Automated Transmitter Beam Size and Divergence Control in the SLR2000 System

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Goals

- SLR2000 adjusts transmitter beam divergence based on satellite altitude and orbital knowledge
  - Narrow for high satellites (± 4 arcsec min)
  - Wider for low satellites (± 13 arcsec max)

- Gaussian beam diameter must remain fixed at optimum value* of 35.7 cm (40 cm/1.12) at the telescope exit aperture to achieve maximum far field gain at lowest divergence setting and to maintain eye safety for all other divergence settings. Final divergence is set by adjusting phasefront curvature at the telescope exit window.

- The beam expander is located in the transmitter path and the beam must then pass through two telescopes with a total magnification of 30.48 before exiting the system.

SLR2000 Optical System

Telescope

SLR2000 TELESCOPE (NOT TO SCALE)

Transceiver

Optical Bench
Ray Matrices and Gaussian Beams

Paraxial ray matrix theory can be applied to gaussian beam propagation if we define the following complex parameter:

\[
\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi \omega^2(z)}
\]

If propagation from a point \( z_0 \) to \( z \) can be described by the ray matrix

\[
M = \begin{vmatrix}
A & B \\
C & D
\end{vmatrix}
\]

then the gaussian beam properties at \( z \) are given by

\[
\frac{1}{q(z)} = \frac{1}{q(z_0)} \left( \frac{C + D}{A + B} \frac{1}{q(z_0)} \right)
\]
Technical Approach

1. Measure transmitter gaussian beam radius (.969 mm) at entrance plane to beam expander, raw beam half divergence, and compute gaussian complex q-parameter for the input beam.

2. Choose COTS beam expander with adequate
   - exit aperture (>40 cm/30.48 = 13.1 mm)
   - magnification range (~13.1 mm/ 2 mm = 6.5) and
   - at least two control elements for independently adjusting beam size and phasefront curvature at the output.

   and replace vendor-provided controller and software with more flexible control system from National Aperture.

3. Develop dynamic ray model for unit including variable lens spacings.

4. Test dynamic ray model against sophisticated ray tracing program such as ZEMAX.

5. Calibrate beam expander servo controllers at various magnifications.

6. For each divergence value, use the gaussian beam propagation law to compute the complex q-parameter of the expander output beam and the lens spacings which produce that parameter.

7. Compute lookup table specific to laser transmitter.
1. Characterize Transmitter Beam

a. Spot Size Measurement

\[ \omega_{in} \] Input beam radius (to $1/e^2$ intensity point)

b. Divergence measurement

\[ \theta_{in} = \frac{\omega_f}{f} \] Divergence half-angle

Compute complex gaussian beam parameter at expander input

\[
\frac{1}{q_{in}} \equiv \frac{1}{R_{in}} - j \frac{\lambda}{\pi \omega^2_{in}} = \frac{\lambda}{\pi \omega^2_{in}} \left( \sqrt{\frac{\pi \omega_{in} \omega_f}{\lambda f}}^2 - 1 - j \right)
\]

GSFC currently uses the long focal length lens and CCD camera to monitor the divergence of the beam expander output.
2. Choose Beam Expander

Special Optics Beam Expander Model 56C-30-2-8X@450-700
Magnification: 2 to 8; exit aperture: 30 mm; input: 10 mm
3a. Develop “Dynamic” Ray Model
(d₁ and d₂ are variable)

<table>
<thead>
<tr>
<th>Description</th>
<th>Lens #</th>
<th>R₁</th>
<th>R₂</th>
<th>n</th>
<th>t</th>
<th>Eff. Focal length, f (m)</th>
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<tbody>
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<td>d₁</td>
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<td>0.135</td>
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</table>
3b. Ray Model Coefficients

\[
M_{SO} = \begin{bmatrix} A_{SO} & B_{SO} \\ C_{SO} & D_{SO} \end{bmatrix}
\]

\[
A_{SO} (d_1, d_2) = A_0 + A_1 d_1 + A_2 d_2 + A_{12} d_1 d_2
\]

\[
B_{SO} (d_1, d_2) = B_0 + B_1 d_1 + B_2 d_2 + B_{12} d_1 d_2
\]

\[
C_{SO} (d_1, d_2) = C_0 + C_1 d_1 + C_2 d_2 + C_{12} d_1 d_2
\]

\[
D_{SO} (d_1, d_2) = D_0 + D_1 d_1 + D_2 d_2 + D_{12} d_1 d_2
\]

<table>
<thead>
<tr>
<th>Sum</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</table>
4. Test Ray Model vs ZEMAX

Perfectly focused magnifier implies:

\[ A_{SO}(d_1, d_2) = m_{SO} \]

\[ C_{SO}(d_1, d_2) = 0 \]

or

\[ d_0^0(m_{SO}) = \frac{-0.02807695}{m_{SO}} + 0.104059 \]

\[ d_2^0(m_{SO}) = \frac{-637121.954 + 6325663.046 d_1^0(m_{SO})}{-16895097 + 162361136 d_1^0(m_{SO})} \]

which is compared to ZEMAX in above plots.
5. Calibrate Servo Controllers

Motor Positions
\[ M_1(m_{SO}) = a - d_1^0(m_{SO}) \]
\[ M_2(m_{SO}) = b - d_1^0(m_{SO}) - d_2^0(m_{SO}) \]

Vendor values: \(a = 88\) mm, \(b = 90\) mm

Calibration Procedure
1. Using vendor values for \(a\) and \(b\), set lens positions for integer magnifications \((m_{SO} = 2, 3, \ldots 7)\)
2. Using servo controller, adjust doublet and singlet lens positions for sharpest image and record motor encoder counts.
3. Apply least squares analysis to set of 4 equations on previous slide to obtain updated values for \(a\), \(b\), and \(m_{SO}\)
4. Repeat experiment and least squares analysis with updated constants until clear images are obtained at all magnification settings without significant correction.
5. Our final values: \(a = 88.7142\) mm, \(b = 92.9858\) mm
6. Use Gaussian Propagation Law

Spot size at primary: \[ \omega\left(\omega_0, R_0\right) = m_i \omega_0 \sqrt{1 + 2 \frac{d_t}{m_i R_0} + \left(\frac{d_t \lambda}{\pi m_i \omega_0^2}\right)^2 \left[1 + \left(\frac{\pi \omega_0^2}{\lambda R_0}\right)^2\right]} \equiv 0.179 \text{ m} \]

Far Field Half-Divergence: \[ \theta_i \left(\omega_0, R_0\right) = \frac{\lambda}{\pi m_i \omega_0} \sqrt{1 + \left(\frac{\pi \omega_0^2}{\lambda R_0}\right)^2} \]

Solving for the beam waist radius and phasefront curvature out of the beam expander, \( \omega_0 \) and \( R_0 \), we obtain for SLR2000:

\[
\omega_0 \left(\theta_i\right) \equiv \frac{\omega}{m_i} - d_i \theta_i = 0.00585 - 30.84 \theta_i \text{ (rad)}
\]

\[
\frac{1}{R_0 \left(\theta_i\right)} \equiv \frac{m_i^2 \theta_i}{\omega - m_i d_i \theta_i} = \frac{929.0 \theta_i \text{ (rad)}}{0.1785 - 940.0 \theta_i \text{ (rad)}} \text{ m}^{-1}
\]

For each divergence, \( \theta_i \), we then use the beam expander ray matrix to compute the expander lens positions, \( d_1 \) and \( d_2 \), which yield the above values of \( \omega_0 \) and \( R_0 \).
7. Generate Lookup Table Source: Phase II SLR2000 Transmitter

<table>
<thead>
<tr>
<th>Divergence Half-Angle, arcsec</th>
<th>Interlens Distance (d_1) (mm)</th>
<th>Interlens Distance (d_2) (mm)</th>
<th>Motor 1 Position (M_1) (mm)</th>
<th>Motor 2 Position (M_2) (mm)</th>
<th>Motor 1 Encoder Counts (C_1)</th>
<th>Motor 2 Encoder Counts (C_2)</th>
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<tbody>
<tr>
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</tbody>
</table>

“0” = 0.25 arcsec

One encoder count = 0.304 microns of lens movement
Lookup Table Plots

Central Airspace (d1) vs Divergence

Zoom Distance vs Divergence
Summary

- Using a computer lookup table, the SLR2000 computer can set two lens spacings in the transmit beam expander to provide a fixed beam diameter (35.8 cm) at the telescope exit aperture for eye safety while adjusting the phasefront curvature to give the desired final divergence.
- Lookup table must be adjusted for different transmitters but is an automated process.
- Optical half-divergence range of the final transmit beam is theoretically 0.25 arcseconds to 13 arcseconds (1.3 to 65 microradians) but atmosphere will set the actual lower limit.
- For verification, GSFC monitors the divergence of the beam expander output via a long focal length lens and CCD camera. Final beam divergence is reduced relative to the measured value by the transmitter magnification (3 x \(10.16 = 30.48\)).
- q-parameter technique was successfully used to couple transmitter to the 1.2 meter telescope in GSFC to Mars laser link (Sept. 2005).