New accurate atmospheric correction of SLR observations

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Motivation

- Contribution to Refraction Study Group (RSG) of ILRS:
  
  Accuracy goal was set to achieve $\sigma_S \approx 1$mm

- Provide an accurate atmospheric correction formula for two frequency observations:
  
  Two frequency SLR & Co-located SLR-GPS

- New formula has been developed based on the theory of two frequency range correction (Gu and Brunner, 1990)
1. Theoretical basis: optical path length equations $R_1$ & $R_2$

Projection of the second optical path $R_2(\lambda_2)$ onto the first path $R_1(\lambda_1)$

Accepted assumption: - Geometrical optics formulation is applicable
- Atmospheric dispersion & no turbulence

Original paths

Path projection using perturbation technique:

• Calculations along two different paths
• Calculations along the reference path
1. Theoretical basis: optical path length equations $R_1$ & $R_2$

• Equations for original paths

\[
R_1 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_1) \, ds_1 + K_1
\]

\[
R_2 = S + 10^{-6} \int_{p_2} N_g(\vec{r}_2, \lambda_2) \, ds_2 + K_2
\]

$K_i$: Arc-to-chord correction

$P_{21}$: Propagation correction

$N_g$: the group refractivity

$r_1, r_2$: vector of position

$\lambda_1, \lambda_2$: wavelength

• Equations after path projection

\[
R_1 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_1) \, ds_1 + K_1
\]

\[
R_2 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_2) \, ds_1 + K_1 + P_{21}
\]

(Calculated along the path $p_1$)
1. Theoretical basis: the refractivity $N_g$ & elimination of the density $\rho_t$

- The refractivity model (Separation of density from dispersion effect)

$$N_g = D(\lambda)\rho_t + W(\lambda)\rho_v$$

$\rho_t, \rho_v$: the total/vapour density of air  
$D, W$: the dispersion factors for dry and wet air

- Substituting $N_g$ yields:

$$R_1 = S + 10^{-6} D(\lambda_1) \int_{p_1}^{p_1} \rho_t \, ds_1 + 10^{-6} W(\lambda_1) \int_{p_1}^{p_1} \rho_v \, ds_1 + K_1$$

$$R_2 = S + 10^{-6} D(\lambda_2) \int_{p_1}^{p_1} \rho_t \, ds_1 + 10^{-6} W(\lambda_2) \int_{p_1}^{p_1} \rho_v \, ds_1 + K_1 + P_{21}$$

Now $\int_{p_1}^{p_1} \rho_t \, ds_1$ including the gradients can rigorously be eliminated
1. Theoretical basis: new atmospheric correction formula

\[ S = R_1 + \nu x (R_2 - R_1) - (\nu P_{21} + K_1) + H x SIWV \]

- \( T_1 \): dispersion
- \( T_2 \): curvature
- \( T_3 \): water vapour

- The derivation considers all propagation effects, except turbulence

- Applications:
  - Two frequency SLR observations
  - Co-located observations of two frequency SLR and GPS
2. Application: Two frequency SLR observations (Graz station)

\[ S = R_{SLR-1} + \frac{\nu (R_{SLR-2} - R_{SLR-1})}{T_1} - \frac{(\nu P_{21} + K_1)}{T_2} + H \times SIWV \]

- **Simulation method:**
  2D Ray tracing technique

- **SLR Station:**
  Graz

- **Frequency:**
  \( \lambda_1 = 0.532 \ \mu \text{m} \) & \( \lambda_2 = 1.0684 \ \mu \text{m} \)

- **Meteorological data:**
  ECMWF in Jan-March 2006
2. Application: Two frequency SLR observations (Graz station)

\[ S = R_{SLR-1} + \nu \times (R_{SLR-2} - R_{SLR-1}) - (\nu P_{21} + K_1) + H \times SIWV \]

- **Dispersion term:**
  \[ \nu = 65.86 \]

- **Humidity term:**
  \[ H = 1.5 \times 10^{-4} \text{ m}^3/\text{kg} \]

- **Curvature term:**
  \[ K_1 \gg \nu \times P_{21} \]

**Error (Model – Ray tracing)**

![Graph showing error comparison between model and ray tracing for different elevations.](image)
### 2. Application: Two frequency SLR observations (TIGO-Wettzell)

\[
S = R_{SLR-1} + \frac{\nu \times (R_{SLR-2} - R_{SLR-1})}{T_1} - \frac{(\nu P_{21} + K_1)}{T_2} + H \times SIWV
\]

- **SLR Station:**
  
  **TIGO Wettzell**

- **Frequency:**
  
  \( \lambda_1 = 0.4235 \, \mu m \) \& \( \lambda_2 = 0.8470 \, \mu m \)

- **Dispersion term:**
  
  \( \nu = 41.25 \)

- **Humidity factor:**
  
  \( H = 1.4 \times 10^{-4} \, m^3/Kg \)
3. Critical issues (1) : SLR observations $R_{SLR}$

$$S = R_{SLR-1} + \nu \times (R_{SLR-2} - R_{SLR-1}) - (\nu P_{21} + K_1) + H \times SIWV$$

• Requirement for the observation accuracy
  
  • Accuracy for SLR observations : $\sim 0.03$ mm
  
  • Hard to achieve using the current SLR systems.

• The dispersion factor

  • Typical values for $\nu$ : $20 - 300$

  $$\nu = \frac{D(\lambda_1)}{D(\lambda_1) - D(\lambda_2)}$$

<table>
<thead>
<tr>
<th>SLR Station</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graz</td>
<td>65.86</td>
</tr>
<tr>
<td>TIGO-Wettzell</td>
<td>41.25</td>
</tr>
<tr>
<td>Zimmerwald</td>
<td>41.15</td>
</tr>
</tbody>
</table>
3. Critical issues (2) : Curvature $K_1$ and propagation terms $\nu P_{21}$

\[
S = R_{SLR-1} + \nu x (R_{SLR-2} - R_{SLR-1}) - (\nu P_{21} + K_1) + H x SIWV
\]

- Formulae and values for $K_1$ and $P_{21}$

\[
K_1 = \int_{p_1} ds - S
\]

\[
P_{21} = \int_{p_2} n(\bar{r}_2, \lambda_2) ds_2 - \int_{p_1} n(\bar{r}_1, \lambda_2) ds_1
\]

- Accuracy requirement

$\sigma_{K1} \approx 1 \text{ mm}$  \quad $\sigma_{P21} \approx 0.03 \text{ mm}$
3. Critical issues (3) : Slant integrated water vapour $SIWV$ and humidity factor $H$

\[ S = R_{SLR-1} + \nu (R_{SLR-2} - R_{SLR-1}) - (\nu P_{21} + K_1) + H \times SIWV \]

- $SIWV$ estimation

\[
SIWV = \int_{\rho_1}^{\rho_2} \rho_v \, ds_1
\]

- Accuracy requirement : 5 Kg/m$^2$ of SIWV ( ~ 5 cm Slant Wet Delay)

- GPS is very useful and readily available technique

- Calculation for humidity factor $H$

\[
H = 10^{-6} W(\lambda_1) \left[ \frac{D(\lambda_2)}{D(\lambda_1)} - \frac{W(\lambda_2)}{W(\lambda_1)} \right]
\]

  - Typical values: $1.4 \times 10^{-4}$ m$^3$/kg
  - Dispersion formula of Ciddor (1996)
3. Critical issues (3) : Water vapour estimation by GPS

Co-located observations of two frequency SLR and GPS

- **Purpose:**
  - Elimination of the vapour density effects in two frequency SLR observations by GPS

- **Ideal conditions**
  - SLR telescope and GPS receiver are very close to each other
  - SLR and GPS signals travel through the same atmospheric condition

**Assumption for the computation:** SLR and GPS receiver positions are identical

- **Microwave refractivity for Non-hydrostatic part**

\[
N_{nh} \equiv k_3 R_v \left( \frac{1}{T} + k_5 \right) \rho_v
\]

\( k_{3,5} \) : refractivity constants
\( R_v \) : gas constant

Approximation by the mean temperature, \( T \approx T_m \)
3. Critical issues (3) : Water vapour estimation by GPS

• GPS Slant wet delay (SWD):

\[ SWD \approx 10^{-6} k_3 R_v \left( \frac{1}{T_m} + k_5 \right) \int_{p_1}^{\rho_v} ds_1 \]

\[ SIWV = \int_{p_1}^{\rho_v} ds_1 \]

• T₃ correction term:

\[ T_3 = H_{swd} \times SWD \]

\[ H_{swd} = \frac{10^6 H}{k_3 R_v \left( \frac{1}{T_m} + k_5 \right)} \approx 0.02 \]

• Two observation scenarios:

1. SLR and GPS observations to GPS satellites equipped with retroreflector

   SWD is calculated from GPS-35/36 signals

2. SLR observations to the other retroreflectors

   SWD is calculated by interpolation from all GPS signals
4. Summary

- Rigorous derivation of atmospheric corrections
- The total density effects and their gradients are eliminated
- Curvature and propagation terms are considered
- Water vapour effects and their gradients can be calculated from GPS
- Potential accuracy of the range correction is better than 1 mm
5. Closing remarks

**Advantage:**
The systematic effects in SLR observations are eliminated

**Remaining problem:**
Very high accuracy requirement for SLR observations

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Thank you for your attentions...!!!!

**Next development:**
Averaging technique could help to improve precision of the results