

## SLR-derived terrestrial reference frame using the observations to LAGEOS-1/2, Starlette, Stella, and AJISAI

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**Abstract.** *Currently, the contributions of Starlette, Stella, and AJISAI are not taken into account when defining the International Terrestrial Reference Frame (ITRF), despite the large amount of data collected in a long time-span. Consequently, the SLR-derived parameters and the SLR part of the ITRF are almost exclusively defined by LAGEOS-1 and LAGEOS-2.*

*We investigate the potential of combining the observations to several SLR satellites with different orbital characteristics. Ten years of SLR data are homogeneously processed using the development version 5.3 of the Bernese GNSS Software. Special emphasis is put on orbit parameterization and the impact of LEO data on the estimation of the geocenter coordinates, Earth rotation parameters, Earth gravity field coefficients, and the station coordinates in one common adjustment procedure.*

*We find that the parameters derived from the multi-satellite solutions are of better quality than those obtained in single satellite solutions or solutions based on the two LAGEOS satellites. A spectral analysis of the SLR network scale w.r.t. SLRF2008 shows that artifacts related to orbit perturbations in the LAGEOS-1/2 solutions, i.e., periods related to the draconitic years of the LAGEOS satellites, are greatly reduced in the combined solutions.*

### Introduction

Today, the contributions of Starlette, Stella, and AJISAI are not considered when defining the ITRF, despite a huge amount of data collected within the long time-span of precise SLR observations. The ILRS does not routinely deliver products related to geodetic Low Earth Orbiters (LEO). The SLR-derived parameters and ITRF are, therefore, almost solely defined by LAGEOS-1, and LAGEOS-2, because the contribution of very high-orbiting Etalon satellites is minor (Thaller et al., 2013).

Therefore, the question has to be answered: Can the SLR-derived reference frame be improved by incorporating SLR observations to Starlette, Stella, and AJISAI?

We process 10 years of SLR data to 5 geodetic satellites: LAGEOS-1, LAGEOS-2, Starlette, Stella, and AJISAI and we compare the LAGEOS-1/2 solutions with the combined multi-SLR solutions. SLR data are homogeneously processed using the development version of the Bernese GNSS Software (Dach et al., 2007).

### Orbit modeling

The precise orbit determination of LEO, such as Starlette, Stella, and AJISAI is more demanding than the determination of the LAGEOS orbits, because of:

- a larger sensitivity to the Earth's gravity field and to its temporal variations,
- deficiencies in atmospheric drag models and variations of air density in the upper atmosphere,

- deficiencies of SLR station-specific range biases due to different laser systems used at SLR stations.

The issue of uncertainties and the sensitivity to time varying Earth's gravity field is addressed by using one of the latest state-of-the-art gravity field model EGM2008 (Pavlis et al., 2012) and the estimation of time variable low degree gravity field coefficients from SLR. EGM2008 is used up to degree 30 for LAGEOS and up to degree 90 for LEO satellites.

The issue related to deficiencies in atmospheric drag models is addressed by estimation the empirical orbit parameters and pseudo-stochastic orbit parameters in along-track once-per-revolution with a priori sigma  $10^{-7}$  ms<sup>-2</sup> (see Table 1).

The station specific center-of-mass corrections for LEO were estimated for every SLR station following the method from Sośnica et al., (2012).

**Table 1.** List of parameters in the LAGEOS and LEO solutions, after Sośnica (2014).

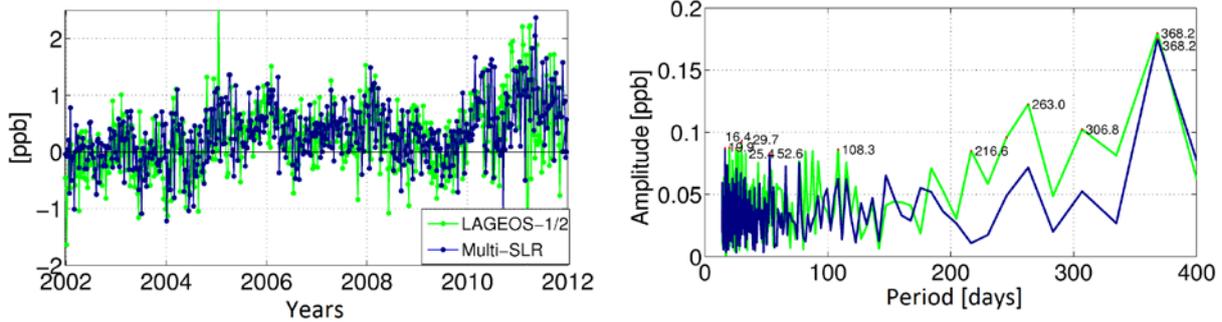
<b>Parameter</b>		<b>LAGEOS-1/2</b>	<b>Starlette, Stella, AJISAI</b>
<b>Station Coordinates</b>		Weekly	Weekly
<b>Earth Rotation Parameters</b>		PWL daily	PWL daily
<b>Geocenter Coordinates</b>		Weekly	Weekly
<b>Gravity field</b>		Up to d/o 4/4	Up to d/o 4/4
<b>Range Biases</b>		Selected stations	Selected stations
<b>Orbit</b>	<b>Osculating Elements</b>	Weekly	Weekly
	<b>Constant along-track <math>S_0</math></b>	Weekly	-
	<b>Air Drag Scaling Factor</b>	-	Daily
	<b>Once-per-rev <math>S_s, S_c</math></b>	Weekly	Daily
	<b>Once-per-rev <math>W_s, W_c</math></b>	-	Daily
	<b>Pseudo-Stochastic Pulses</b>	-	Once-per-rev in along-track

### Scale of the reference frame

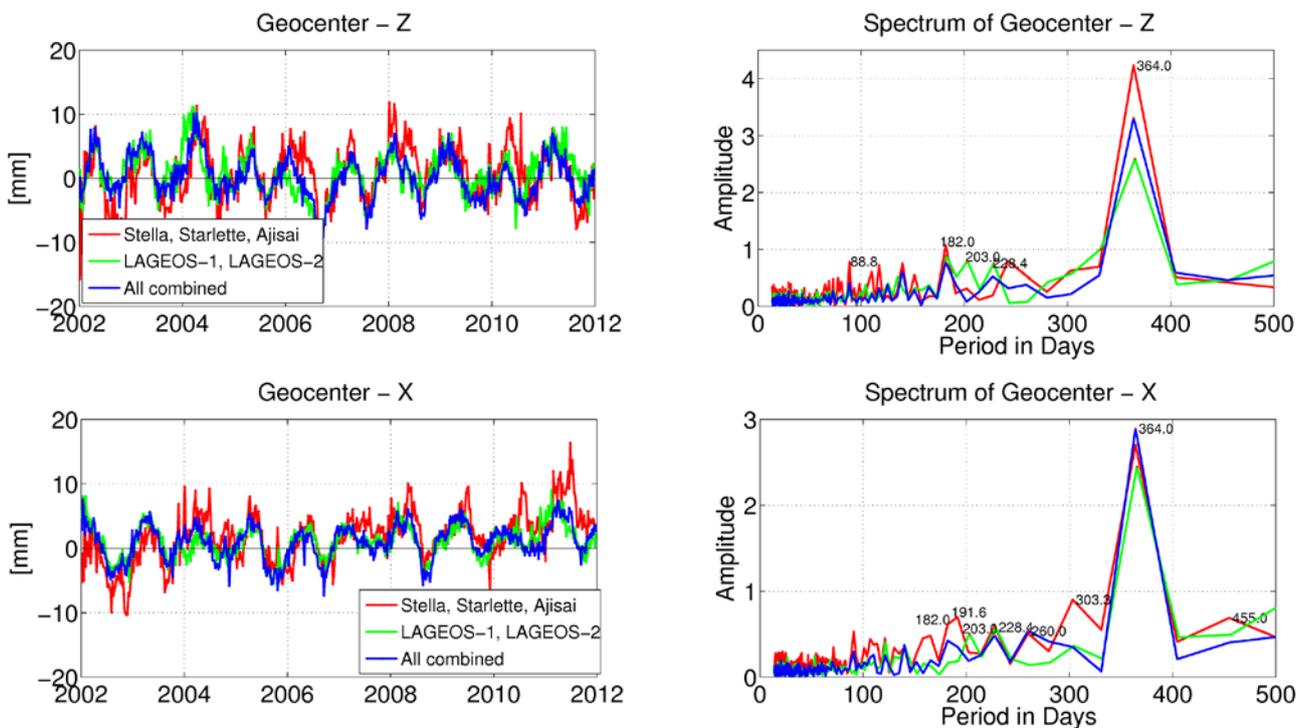
Figure 1 shows the weekly scale estimates from the Helmert transformation w.r.t. the a priori reference frame SLRF2008. Both scale estimates, from the LAGEOS-only and the multi-SLR solutions with 5 geodetic satellites, agree within 1 ppb for most of the epochs. The mean values of scale differences w.r.t. SLRF2008 are 0.24 ppb and 0.33 ppb for the LAGEOS and the multi-SLR solutions, respectively. These values correspond to 1.5 mm and 2.1 mm w.r.t. Earth radius.

The scale differences in LAGEOS and in combined solutions are not statistically significant from the SLRF2008 scale, because the WRMS of scale differences are 0.58 ppb, and 0.57 ppb, respectively.

The spectral analysis shows that the artifactual peaks from the LAGEOS solution due to orbit modeling deficiencies are remarkably reduced, e.g., the peak of 216.6 days, corresponding to the draconitic year of LAGEOS-2 (the time interval between two consecutive passes of the Sun through the orbital plane), the 108.3 day period and the 263.0 day period corresponding to eclipsing periods of LAGEOS-2 and LAGEOS-1, respectively, and the 306.8 day period corresponding to a drift of LAGEOS-2 perigee and ascending node w.r.t. the Sun. The scale derived from the multi-SLR solutions is, thus, less affected by the orbit modeling problems than that from the LAGEOS-1/2 solutions. A similar improvement of the scale estimates is observed when combining LAGEOS with LARES solutions (see Sośnica et al., 2014).



**Figure 1.** Scale of the reference frame from the Helmert 7-parameter transformation w.r.t. SLRF2008 in the LAGEOS-1/2 solutions and the multi-SLR solutions.



**Figure 2.** The Z and X components of the geocenter coordinate from the LAGEOS-1/2 solutions, LEO-only solution, and multi-SLR solution.

### Geocenter coordinates

Figure 2 shows the time series of geocenter coordinates from LEO, LAGEOS, and combined solutions for the Z and X components. All series represent a similar signal for the geocenter motion, but the LEO solution is noisier than the solutions including the LAGEOS satellites.

The Z geocenter component is of special importance, because estimation of the Z component is affected by, e.g., by solar radiation pressure mismodelings (Meindl et al., 2013). Therefore, reliable estimates of the Z geocenter coordinate cannot be derived from GPS, GLONASS, or DORIS, whereas SLR is the only technique of the space geodesy, which allows deriving well-established geocenter coordinates (Thaller et al., 2014). Moreover, using double differences and estimating troposphere delay parameters in GNSS solutions may absorb some geophysical variations in geocenter coordinate series (Sośnica et al, 2013a).

The a posteriori errors of the Z geocenter coordinate are smaller in the multi-SLR solutions than in the LAGEOS solutions, amounting on average 0.92 and 1.31 mm, respectively. The Z component benefits especially from the Stella's orbit, which is almost a polar orbit.

For all geocenter components, the amplitude of the annual signal is larger in the combined solution than in the LAGEOS solution, on average by 0.45 mm.

The spectral analysis of the Z geocenter component shows that the amplitude of period related to draconitic year of LAGEOS-2 is reduced from 0.60 mm for the LAGEOS-only solution to 0.35 mm in the combined multi-SLR solution. Therefore, the Z component is better defined in the multi-SLR solution.

The decomposition of the accelerations caused by the gravity field coefficient  $C_{10}$  into the R-S-W (radial, along-track, out-of-plane) system, following Meindl et al., (2013), reads as:

$$\begin{pmatrix} R \\ S \\ W \end{pmatrix} = C_{10} \frac{GMa_e}{r^3} \begin{pmatrix} -2 \sin i \sin u \\ \sin i \cos u \\ \cos i \end{pmatrix},$$

where  $C_{10}$  corresponds to the Z geocenter coordinate (see Meindl et al., 2013),  $GM$  is gravitational constant times Earth mass,  $a_e$  is Earth radius,  $r$  is length of the satellite state vector,  $i$  is inclination angle,  $u$  is argument of latitude of a satellite.

The estimated once-per-revolution parameter in the along-track direction  $S_C$  may, thus, absorb some geocenter variations, because of the correlation with the geocenter-induced perturbing acceleration. Indeed, the spectral analysis of the  $S_C$  parameter (see, e.g., Sošnica et al., 2011) reveals peaks corresponding not only to the draconitic year of LAGEOS satellites, but also to the annual period in the LAGEOS-1/2 solutions. The correlation coefficients between the Z geocenter coordinate and the  $S_C$  parameter are -0.83 and 0.58 for LAGEOS-1 and LAGEOS-2, respectively in the LAGEOS-1/2 solutions. These correlations are reduced to -0.23 and 0.15 in the multi-SLR solutions.

Thus, we conclude that in the LAGEOS-only SLR solutions with the estimation of the standard set of empirical parameters (including  $S_C$ , see Table 1), some of the geocenter signals can be absorbed by the empirical orbit parameters. In particular, the amplitude of annual signal is underestimated for the Z geocenter coordinate in the LAGEOS-1/2 solutions (see Figure 2). In the multi-SLR solutions the correlations between empirical parameters and geocenter coordinates are substantially reduced, and the amplitudes of the annual signal are increased.

From the spectral analysis, the amplitudes of other than the annual and semiannual signals in X and Y geocenter components can be slightly increased in the multi-SLR solutions, but none of the amplitudes exceed the value of 0.5 mm (see Figure 2 for the X component). Therefore, we conclude that the quality of estimated geocenter coordinates is the same for the X and Y components in LAGEOS-only and the multi-SLR solutions, whereas the quality of the Z geocenter coordinate is remarkably improved in the multi-SLR solution.

## Station coordinates

Some of the SLR stations within the ILRS network are not capable of observing LAGEOS satellites or the number and quality of LAGEOS observations is not sufficient. The only way to estimate reliable positions of such stations is to use observations of LEO satellites.

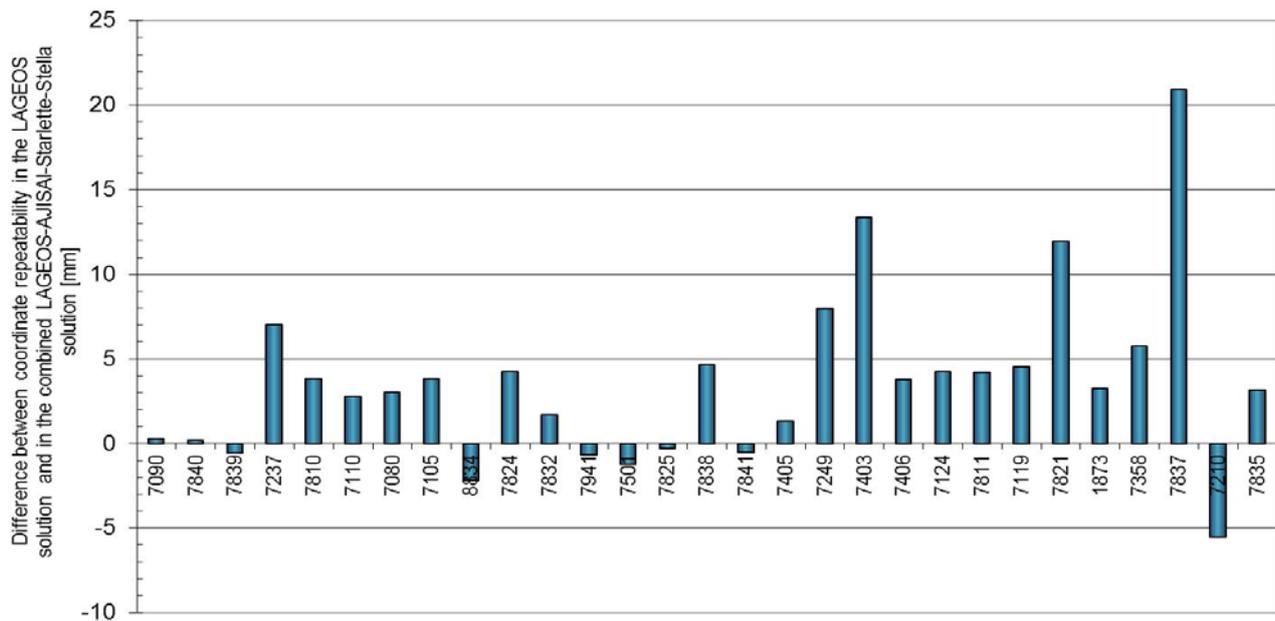
In our solution, the positions of six SLR stations are estimated exclusively on the basis of LEO observations: Mendeleevo, Russia (1870), Helwan, Egypt (7831), Lhasa (7356) and Beijing-A, China (7357), Cagliari, Italy (7548) and the mobile French Transportable Laser Ranging Station in Burnie, Tasmania (7370). Some of these stations are considered in SLRF2008, but not in ITRF2008.

The Mendeleev station, e.g., neither is included in the official release of ITRF2008 nor in SLRF2008, despite, e.g., ten years of observations to LEO satellites.

The difference of repeatability in the LAGEOS solution and in the combined multi-SLR solution is presented in Figure 3. Positive values denote a better repeatability in the combined solution, negative values in the LAGEOS solution. The SLR stations are sorted by the number of weekly solution.

In general, the repeatability of station coordinates can be improved when combining LAGEOS solutions with Starlette, Stella, and AJISAI solution. The improvement is largest for horizontal components and for non-core SLR stations. However, the vertical component shows worse characteristic of repeatability for some stations, when additionally including LEO data, but this degradation is usually due to weeks with LEO-only solutions.

We conclude that combining LEO with LAGEOS satellites does not worsen the LAGEOS-derived coordinates, but improves especially the estimates of station coordinates, due to a better observation geometry. Moreover, for some of SLR stations the coordinates can only be obtained using the SLR observations to LEO satellites, because of the lack of LAGEOS observations.



**Figure 3.** Difference between coordinate repeatability in the LAGEOS-1/2 solutions and in the multi-SLR solutions, after Sośnica et al., (2013b).

## Summary

We found that the repeatability of station coordinates is improved when combining LAGEOS solutions with low-orbiting SLR satellites. The Z geocenter coordinate is of superior quality in the multi-SLR solutions as compared to the LAGEOS-1/2 solutions, because the mean a posteriori error is smaller (from 1.3 mm to 0.9 mm), and the variations related to the draconitic year of LAGEOS-2 are reduced due to the much smaller correlations with  $S_C$  empirical orbit parameter. The correlation coefficient between  $S_C$  of LAGEOS-1 and the Z geocenter coordinate is  $-0.83$  in the LAGEOS-1/2 solution and  $-0.23$  in the multi-SLR solution.

The spectral analysis of the scale of the SLR network shows that the artefacts related to orbit perturbations in the LAGEOS-1/2 solutions are significantly reduced in the multi-SLR solutions.

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