Earth Orientation Parameters from Lunar Laser Ranging

Liliane Biskupek
Jürgen Müller
Contents

• Model and analysis
• Nutation
• Earth rotation from LLR data by daily decomposition method
• Conclusions
Model

- model based upon Einstein's theory

\[ d = c \frac{\tau}{2} = \left| \mathbf{r}_\text{EM}^i - \mathbf{r}_\text{station}^i + \mathbf{r}_\text{reflector}^i \right| + c\Delta\tau \]

\[ \frac{d^2 \mathbf{r}_\text{EM}^i}{dt^2} = -\frac{GM_{E+M}}{r_{EM}^3} \mathbf{r}_\text{EM}^i + b_{\text{Newtonian}} + b_{\text{Relativity}} \]

- transformation between reference systems (Earth, Moon, inertial)
- transformation between time systems
- orbital motion of the solar system bodies
- rotation of Earth and Moon
- gravitational time delay (Shapiro effect)
Analysis

- weighted least-squares adjustment:
  - determination of the parameter of Earth-Moon system (ca. 180)
  - coordinates of LLR stations and retro-reflector arrays
  - parameters of physical librations and orbit of the Moon
  - orbit / mass of the Earth-Moon system,
  - lowest mass multipole moments of the Moon
  - long-periodic nutation parameters
  - relativistic parameters
Analysis

- weighted least-squares adjustment:
  → determination of the parameter of Earth-Moon system (ca. 180)
  - coordinates of LLR stations and retro-reflector arrays
  - parameters of physical librations and orbit of the Moon
  - orbit / mass of the Earth-Moon system,
  - lowest mass multipole moments of the Moon
  - long-periodic nutation parameters
  - relativistic parameters

residuals of standard solution
Standard solution 2008

- weighted residuals of data 12.1969 - 03.2008 (16230 normal points)
**Nutation**

- Fitting for different period of nutation coefficients
  - 18.6 years, 9.3 years, 1 year, 182.6 days, 13.6 days

\[
\Delta \psi = \sum_{i=1}^{N} \left( A_i + A_i't \right) \sin(\text{ARG}) + \left( A_i'' + A_i'''t \right) \cos(\text{ARG})
\]

\[
\Delta \epsilon = \sum_{i=1}^{N} \left( B_i + B_i't \right) \cos(\text{ARG}) + \left( B_i'' + B_i'''t \right) \sin(\text{ARG})
\]

\[
\text{ARG} = \sum_{i=1}^{5} N_j F_j \quad N_j : \text{multiplier, } F_j : \text{Delaunay parameter}
\]
## Nutation

<table>
<thead>
<tr>
<th>MHB2000 model</th>
<th>$A_i$ [mas]</th>
<th>$B_i$ [mas]</th>
<th>$A_i''$ [mas]</th>
<th>$B_i''$ [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6 year</td>
<td>-17206.42</td>
<td>9205.23</td>
<td>3.34</td>
<td>1.54</td>
</tr>
<tr>
<td>182.6 days</td>
<td>-1317.09</td>
<td>573.03</td>
<td>-1.37</td>
<td>-0.46</td>
</tr>
<tr>
<td>13.6 days</td>
<td>-227.64</td>
<td>97.85</td>
<td>0.28</td>
<td>0.14</td>
</tr>
<tr>
<td>9.3 year</td>
<td>207.46</td>
<td>-89.75</td>
<td>-0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>1 year</td>
<td>147.59</td>
<td>7.39</td>
<td>1.18</td>
<td>-0.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>own results</th>
<th>$A_i$ [mas]</th>
<th>$B_i$ [mas]</th>
<th>$A_i''$ [mas]</th>
<th>$B_i''$ [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6 year</td>
<td>-17201.93</td>
<td>9203.41</td>
<td>3.84</td>
<td>3.88</td>
</tr>
<tr>
<td>182.6 days</td>
<td>-1316.88</td>
<td>572.98</td>
<td>-3.25</td>
<td>-0.98</td>
</tr>
<tr>
<td>13.6 days</td>
<td>-230.54</td>
<td>99.26</td>
<td>0.16</td>
<td>0.31</td>
</tr>
<tr>
<td>9.3 year</td>
<td>207.13</td>
<td>-90.75</td>
<td>1.63</td>
<td>-0.21</td>
</tr>
<tr>
<td>1 year</td>
<td>146.83</td>
<td>7.86</td>
<td>0.27</td>
<td>-0.58</td>
</tr>
</tbody>
</table>
## Nutation

<table>
<thead>
<tr>
<th>MHB2000 model</th>
<th>$A_i$ [mas]</th>
<th>$B_i$ [mas]</th>
<th>$A_i''$ [mas]</th>
<th>$B_i''$ [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6 year</td>
<td>-17206.42</td>
<td>9205.23</td>
<td>3.34</td>
<td>1.54</td>
</tr>
<tr>
<td>182.6 days</td>
<td>-1317.09</td>
<td>573.03</td>
<td>-1.37</td>
<td>-0.46</td>
</tr>
<tr>
<td>13.6 days</td>
<td>-227.64</td>
<td>97.85</td>
<td>0.28</td>
<td>0.14</td>
</tr>
<tr>
<td>9.3 year</td>
<td>207.46</td>
<td>-89.75</td>
<td>-0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>1 year</td>
<td>147.59</td>
<td>7.39</td>
<td>1.18</td>
<td>-0.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>own results</th>
<th>$A_i$ [mas]</th>
<th>$B_i$ [mas]</th>
<th>$A_i''$ [mas]</th>
<th>$B_i''$ [mas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6 year</td>
<td>-17201.93</td>
<td>9203.41</td>
<td>3.84</td>
<td>3.88</td>
</tr>
<tr>
<td>182.6 days</td>
<td>-1316.88</td>
<td>572.98</td>
<td>-3.25</td>
<td>-0.98</td>
</tr>
<tr>
<td>13.6 days</td>
<td>-230.54</td>
<td>99.26</td>
<td>0.16</td>
<td>0.31</td>
</tr>
<tr>
<td>9.3 year</td>
<td>207.13</td>
<td>-90.75</td>
<td>1.63</td>
<td>-0.21</td>
</tr>
<tr>
<td>1 year</td>
<td>146.83</td>
<td>7.86</td>
<td>0.27</td>
<td>-0.58</td>
</tr>
</tbody>
</table>
Earth rotation from LLR data

- use of different EOP series (IERS EOP C04, COMB2006) as input in global adjustment
- analysis of the post-fit residuals to determine corrections for Earth rotation ΔUT0 and variation of latitude VOL
- use of determined VOL corrections in own analysis to improve the results
## Data sets of EOP (1)

<table>
<thead>
<tr>
<th>IERS EOP C04</th>
<th>COMB2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td></td>
</tr>
<tr>
<td>VLBI</td>
<td>VLBI</td>
</tr>
<tr>
<td>GPS</td>
<td>GPS</td>
</tr>
<tr>
<td>SLR</td>
<td>SLR</td>
</tr>
<tr>
<td>LLR</td>
<td>LLR</td>
</tr>
<tr>
<td>optical observation</td>
<td>optical observation</td>
</tr>
<tr>
<td>tidal effects</td>
<td></td>
</tr>
<tr>
<td>5 days - 18.6 years</td>
<td>5 days - 35 days</td>
</tr>
</tbody>
</table>
Data sets of EOP (2)

- differences between IERS EOP C04 and COMB2006
Daily decomposition (1)

• merge of post-fit residuals to daily sets of station-reflector combinations (min. 3)
  - 1179 daily sets for OCA, Grasse
  - 752 daily sets for Mc Donald
  - daily-decomposition method (least-squares adjustment)

\[ r(t) = r_{\Delta \phi} + r_{\Delta UT0} + r_n \]
Daily decomposition (2)

\[ r_{\Delta \phi} = 2 \Delta \phi a_e \left( \sin \phi \cos \delta \cos H - \sin \delta \cos \phi \right) \]
\[ r_{\Delta UT0} = 2 \Delta UT0 a_e \cos \phi \sin H \cos \delta \]
\[ \Delta \phi = x_p \cos \lambda - y_p \sin \lambda \]
\[ \Delta UT0 = \Delta UT1 + \tan \phi \left( x_p \sin \lambda + y_p \cos \lambda \right) \]

\( \delta \): declination  
\( H \): hour angle  
\( \phi \): latitude  
\( a_e \): distance to rotation axis
Results for VOL

- Mc Donald, Texas

**IERS EOP C04**

\[
\text{std} = 6.1 \text{ mas}
\]

**COMB2006**

\[
\text{std} = 6.4 \text{ mas}
\]
Corrections in global adjustment

- use of VOL as correction in calculation
Conclusions

• Long-periodic nutation coefficients
  - differences to the model not yet understood, further investigation needed
  - comparison with VLBI results

• Corrections for $\Delta UT0$ and VOL are calculated
  - use of VOL in own analysis does not show significant improvements in the results
  - test of different filters
  - calculation of LOD to compare with VLBI results