New accurate atmospheric correction of SLR observations

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Motivation

- Contribution to Refraction Study Group (RSG) of ILRS :

  Accuracy goal was set to achieve $\sigma_S \cong$ 1mm

- Provide an accurate atmospheric correction formula for two frequency observations :

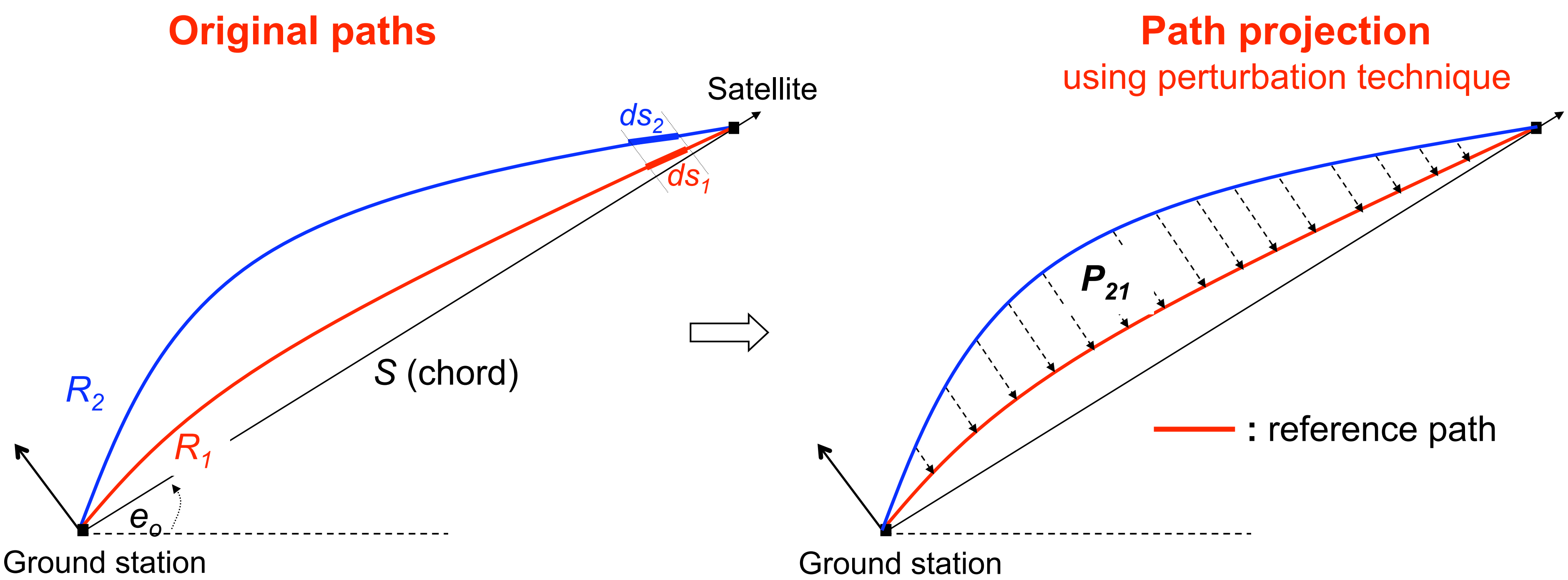
  Two frequency SLR & Co-located SLR-GPS

- New formula has been developed based on the theory of two frequency range correction (Gu and Brunner, 1990)

Projection of the second optical path $R_2(\lambda_2)$ onto the first path $R_1(\lambda_1)$

Accepted assumption: - Geometrical optics formulation is applicable

- Atmospheric dispersion & no turbulence



- **Calculations along two different paths**

- **Calculations along the reference path**

1. Theoretical basis : optical path length equations R_1 & R_2

- **Equations for original paths**

$$R_1 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_1)\, ds_1 + K_1$$

$$R_2 = S + 10^{-6} \int_{p_2} N_g(\vec{r}_2, \lambda_2)\, ds_2 + K_2$$

K_i : Arc-to-chord correction

P_{21} : Propagation correction

N_g : the group refractivity

r_1, r_2 : vector of position

λ_1, λ_2 : wavelength

- **Equations after path projection**

$$R_1 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_1)\, ds_1 + K_1$$

$$R_2 = S + 10^{-6} \int_{p_1} N_g(\vec{r}_1, \lambda_2)\, ds_1 + K_1 + P_{21}$$

(Calculated along the path p_1)

1. Theoretical basis : the refractivity N_g & elimination of the density ρ_t

- **The refractivity model (Separation of density from dispersion effect)**

$$N_g = D(\lambda)\rho_t + W(\lambda)\rho_v$$

ρ_t , ρ_v : the total/vapour density of air D, W : the dispersion factors for dry and wet air

- **Substituting N_g yields:**

$$R_1 = S + 10^{-6} D(\lambda_1) \int_{p_1} \rho_t \, ds_1 + 10^{-6} W(\lambda_1) \int_{p_1} \rho_v \, ds_1 + K_1$$

$$R_2 = S + 10^{-6} D(\lambda_2) \int_{p_1} \rho_t \, ds_1 + 10^{-6} W(\lambda_2) \int_{p_1} \rho_v \, ds_1 + K_1 + P_{21}$$

Now $\int_{p_1} \rho_t \, ds_1$ including the gradients can rigorously be eliminated

1. Theoretical basis : new atmospheric correction formula

$$S = R_1 + \nu \times (R_2 - R_1) - (\nu P_{21} + K_1) + H \times SIWV$$

T_1: dispersion T_2: curvature T_3: water vapour

- The derivation considers all propagation effects, except turbulence

- Applications:
  - Two frequency SLR observations
  - Co-located observations of two frequency SLR and GPS

2. Application : Two frequency SLR observations (Graz station)

$$S = R_{SLR-1} + \underbrace{\nu \times (R_{SLR-2} - R_{SLR-1})}_{T_1} - \underbrace{(\nu P_{21} + K_1)}_{T_2} + \underbrace{H \times SIWV}_{T_3}$$

- Simulation method:

  2D Ray tracing technique

- SLR Station :

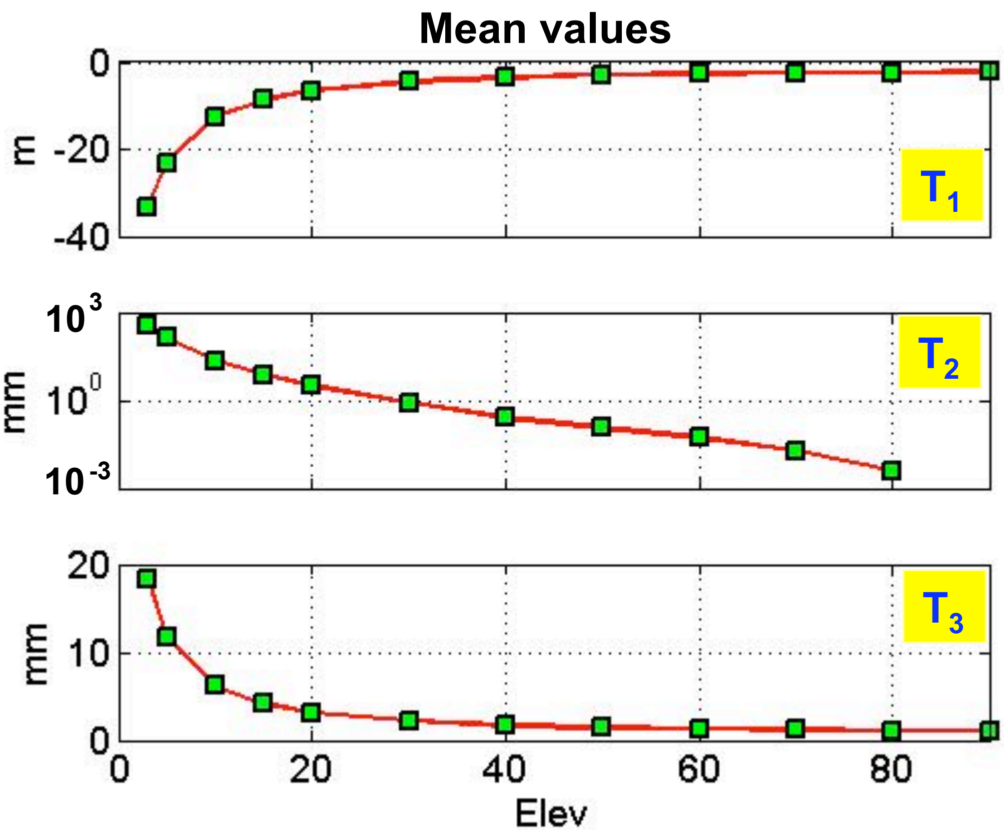
  Graz

- Frequency:

  λ_1= 0.532 μm & λ_2= 1.0684 μm

- Meteorological data:

  ECMWF in Jan-March 2006



2. Application : Two frequency SLR observations (Graz station)

$$S = R_{SLR-1} + \underline{\nu} \times (R_{SLR-2} - R_{SLR-1}) - (\underline{\nu P_{21}} + \underline{K_1}) + \underline{H} \times SIWV$$

- **Dispersion term**:

  $\nu = 65.86$

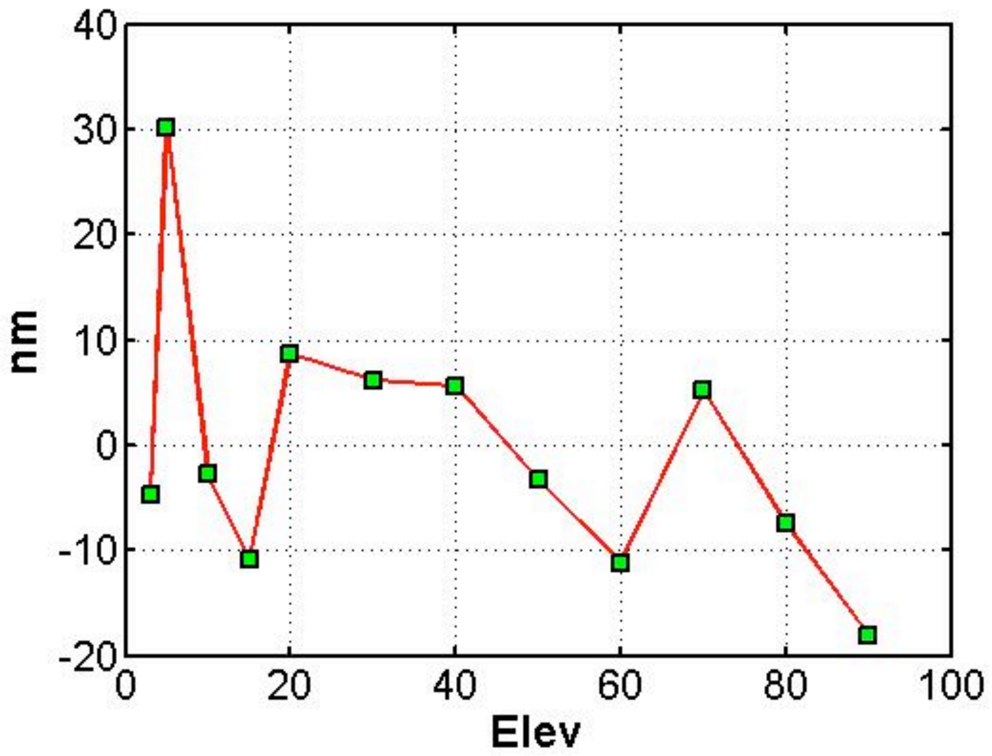
- **Humidity term**:

  $H = 1.5 \times 10^{-4}$ m^3/kg

- **Curvature term:**

  $K_1 >> \nu \times P_{21}$

- **Error (Model – Ray tracing)**



2. Application : Two frequency SLR observations (TIGO-Wettzell)

$$S = R_{SLR\text{-}1} + \underbrace{\nu \times (R_{SLR\text{-}2} - R_{SLR\text{-}1})}_{T_1} - \underbrace{(\nu P_{21} + K_1)}_{T_2} + \underbrace{H \times SIWV}_{T_3}$$

- SLR Station :

TIGO Wettzell

- Frequency:

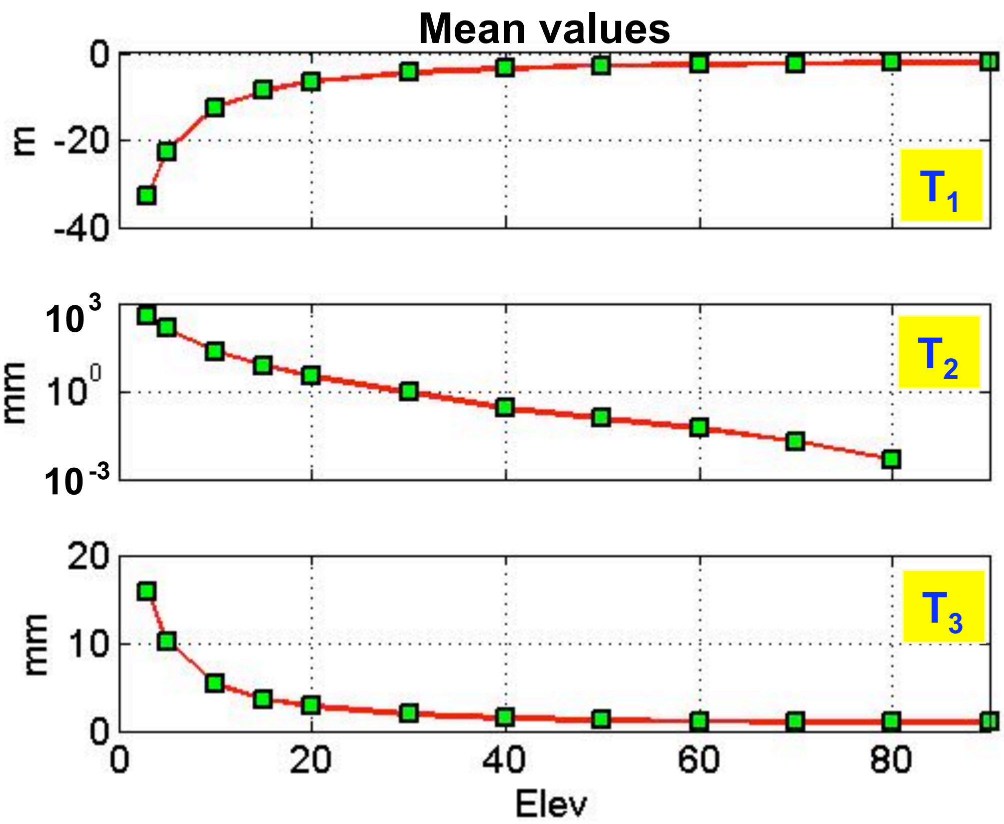
$\lambda_1 = 0.4235\ \mu m$ & $\lambda_2 = 0.8470\ \mu m$

- Dispersion term:

$\nu = 41.25$

- Humidity factor:

$H = 1.4 \times 10^{-4}\ m^3/Kg$



3. Critical issues (1) : SLR observations R_{SLR}

$$S = \underline{R_{SLR\text{-}1}} + \underline{\nu} \times (\underline{R_{SLR\text{-}2}} - \underline{R_{SLR\text{-}1}}) - (\nu P_{21} + K_1) + H \times SIWV$$

• Requirement for the observation accuracy

- Accuracy for SLR observations : **~ 0.03 mm**
- Hard to achieve using the current SLR systems.

• The dispersion factor

- Typical values for ν : **20 – 300**

$$\nu = \frac{D(\lambda_1)}{D(\lambda_1) - D(\lambda_2)}$$

SLR Station	ν
Graz	65.86
TIGO-Wettzell	41.25
Zimmerwald	41.15

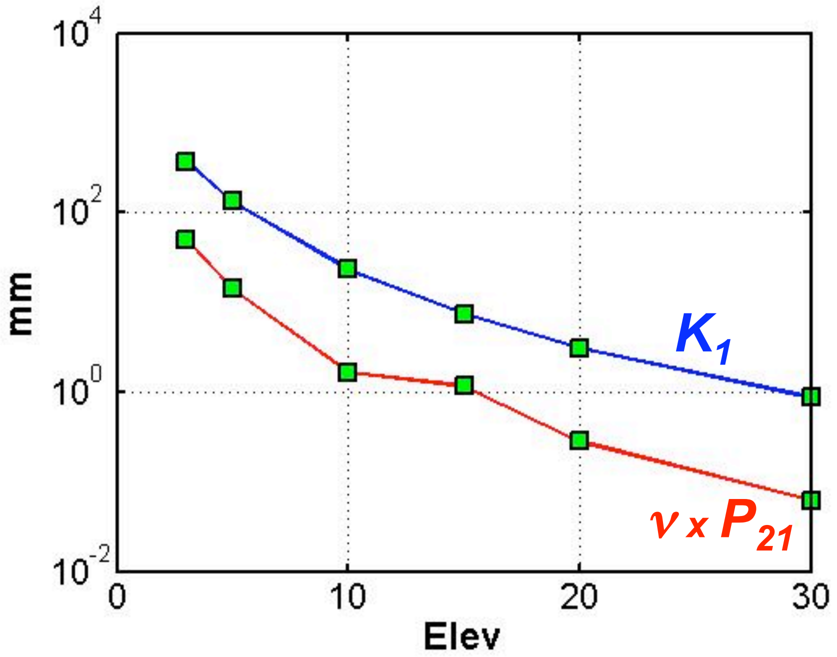
3. Critical issues (2) : Curvature K_1 and propagation terms νP_{21}

$$S = R_{SLR\text{-}1} + \nu \times (R_{SLR\text{-}2} - R_{SLR\text{-}1}) - (\underline{\nu P_{21}} + \underline{K_1}) + H \times SIWV$$

- **Formulae and values for K_1 and P_{21}**

$$K_1 = \int_{p_1} ds - S$$

$$P_{21} = \int_{p_2} n(\vec{r}_2, \lambda_2)\, ds_2 - \int_{p_1} n(\vec{r}_1, \lambda_2)\, ds_1$$



- **Accuracy requirement**

$\sigma_{K1} \cong 1$ mm $\sigma_{P21} \cong$ 0.03 mm

3. Critical issues (3) : Slant integrated water vapour *SIWV* and humidity factor *H*

$$S = R_{SLR\text{-}1} + \nu \times (R_{SLR\text{-}2} - R_{SLR\text{-}1}) - (\nu P_{21} + K_1) + \underline{H} \times \underline{SIWV}$$

- **SIWV estimation**

$$SIWV = \int_{p_1} \rho_v \, ds_1$$

- **Accuracy requirement : 5 Kg/m^2 of SIWV (~ 5 cm Slant Wet Delay)**
- **GPS is very useful and readily available technique**

- **Calculation for humidity factor *H***

$$H = 10^{-6} W(\lambda_1) \nu \left[\frac{D(\lambda_2)}{D(\lambda_1)} - \frac{W(\lambda_2)}{W(\lambda_1)} \right]$$

- **Typical values: 1.4 x 10^{-4} m^3/kg**
- **Dispersion formula of Ciddor (1996)**

3. Critical issues (3) : Water vapour estimation by GPS

Co-located observations of two frequency SLR and GPS

- **Purpose:**
  - **Elimination of the vapour density effects in two frequency SLR observations by GPS**

- **Ideal conditions**
  - **SLR telescope and GPS receiver are very close to each other**
  - **SLR and GPS signals travel through the same atmospheric condition**

Assumption for the computation: SLR and GPS receiver positions are identical

- **Microwave refractivity for Non-hydrostatic part**

$$N_{nh} \cong k_3 R_v \left(\frac{1}{T} + k_5 \right) \rho_v$$

$k_{3,5}$: refractivity constants

R_v : gas constant

Approximation by the mean temperature, $T \cong T_m$

3. Critical issues (3) : Water vapour estimation by GPS

- **GPS Slant wet delay (SWD):**

$$SWD \cong 10^{-6} k_3 R_v \left(\frac{1}{T_m} + k_5 \right) \int_{p_1} \rho_v ds_1 \qquad SIWV = \int_{p_1} \rho_v ds_1$$

- **T_3 correction term:**

$$T_3 = H_{swd} \times SWD \qquad H_{swd} = \frac{10^6 H}{k_3 R_v \left(\frac{1}{T_m} + k_5 \right)} \cong 0.02$$

- **Two observation scenarios:**

1. **SLR and GPS observations to GPS satellites equipped with retroreflector**

   SWD is calculated from GPS-35/36 signals

2. **SLR observations to the other retroreflectors**

   SWD is calculated by interpolation from all GPS signals

4. Summary

- Rigorous derivation of atmospheric corrections
- The total density effects and their gradients are eliminated
- Curvature and propagation terms are considered
- Water vapour effects and their gradients can be calculated from GPS
- Potential accuracy of the range correction is better than 1 mm

5. Closing remarks

Advantage:

The systematic effects

in SLR observations are eliminated

Remaining problem:

Very high accuracy requirement

for SLR observations

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Next development:

Averaging technique could help

to improve precision of the results

Thank you

for your attentions…!!!!