

## Lunar Laser Ranging - What is it Good for?

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**Abstract.** *In 1969, a new era for studying Earth-Moon dynamics has started. Since the first returns of laser pulses sent from observatories on Earth to reflector arrays on the Moon, a new space geodetic technique – Lunar Laser Ranging (LLR) – provides an ongoing time series of highly accurate Earth-Moon distance measurements. This data can be used to carry out relativity tests and to potentially support various GGOS objectives.*

*LLR data analysis is realized at the cm level of accuracy, for which the whole measurement process is modeled at appropriate post-Newtonian approximation, i.e., the orbits of the major bodies of the solar system, the rotation of Earth and Moon, signal propagation, but also the involved reference and time systems.*

*By analysing the 43-year record of range data, LLR is able to provide, among others, a dynamical realization of the International Celestial Reference System, parameters related to the selenocentric and terrestrial reference frames, (long-periodic) Earth Orientation Parameters as well as quantities testing General Relativity (e.g. strong equivalence principle, Yukawa-like perturbations or time-variability of the gravitational constant).*

*We will present results for relativistic parameters as well as first results from our software extension, which is able to generate and analyse simulated LLR data.*

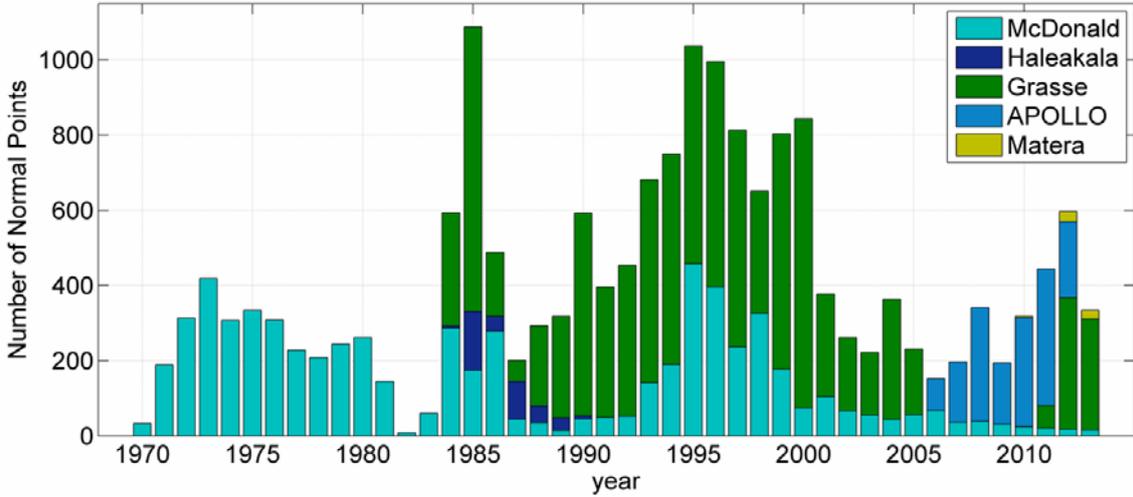
### Introduction

The first retroreflector for LLR was deployed on the Moon by the Apollo 11 astronauts on July 21, 1969. With Apollo 14 and Apollo 15 in the early Seventies, two further reflector arrays were brought onto the Moon by US missions. Unmanned Soviet missions, Luna 17 and Luna 21, completed the lunar network by 1973 with their lunar rovers Lunokhod 1 and 2. The unique time series of highly accurate Earth-Moon distance measurements was assembled by just a few observatories on Earth. Currently, four active LLR sites track the Moon on a routine basis: the McDonald Observatory in Texas, USA, the Observatoire de la Côte d'Azur, France, the APOLLO site in New Mexico, USA and the Matera Laser Ranging station in Italy.

LLR tracking is quite similar to SLR (Satellite Laser Ranging) tracking. A series of short laser pulses is sent from an observatory towards a reflector on the Moon. The weak reflected signal is registered at the observatory. The round trip travel times of the laser pulses are measured and combined to a so-called normal point (NP) which is used in the analysis. From 1970 to 2013, about 18100 NPs were collected. The annual distribution is shown in figure 1. In addition, there are also few measurements from the observatories at Orroal and Wettzell which are not shown here.

Over long periods often only one observatory carried out LLR measurements, which is related to the difficulty to get successful lunar returns. The measurement is affected by a huge signal loss due to beam divergence from the outgoing beam at the telescope and especially from the reflector array on the Moon in combination with the large distance and atmospheric effects.

The Institut für Erdmessung (IfE) LLR analysis model consists of a collection of sophisticated software modules at the cm-level of accuracy. The whole measurement process is modeled at appropriate post-Newtonian approximation, including the orbits of the major bodies of the solar system, relativistic precession of the Moon, signal propagation, but also the involved reference and time systems as well as the time-variable positions of the observatories and reflectors. Most prominently, LLR is one of the best tools to test General Relativity within the solar system. It allows for constraining gravitational physics parameters related to the strong equivalence principle, Yukawa-like perturbations, preferred-frame effects, or the time variability of the gravitational constant. In this paper, we focus on Yukawa-like perturbations and study the benefit when adding further ground stations on Earth to the LLR network.



**Figure 1.** NP distribution with respect to LLR observatories covering the years 1970 to 2013

### Relativity tests with LLR - Yukawa like perturbations

The gravitational acceleration between two bodies decreases proportional to the inverse square of the distance between the bodies. This is also called Newton's inverse square law. A possible violation can be parametrized by the introduction of an additional Yukawa term in the model for the combined gravitational field of Moon and Earth (Adelberger, 2001):

$$V_{EM}(r) = -\frac{GM_E M_M}{r} (1 + \alpha e^{-r/\lambda}). \quad (1)$$

$M_E$  and  $M_M$  denote the masses of Earth and Moon,  $G$  the gravitational constant,  $\alpha$  and  $\lambda$  are the coupling constant and interaction range, respectively. The interaction range is fixed to 380000 km while the coupling constant is estimated in the LLR analysis. Therefore, an additional differential acceleration  $\Delta \ddot{\mathbf{r}}_{yuk} = \ddot{\mathbf{r}}_{Myuk} - \ddot{\mathbf{r}}_{Eyuk}$  between Earth E and Moon M is introduced in the equations of motion:

$$\Delta \ddot{\mathbf{r}}_{yuk} = -\frac{\mathbf{r}_{EM}}{r_{EM}} \frac{GM_{E+M}}{r_{EM}} \alpha e^{-r_{EM}/\lambda} \left( \frac{1}{r_{EM}} + \frac{1}{\lambda} \right). \quad (2)$$

The power spectrum of the effect of such an additional acceleration term on the Earth-Moon distance is shown in figure 2, where the strongest signals appear at frequencies of the anomalistic and synodic months and of their linear combinations.

The estimation of the coupling constant  $\alpha$  yields

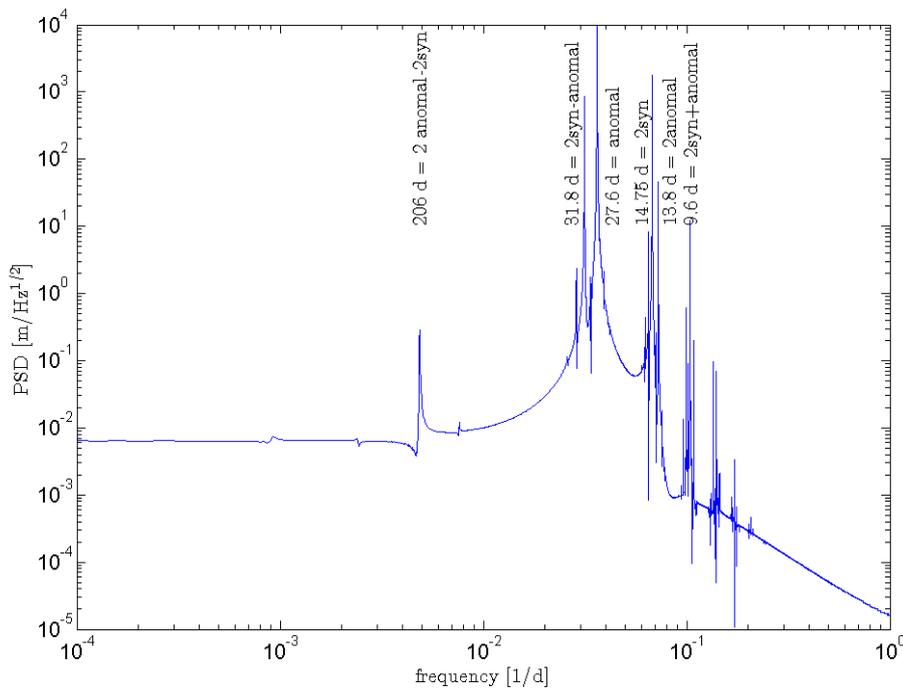
$$\alpha = (-1.8 \pm 0.5) \times 10^{-11}.$$

The given accuracy is the  $3\sigma$  value, i.e., 3 times the standard deviation which is obtained in the least squares adjustment. The possible non-null result should not be misinterpreted. We do not consider this result as a significant deviation from the predictions of general relativity, where  $\alpha = 0$ . The given error might be too small due to effects from possible systematics or insufficient modelling. The Yukawa potential introduces an additional precession of the lunar orbit (Adelberger, 2001), analog to the geodetic precession. If the Yukawa term is estimated together with a parameter for the geodetic precession, we obtain, for the coupling constant  $\alpha$ , a value of

$$\alpha = (-0.6 \pm 1.8) \times 10^{-11},$$

which is highly correlated with the geodetic precession. This result seems to give a more realistic error estimation for  $\alpha$ .

An overview about the results for other relativistic quantities which were estimated within our LLR analysis is given in table 1. Some discussion on these parameters is given in Müller et al. (2014) and the references therein.



**Figure 2.** Power spectrum of the difference in the Earth-Moon distance with and without a perturbation due to an additional Yukawa term with  $\alpha = 1.8 \times 10^{-11}$ . Here, „syn” denotes the synodic frequency and „anomal” the anomalistic frequency.

**Table 1.** IfE results for various relativistic parameters and estimated „realistic” errors

Parameter	Results
Nordtvedt parameter $\eta$ (test of the strong equivalence principle)	$(2.0 \pm 4.0) \times 10^{-4}$
Time variable gravitational constant $\dot{G}/G$ [ $\text{yr}^{-1}$ ]	$(1.4 \pm 1.5) \times 10^{-13}$
$\ddot{G}/G$ [ $\text{yr}^{-2}$ ]	$(4.0 \pm 5.0) \times 10^{-15}$
Geodetic precession (difference to predicted value of $1.92''/\text{cy}$ in general relativity) [ $''/\text{cy}$ ]	$(-0.6 \pm 1.0) \times 10^{-2}$
Metric parameter $\gamma - 1$ (space curvature)	$(3.0 \pm 4.0) \times 10^{-3}$
Metric parameter $\beta - 1$ (non-linearity)	$(1.7 \pm 2.0) \times 10^{-3}$
$\beta - 1$ using $\eta = 4\beta - \gamma_{\text{Cassini}} - 3$ with $\gamma_{\text{Cassini}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$	$(0.6 \pm 1.1) \times 10^{-4}$
Preferred-frame within special relativity $\zeta_1 - \zeta_0 - 1$	$(-0.5 \pm 1.2) \times 10^{-4}$
Preferred-frame effect $\alpha_1$	$(3.0 \pm 3.0) \times 10^{-5}$
$\alpha_2$	$(2.0 \pm 2.0) \times 10^{-5}$
(coupled with velocity of the solar system)	
Preferred-frame effect $\alpha_1$	$(1.6 \pm 3.0) \times 10^{-3}$
(coupled with dynamics within the solar system)	
Influence of dark matter $\delta_{\text{gc}}$ [ $\text{cm}/\text{s}^2$ ]	$(0.0 \pm 2.0) \times 10^{-14}$
(in the direction of the galactic center, equivalence principle test)	

## Simulation results

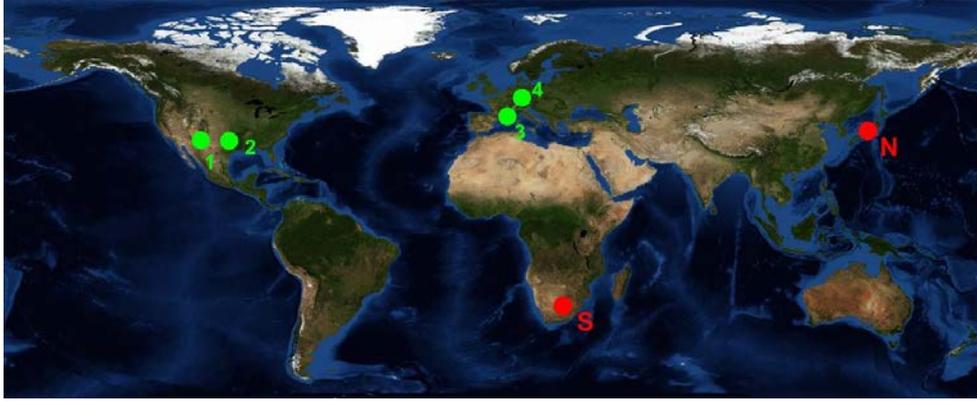
The IfE LLR analysis program was extended to simulate and analyze artificial laser ranging data to the Moon. In this study, we simulated the effect of further ground stations on selected parameters. A basis solution was computed with the simulation of data from four existing observatories (APOLLO, McDonald, Grasse, Wettzell) to the five usual lunar retroreflectors. The assumptions for the simulation were

- White noise, specifically such that the annual wrms becomes  $\sim 3\text{-}5$  cm,
- 40 years of data, homogeneously distributed (non-realistic, compare figure 1),
- Lunar elevation above  $40^\circ$ ,
- Case 1: only reflectors which are on the lunar night side,
- Case 2: all reflectors are used.

With the simulated data, a standard LLR analysis was carried out with the following estimated parameters:

- Initial positions and velocities for lunar orbit and rotation,
- Reflector and station coordinates (one site fixed),
- Some lunar gravity field coefficients, tidal parameters (lunar  $k_2$ , time delay  $D$ ),
- Mass of the Earth-Moon system.

In this paper, the solution with four ground stations is called “basis solution”. In addition to the four basis observatories, LLR data is simulated for one further ground station. In one case, the station is located in the northern hemisphere (e.g. in Japan) and, in another case, it is located in the southern hemisphere, e.g. in South Africa, where the old French OCA system will be installed, see Combrinck and Botha (2014). The geographical distribution of the chosen basis stations and the additional ground stations is shown in figure 3.

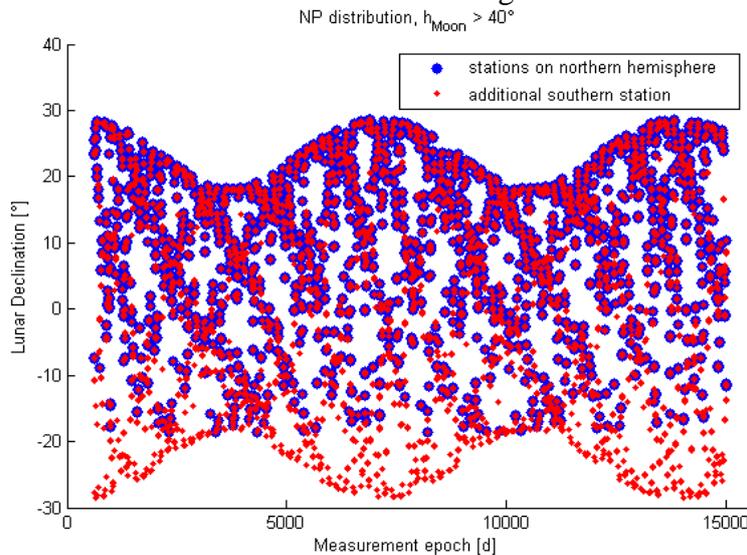


**Figure 3.** Distribution of 4 existing LLR ground stations of the basis solution (green dots) and 2 simulated additional ground stations (red dots) in the northern (N) and southern (S) hemisphere

The results for the basis solution as well as for the solutions with an additional observatory in the northern (N) and southern (S) hemisphere is shown in table 2 (test case 1) and table 3 (test case 2). The comparison is based on the single standard deviations of the above mentioned parameters, which are obtained from the fit.

The increasing accuracy from case 1 to case 2 implies, that it is advisable to observe as many reflectors as possible at a LLR session, if possible on the non-illuminated and illuminated area of the lunar disk. Adding a new observation site in the "opposite" northern or southern hemisphere leads to an improvement of about 10%-15% for the considered parameters in this simulated test case. This result is strongly affected by the assumed measurement accuracy of the new station. Here, the accuracy was chosen as the average measurement accuracy of the basis observatories. A higher accuracy of a new station would further improve the solution.

The resulting accuracies of the solution with the additional southern station seem to be slightly better than the solution with an additional northern station. In the simulation, this is mainly caused by a better coverage of the whole lunar orbit from a southern station. Figure 4 shows how the NP distribution depends on the lunar declination. With the given elevation mask, the southernmost lunar declinations can only be reached by a southern station. Furthermore, the simulation actually does not account for atmospherically caused loss of accuracy of observations at lower elevations. In this case, an additional southern ground station has, in combination with the northern sites, the advantage, that the full lunar orbit can be observed under high elevations with the highest accuracy.



**Figure 4.** NP distribution w.r.t. lunar declination and a minimum measurement elevation of  $40^\circ$

**Table 2.** Standard deviations for reflector coordinates, initial lunar rotation angles and mass of the Earth-Moon system (case 1, only reflectors in non-illuminated area of the lunar disk)

		basis solution	basis + N	basis + S
$X_{\text{ref}}$	x [mm]	242	216	195
	y [mm]	72	63	60
	z [mm]	243	219	204
Euler angles	$\varphi$ [as]	1.15	1.03	0.97
	$\theta$ [as]	0.012	0.010	0.010
	$\psi$ [as]	1.15	1.03	0.97
$GM_{\text{E+M}}$	[ $\text{km}^3\text{s}^{-2}$ ]	$6.73 \times 10^{-4}$	$6.01 \times 10^{-4}$	$5.34 \times 10^{-4}$

**Table 3.** Like table 2 but for case 2, i.e., all reflectors are used.

		Basis solution	Basis + N	Basis + S
$X_{\text{ref}}$	x [mm]	73	65	61
	y [mm]	23	21	20
	z [mm]	132	118	114
Euler angles	$\varphi$ [as]	0.63	0.57	0.53
	$\theta$ [as]	0.006	0.006	0.005
	$\psi$ [as]	0.63	0.57	0.53
$GM_{\text{E+M}}$	[ $\text{km}^3\text{s}^{-2}$ ]	$1.45 \times 10^{-4}$	$1.28 \times 10^{-4}$	$1.22 \times 10^{-4}$

## Summary

The ongoing LLR activities at IfE have been reported in this paper. We updated our analysis software including new measurements from 2012 and 2013 and estimated a new IfE standard solution for a multitude of relativistic parameters, see table 1. We discussed a possible Yukawa-like perturbation in some detail, where we did not obtain a significant deviation from the prediction of general relativity. We also extended the software for generating simulated LLR data and showed the positive effect of a further ground station in the northern or southern hemisphere.

## Acknowledgements

Current LLR data are collected, archived and distributed under the auspices of the International Laser Ranging Service (ILRS) (Pearlman et al., 2002). We acknowledge with thanks, that the more than 43 years of used LLR data has been obtained under the efforts of the personnel at the Observatoire de la Côte d’Azur in France, the LURE Observatory in Maui, Hawaii, the McDonald Observatory in Texas as well as the Apache Point Observatory in New Mexico. This research was funded by the Centre for Quantum Engineering and Space-Time Research QUEST and the DFG, the German Research Foundation, within the research unit FOR584 “Earth rotation and global dynamic processes” as well as within the research unit FOR1503 “Space-Time Reference Systems for Monitoring Global Change and for Precise Navigation in Space”.

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