</td>
<td>Free constraints</td>
</tr>
<tr>
<td>DORIS</td>
<td>IDS/IGN</td>
<td>IGN, LCA, ESA, GAU, GOP, etc.</td>
<td>1983-2012</td>
<td>1 WEEK</td>
<td>STA+EOP</td>
<td>Minimum constraints</td>
</tr>
</tbody>
</table>
2、Methods

Data processing flow

- Normal Equations Sys1
  \( (x_{VLBI}^{e}(J2005.0), \dot{x}_{VLBI}^{e}, \dot{y}_{VLBI}^{e}, \Delta U, \text{LOD}) \)

- Normal Equations Sys2
  \( (x_{GPS}^{e}(J2005.0), \dot{x}_{GPS}^{e}, \dot{y}_{GPS}^{e}, \text{LOD}) \)

- Normal Equations Sys3
  \( (x_{SLR}^{e}(J2005.0), \dot{x}_{SLR}^{e}, \dot{y}_{SLR}^{e}) \)

- Normal Equations Sys4
  \( (x_{DORIS}^{e}(J2005.0), \dot{x}_{DORIS}^{e}, \dot{y}_{DORIS}^{e}) \)

Combined Normal Equation Sys

\( (x_{\text{in}}^{e}(J2005.0), \dot{x}_{\text{in}}^{e}, \dot{y}_{\text{in}}^{e}(t), \Delta U(t), \text{LOD}(t)) \)

SHA0: Combined TRF solution + EOP time series

Local-ties Information
Core-sites' Orientation constraints

Re-edit Unknown parameters
Combination methods

\[
\begin{bmatrix}
X_s \\
Y_s \\
Z_s
\end{bmatrix} = 
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} + 
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix} + 
\begin{pmatrix}
D & -R_3 & R_2 \\
R_3 & D & -R_1 \\
-R_2 & R_1 & D
\end{pmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

\[X(t) = X_c(2005.0) + \dot{X}_c(t - 2005.0)\]

\[\nu_s = (A_{1s} \quad A_{2s}) \begin{bmatrix}
\hat{X}_c^s \\
\hat{X}_c^{et,s}
\end{bmatrix} - l_s \rightarrow \begin{pmatrix}
\sum_{i=1}^{k} A_{11}^T P_i A_{11} & A_{11}^T P_1 A_{21} & A_{12}^T P_2 A_{22} & \cdots & A_{1k}^T P_k A_{2k} \\
A_{21}^T P_1 A_{11} & A_{21}^T P_1 A_{21} & 0 & \cdots & 0 \\
A_{22}^T P_2 A_{12} & 0 & A_{22}^T P_2 A_{22} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_{2k}^T P_k A_{1k} & 0 & 0 & \cdots & A_{2k}^T P_k A_{2k}
\end{pmatrix} \cdot \hat{\chi} = \begin{pmatrix}
\sum_{i=1}^{k} A_{11}^T P_i l_i \\
A_{21}^T P_1 l_1 \\
A_{22}^T P_2 l_2 \\
\vdots \\
A_{2k}^T P_k l_k
\end{pmatrix}\]
2、Methods

Datum definition and local-ties model

<table>
<thead>
<tr>
<th>Datum definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Origin</strong></td>
</tr>
<tr>
<td><strong>Scale</strong></td>
</tr>
<tr>
<td><strong>Orientation</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Local-ties model: baseline vector

\[
\begin{align*}
\Delta x_s &= x_s^i - x_s^j \\
\Delta y_s &= y_s^i - y_s^j \\
\Delta z_s &= z_s^i - z_s^j \\
\end{align*}
\]

\[
D_{\Delta,s} = K \cdot D_{ij,s} \cdot K^T, K = \\
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 \\
\end{bmatrix}
\]
2、Methods

Choice of weighting

\[
D_{ll} = \begin{pmatrix}
\sigma_1^2 Q_1 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & \sigma_2^2 Q_2 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & \sigma_k^2 Q_k
\end{pmatrix}
\]

\[
\hat{\sigma}_{i,D}^2 = \hat{s}_{i,F} = \frac{v_i^T P_i v_i}{n_i - \text{tr}(N^{-1} A_i^T P_i A_i)}
\]

Result:

<table>
<thead>
<tr>
<th>Technique</th>
<th>SLR</th>
<th>GPS</th>
<th>VLBI</th>
<th>DORIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance Factor</td>
<td>5.5</td>
<td>6.4</td>
<td>1.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>
2、Methods

Processing of discontinuity
## Application of parallel algorithm

Unknown parameters: >60000

<table>
<thead>
<tr>
<th>Test (data span)</th>
<th>Steps</th>
<th>Serial Program time consuming (min)</th>
<th>OpenMP parallel program (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005.0-2006.0</td>
<td>SLR normal equation stacking</td>
<td>0.08</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>GPS normal equation stacking</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>VLBI normal equation stacking</td>
<td>0.01</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>DORIS normal equation stacking</td>
<td>0.17</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>Solve the combined normal equation system</td>
<td>2</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>Iteration 4 times</td>
<td>60</td>
<td>24</td>
</tr>
<tr>
<td>all years</td>
<td>with no iteration</td>
<td>227</td>
<td>141</td>
</tr>
</tbody>
</table>
Initial results of SHAO TRF and EOP

Based on all POS+EOP SINEX solutions from GPS, VLBI, SLR and DORIS we solved the site coordinates, velocity at epoch J2005.0 and daily EOP with constraints from the datum definition. And then compare our results with those from ITRF2008 and IERS C04.
3. Data Processing and results
3、Data Processing and results

VLBI

SLR

GPS

DORIS
3. Data Processing and results

Initial results of SHAO TRF and EOP

<table>
<thead>
<tr>
<th>EOPs</th>
<th>SHAO</th>
<th>DGFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMX</td>
<td>0.142mas</td>
<td>0.123mas</td>
</tr>
<tr>
<td>PMY</td>
<td>0.139mas</td>
<td>0.122mas</td>
</tr>
<tr>
<td>LOD</td>
<td>0.020ms</td>
<td>0.022ms</td>
</tr>
<tr>
<td>UT1-UTC</td>
<td>0.010ms</td>
<td>0.012ms</td>
</tr>
</tbody>
</table>

The results show the similar accuracy as that of DGFI. Our TRF results are also similar to ITRF2008 for regular sites. The accuracy is better than 5mm for coordinates and 1mm/yr for velocities with respect to ITRF2008.
3. Data Processing and results
3、Data Processing and results
## 3、Data Processing and results

### Initial results of SHAO TRF and EOP

<table>
<thead>
<tr>
<th>EOP</th>
<th>Technique</th>
<th>WRMS (DGFI)</th>
<th>WRMS (SHAO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMX (mas)</td>
<td>GPS</td>
<td>0.063</td>
<td>0.061</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>0.163</td>
<td>0.205</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>0.205</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>DORIS</td>
<td>0.234</td>
<td>0.850</td>
</tr>
<tr>
<td>PMY (mas)</td>
<td>GPS</td>
<td>0.055</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>0.232</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>0.204</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td>DORIS</td>
<td>0.357</td>
<td>0.853</td>
</tr>
<tr>
<td>UT1-UTC (ms)</td>
<td>VLBI</td>
<td>0.013</td>
<td>0.017</td>
</tr>
<tr>
<td>LOD (ms)</td>
<td>VLBI</td>
<td>0.027</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>GPS</td>
<td>0.022</td>
<td>0.007</td>
</tr>
</tbody>
</table>
3、Data Processing and results

STRF2013

ITRF2008

DTRF2008
4、Error analysis

◆ Scale

SLR

VLBI
4. Error analysis

◆ Origin (translation parameters)
4. Error analysis

◆ Residual series for site coordinates

- **SLR**
  - **SHAO**
    - Mean: 4.43 (mm/m) | Max: 49.41 (mm) | Diff: 0.045 (mm/yr)
  - **ITRF**
    - Mean: 1.74 (mm/m) | Max: 48.12 (mm) | Diff: 0.18 (mm/yr)

- **GPS**
  - **SHAO**
    - Mean: 0.10 (mm/m) | Max: 47 (mm) | Diff: 0.21 (mm/yr)
  - **ITRF**
    - Mean: 0.11 (mm/m) | Max: 14 (mm) | Diff: 0.011 (mm/yr)
4. Error analysis

◆ Residual series for site coordinates

- **SHAQ**
  - East (mm)
    - mean = 0.28 mm
    - std = 0.004 mm/year
  - North (mm)
    - mean = 0.05 mm
    - std = 0.001 mm/year
  - Up (mm)
    - mean = 0.05 mm
    - std = 0.001 mm/year

- **ITRF**
  - East (mm)
    - mean = 0.19 mm
    - std = 0.003 mm/year
  - North (mm)
    - mean = 0.05 mm
    - std = 0.002 mm/year
  - Up (mm)
    - mean = 0.04 mm
    - std = 0.001 mm/year

- **VLBI**
  - East (mm)
    - mean = 0.19 mm
    - std = 0.004 mm/year
  - North (mm)
    - mean = 0.04 mm
    - std = 0.001 mm/year
  - Up (mm)
    - mean = 0.03 mm
    - std = 0.001 mm/year

- **DORIS**
  - East (mm)
    - mean = 0.24 mm
    - std = 0.005 mm/year
  - North (mm)
    - mean = 0.05 mm
    - std = 0.002 mm/year
  - Up (mm)
    - mean = 0.02 mm
    - std = 0.001 mm/year
4、Error analysis

◆ Residual series for co-location site coordinates

- **VLBI**
- **GPS**
- **SLR**
4. Error analysis

- The precision distribution of site coordinate and velocity with the number of SINEX solutions
SHAO has carried out a new TRF and corresponding EOPs based on the SINEX solutions of the space geodetic techniques such as VLBI, SLR, GNSS and DORIS.

The accuracy of EOPs is similar with that of DGFI’s EOPs with respect to IERS 08 C04. The accuracy is about 0.142 mas for PMX, 0.139 mas for PMY, 0.010 ms for UT1-UTC and 0.02 ms for LOD. From the residuals of our EOPs and IERS 08 C04 we can see the mean is very small 0.027 mas for PMX, 0.066 mas for PMY, 0.008 ms for UT1-UTC and 0.001 ms for LOD.
The position and velocity for our TRF are close to that of ITRF2008 for regular space geodetic sites. The accuracy is better than 5mm for coordinates and 1mm/yr for velocities.

After more detail work we hope to improve our TRF and EOP to satisfy the needs of GGOS.
Future plan

◆ Check our EOP and TRF accuracy and find those abnormal solutions. And try to find the reasons of those abnormalities.
◆ Add annual and semi-annual signals into the combination estimation. See if the residuals will be reduced.
◆ Consider the velocity as the different one after the special motions such as earthquakes for some sites. And also import space-based local ties to get better results.
◆ China regional TRF will be completed in future two years.
◆ The EOP, TRF and VLBI/SLR/GNSS solutions will be released at our website in June 2014.
Thank You!

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