

## TRF datum and ILRS network geometry

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### Abstract

*The definition of the ITRF datum is one of the key element to assure a stable reference frame without internal distortion. The last geodetic reference system ITRF2005 was constructed by combining time series from all the space geodesy techniques. The orientation was constrained to the ITRF2000 orientation at epoch 2000.0 and null orientation rates between the two, the translation and its rate were fixed to the SLR solutions and scale and its rate to the VLBI solutions. The SLR time series was not considered in the scale definition mainly for its discontinuity in the time series.*

*The SLR network geometry has been identified as a candidate to explain the discontinuity and possible evidences of the network effect will be investigated.*

### The ITRF datum

The satellite techniques such as SLR, GPS and DORIS are sensitive to the Earth's center of mass, where the ITRF models place the origin of the global geodetic network. SLR is the only one having demonstrated to be able to locate the center of mass relatively to the tracking station network with an accuracy of a few millimetres. Thus the reference frame origin of a SLR loose solution is naturally placed in the Earth's CM; its uncertainty can be measured by the solution degree of looseness.

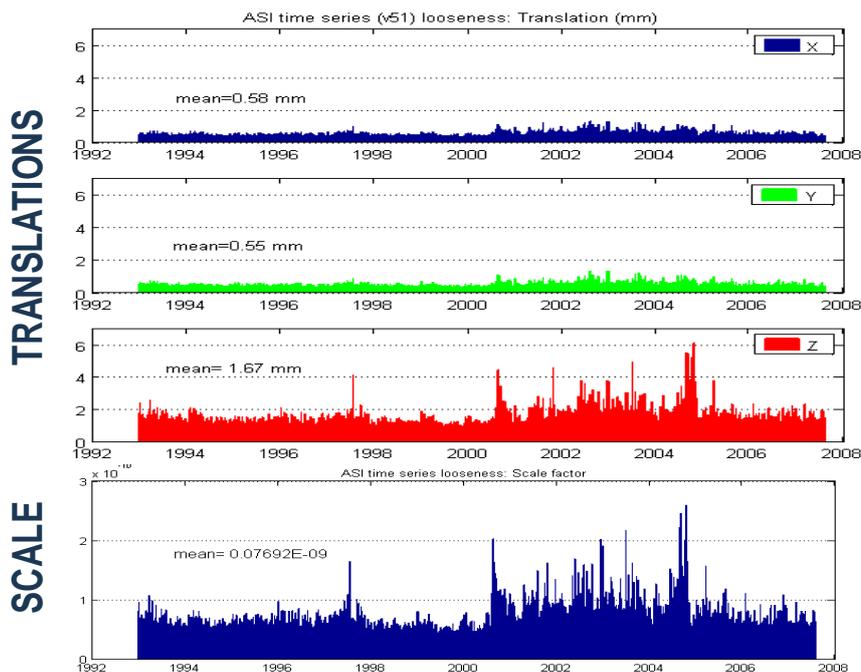
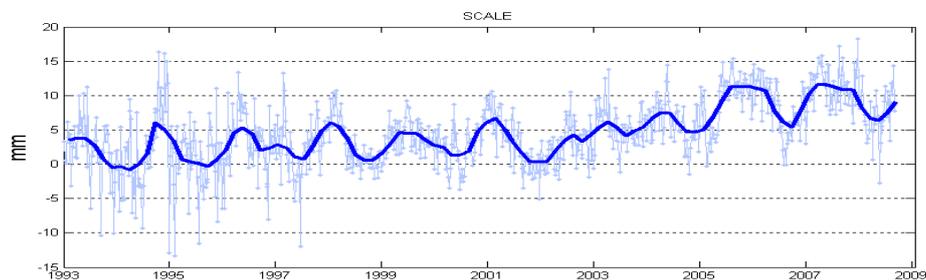


Figure 1. Looseness of the ASI SLR time series

More generally the degree of looseness is the uncertainty of the Helmert parameters of the solution with respect to an ‘error free’ reference frame and is obtained from:  $C_g = (\mathbf{A}^T \mathbf{C}^{-1} \mathbf{A})^{-1}$  (Blewitt, 1998) where C is the solution variance/covariance matrix and A is the Helmert transformation design matrix. Figure 1 is the plot of the time series of the indexes retrieved from the ASI weekly loose solutions of site coordinates and EOP, each bar indicating the value for one week. The mean value is less than one millimetre for the X and Y translation, less than two millimetres for the Z translation.

The time series of the SLR translations, estimated from 1993 on, are quite smooth with variations within one centimeter in X and Y, two centimeters in Z. The ITRF2005 translations and its rates were fixed to the SLR solutions. The situation is a bit different for the scale.

The scale of a frame is dependent on the modeling of some physical parameters: both SLR and VLBI determine the global scale with a higher stability w.r.t. GPS and DORIS. Figure 1 shows the looseness index for the scale: the uncertainty of the SLR scale is less than 0.1 ppb. Despite its accuracy, the scale time series has a discontinuity, roughly around 2002, when compared with VLBI or intrinsically (Altamimi et al., 2007). The direct consequence was that the ITRF2005 scale and its rates were fixed to the VLBI solutions, even if it is much less accurate. Figure 2 is a plot of the scale factor of the ASI SLR solution (in mm at the Earth’s surface) compared with ITRF2005 rescaled (rescaling was due to an inconsistency found between SLR and VLBI, soon after revealed as caused by a mismodeling in the VLBI analysis). The purpose of this study is to find the reason of the discontinuity.



**Figure 2.** Scale of the ASI SLR time series with respect to ITRF2005

### The SLR network

The first candidate identified to explain the scale discontinuity is the network geometry; its lack of spatial uniformity has always been a weakness of the SLR technique. The idea is that the limited number of sites in the network makes it too sensitive to the appearance/disappearance of tracking stations in the years and to the variation of the amount of data tracked by each of them. As a first approach, the number of data acquired by the network tracking the two LAGEOS satellites, from 1993 on, have been computed separately for the two hemispheres: North/South, East/West, +X/-X. Figure 3 shows the amount of data in the years, binned in weekly arcs. The North/South distribution starts changing in 2005 when more data are collected in the southern hemisphere and the balance is better. The data quantity in the East/West is similar until 1999 when more data are collected by the eastern sites, a decrease of “western” data is evident in 2005. The data distribution along X is quite balanced, with a small increase in the positive X. The unbalance of the spatial data coverage led to the idea of a data centroid, a kind of data “center of mass”.

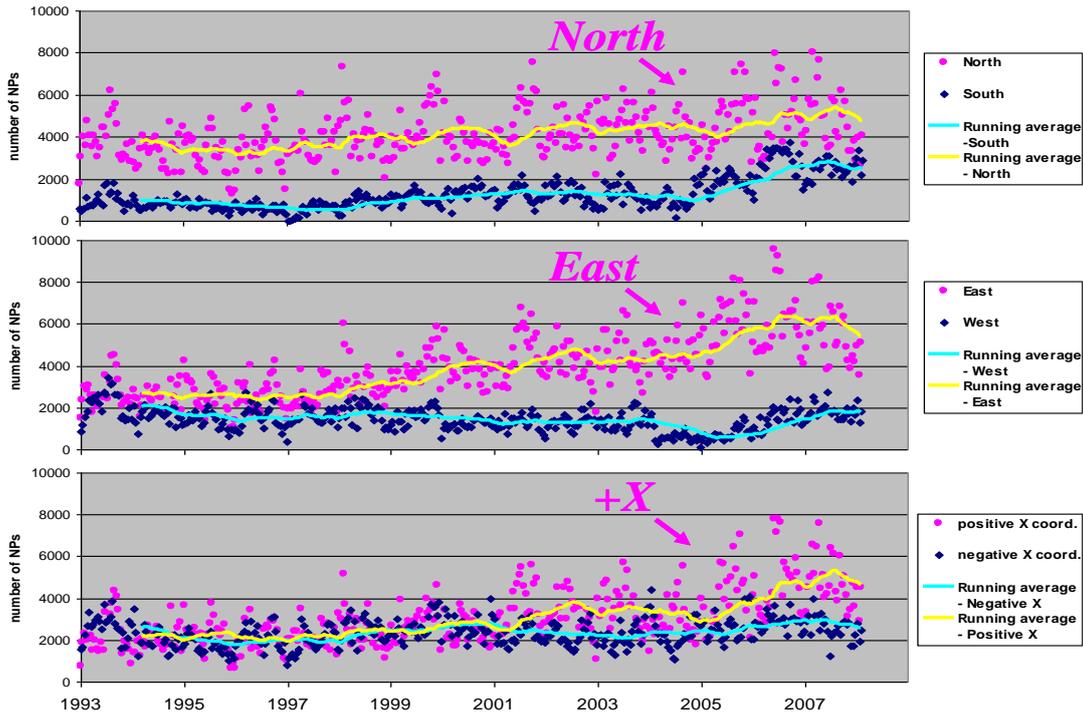


Figure 3. Data distribution in the hemispheres

### Data centroid and scale

The weekly time series of the LAGEOS 1/2 *data centroid* has been computed, expressed as coordinates ( $dX, dY, dZ$ ) in the ITRF reference frame with the following formulas:

$$dX = \sum_i X_i n_i / N_{tot}$$

$$dY = \sum_i Y_i n_i / N_{tot}$$

$$dZ = \sum_i Z_i n_i / N_{tot}$$

where:

$X_i, Y_i, Z_i$  :  $X, Y, Z$  coordinate of the  $i$ -th site of the week

$n_i$  : number of observations of the  $i$ -th site of the week

$N_{tot}$  : total number of observations in the week

We expect a centroid position close to the ITRF origin if the data distribution is spatially well balanced.

Figure 4 is the plot of the centroid coordinates together with the running mean of the values. The time series are quite flat until 2001: the  $X$  and  $Y$  component close to zero and an obviously positive  $Z$  (2-3 km far from the ITRF2005 origin). Afterwards, the centroid moves in the positive  $XY$  quadrant and in 2008 the  $Z$  coordinate becomes smaller, close to  $X$  and  $Y$ . The centroid migration in the space and in the  $XY$  plane is represented in Figure 5. Its position in 2008 is more distant from the ITRF origin than in 1993, in a direction of 45 degrees from the Greenwich Meridian: the distribution of the data is worse in terms of geographic coverage along the longitude.

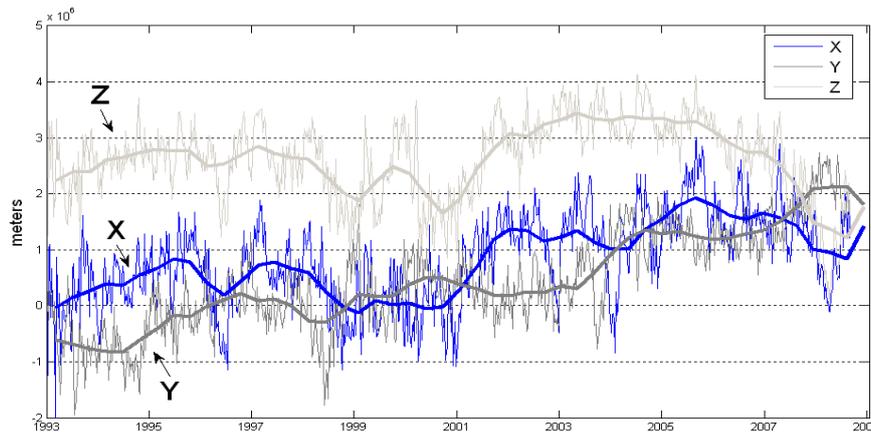
Looking at the plots of the scale variations and the centroid migration, the similarities are evident for the X and Y coordinates. The correlation coefficients between the centroid coordinates and the scale have been computed:

$$\text{scale}/X = 0.32$$

$$\text{scale}/Y = 0.45$$

$$\text{scale}/Z = 0.06$$

and confirm the uncorrelation with the Z coordinates, i.e. with the North/South data distribution.



**Figure 4.** Data centroid coordinates

The correlation between the scale and the length of the centroid vector is 0.27 but, if we consider the vector projection on the equator, the correlation increases up to 0.45.

In Figure 6 the distance, in the XY plane, is plotted together with the scale, expressing the Y-axis for the distance in meters at the right, the Y-axis for the scale in mm (at the Earth's surface) on the left.

## Conclusion

The study of the network geometry in terms of geographic data coverage has put in evidence the unbalanced data distribution since 2001-2002, above all due to the increase of data in the East hemisphere not balanced by data collected in the western part of the world. The correlation coefficients show a possible effect of the longitudinal data distribution on the scale to SLRF2005 while there is no evidence of the North/South data distribution influence. The concept of the data centroid can be useful to monitor the data homogeneity and for the network update, in the definition of the location of new sites.

Further investigations will be done with network simulation to assess the correlation between the scale and the data centroid and to check the reliability of scale estimation, above all in the cases of non uniformity.

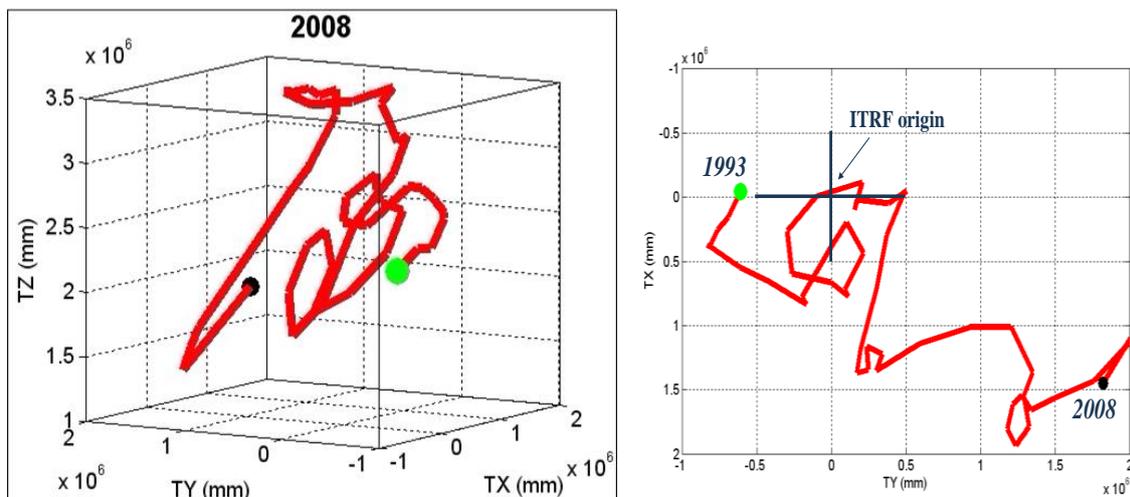


Figure 5. Centroid motion (3-D and XY plane)

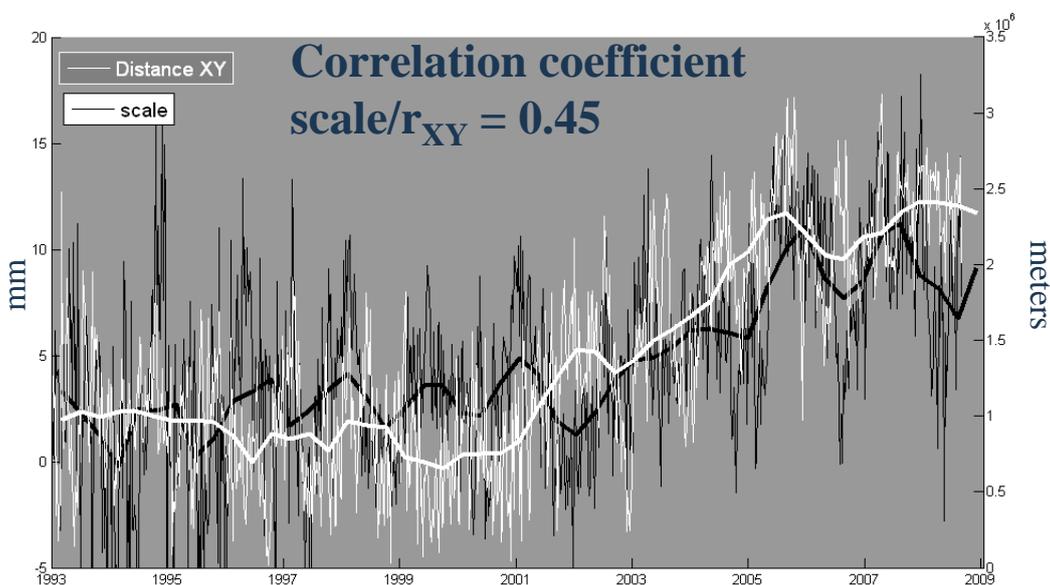


Figure 6. Centroid XY distance and scale

## References

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