LLR Link Efficiency Calibration

Probing the Health of the Lunar Reflectors

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The Basic Link Equation

\[ N_{\text{rx}} = N_{\text{tx}} \eta_c^2 \eta_r Q n_{\text{refl}} \left( \frac{d}{\phi r} \right)^2 \left( \frac{D}{\Phi r} \right)^2 \]

- \( \eta_c \) = one-way optical throughput (encountered twice)
- \( \eta_r \) = receiver throughput (dominated by narrow-band filter)
- \( Q \) = detector quantum efficiency
- \( n_{\text{refl}} \) = number of corner cubes in array (100 or 300)
- \( d \) = diameter of corner cubes (3.8 cm)
- \( \phi \) = outgoing beam divergence (atmospheric “seeing”)
- \( r \) = distance to moon
- \( \Phi \) = return beam divergence (diffraction from cubes)
- \( D \) = telescope aperture (diameter; 3.5 m)

\[ N_{\text{rx}} = 5.4 \left( \frac{E_{\text{pulse}}}{115 \text{ mJ}} \right) \left( \frac{\eta_c}{0.4} \right)^2 \left( \frac{\eta_r}{0.25} \right) \left( \frac{Q}{0.3} \right) \left( \frac{n_{\text{refl}}}{100} \right) \left( \frac{1 \text{ arcsec}}{\phi} \right)^2 \left( \frac{10 \text{ arcsec}}{\Phi} \right)^2 \left( \frac{385000 \text{ km}}{r} \right)^4 \]

- APOLLO should see 5 photons per pulse on Apollo 11 & 14; 15 on Apollo 15
Refining the Estimates, Part I

Terms contributing to the common (two-way) path, $\eta_c \approx 0.51$

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
<th>fractional error</th>
<th># occur</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{\text{atmos}}$</td>
<td>0.87</td>
<td>0.03</td>
<td>1</td>
<td>atmospheric transmission</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.825</td>
<td>0.03</td>
<td>1</td>
<td>Aluminum-coated primary mirror</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>0.875</td>
<td>0.03</td>
<td>1</td>
<td>Aluminum-coated secondary mirror</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>0.825</td>
<td>0.03</td>
<td>1</td>
<td>Aluminum-coated tertiary mirror</td>
</tr>
<tr>
<td>$\eta_4$</td>
<td>0.992</td>
<td>0.01</td>
<td>1</td>
<td>High-power dielectric turning-mirror</td>
</tr>
<tr>
<td>$\eta_5$</td>
<td>0.998</td>
<td>0.01</td>
<td>1</td>
<td>High-power dielectric turning-mirror</td>
</tr>
<tr>
<td>$\eta_L$</td>
<td>0.996</td>
<td>0.01</td>
<td>3</td>
<td>AR-coated lens</td>
</tr>
</tbody>
</table>
Refining the Estimates, Part II

Terms contributing to the receiver, $\eta_r \approx 0.43$

<table>
<thead>
<tr>
<th>symbol</th>
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<th># occur</th>
<th>description</th>
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<tbody>
<tr>
<td>$\eta_{TR}$</td>
<td>0.998</td>
<td>0.01</td>
<td>1</td>
<td>AR-coated Transmit/Receive optic</td>
</tr>
<tr>
<td>$\eta_6$</td>
<td>0.995</td>
<td>0.01</td>
<td>1</td>
<td>broad-band dielectric turning-mirror</td>
</tr>
<tr>
<td>$\eta_7$</td>
<td>0.995</td>
<td>0.01</td>
<td>1</td>
<td>broad-band dielectric turning-mirror</td>
</tr>
<tr>
<td>$\eta_{BS}$</td>
<td>0.93</td>
<td>0.01</td>
<td>1</td>
<td>uncoated fused-silica beam splitter</td>
</tr>
<tr>
<td>$\eta_D$</td>
<td>0.998</td>
<td>0.01</td>
<td>3</td>
<td>AR-coated variable attenuator disks</td>
</tr>
<tr>
<td>$\eta_L$</td>
<td>0.996</td>
<td>0.01</td>
<td>1</td>
<td>AR-coated lens</td>
</tr>
<tr>
<td>$\eta_{\mu L}$</td>
<td>0.95</td>
<td>0.04</td>
<td>1</td>
<td>microlens: uncoated epoxy plus AR side</td>
</tr>
<tr>
<td>$f_{\mu L}$</td>
<td>0.67</td>
<td>0.04</td>
<td>1</td>
<td>measured microlens efficiency</td>
</tr>
<tr>
<td>$f_{APD}$</td>
<td>0.5–1.0</td>
<td>0.10</td>
<td>1</td>
<td>APD fill factor: seeing/source dependent</td>
</tr>
</tbody>
</table>
Refining the Estimates, Part III

Other terms contributing to flux check

<table>
<thead>
<tr>
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<th>value</th>
<th>fractional error</th>
<th># occur</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>0.30</td>
<td>0.12</td>
<td>1</td>
<td>APD photon detection efficiency</td>
</tr>
<tr>
<td>$D$</td>
<td>3.26 m</td>
<td>0.01</td>
<td>2</td>
<td>effective aperture: $A = \pi D^2/4$</td>
</tr>
<tr>
<td>$\Delta \lambda_{\text{NB}}$</td>
<td>0.95 nm</td>
<td>0.05</td>
<td>—</td>
<td>effective filter bandpass</td>
</tr>
<tr>
<td>$\Delta t_{\text{APD}}$</td>
<td>95 ns</td>
<td>0.03</td>
<td>—</td>
<td>APD integration time per gate</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$3.9 \times 10^{-11}$ W/m$^2$/nm</td>
<td>0.03</td>
<td>—</td>
<td>zero-magnitude flux calibration</td>
</tr>
</tbody>
</table>
Checking the one-way flux

- Two ways: use star, or use moon
  - both give consistent results
- Moon around Apollo 15 is 3.60 magnitudes per square arcsec
  - at full-moon illumination
  - 2.87 mag into 1.4×1.4 arcsec APD field of view
  - fill factor \( f_{\text{APD}} \) is 13/16 (three channels missing)
- Measure 0.40 background photons per gate at full moon on Apollo 15
- Calculate 0.40 ± 0.08 using numbers presented above
  - \( Q \) was allowed to vary to match condition
  - came out right at expected value (30%) for uncoated APDs of this structure
- Thus much of link equation is confirmed
  - one-way photon detection efficiency: 2.3%
Apollo 15 Background Count Rate

peak rate = 0.55
dark rate = 0.15
### Additional Parameters for Ranging

<table>
<thead>
<tr>
<th>symbol</th>
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<th>fractional error</th>
<th># occur</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{NB}$</td>
<td>0.35</td>
<td>0.07</td>
<td>1</td>
<td>narrow-band filter throughput</td>
</tr>
<tr>
<td>$f_{\text{launch}}$</td>
<td>0.60</td>
<td>0.05</td>
<td>1</td>
<td>central obstruction on Gaussian beam</td>
</tr>
<tr>
<td>$E_{\text{pulse}}$</td>
<td>0.100 J</td>
<td>0.07</td>
<td>1</td>
<td>typical pulse energy</td>
</tr>
<tr>
<td>$n_{\text{refl}}$</td>
<td>300</td>
<td>—</td>
<td>1</td>
<td># cubes in Apollo 15 (largest) array</td>
</tr>
<tr>
<td>$\eta_{\text{refl}}$</td>
<td>0.93</td>
<td>0.01</td>
<td>1</td>
<td>double-pass through corner-cube face</td>
</tr>
<tr>
<td>$d$</td>
<td>0.038 m</td>
<td>—</td>
<td>2</td>
<td>diameter of individual corner cube</td>
</tr>
<tr>
<td>$r$</td>
<td>$3.85 \times 10^8$ m</td>
<td>0.02</td>
<td>−4</td>
<td>typical earth-moon distance</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.8 arcsec</td>
<td>0.15</td>
<td>−2</td>
<td>best outbound (seeing) divergence</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>10 arcsec</td>
<td>0.15</td>
<td>−2</td>
<td>divergence from return</td>
</tr>
</tbody>
</table>
Results of simple link equation

\[ N_{rx} = N_{tx} \eta_c^2 \eta_r Q n_{refl} \left( \frac{d}{\Phi_r} \right)^2 \left( \frac{D}{\Phi_r} \right)^2 \]

• Using parameters from previous tables, expected \textit{average} return from Apollo 15 array is:
  \[ 12 \pm 6 \text{ photons per pulse} \]

• Example best ranges (December 2005, January 2006) were \(~0.5\) photons per pulse for brief periods (\(~30\) sec)
  – best average rate over several minutes is 0.25 photons/pulse

• Ratio is \[ 12/0.5 = 24 \]

• Estimated uncertainty is 50% of average
  – would have to apply this (multiplicatively) 4.5 times to satisfy result
  – \[ 12 \times 0.5^{4.5} \rightarrow 0.5 \] \( (12 \rightarrow 6 \rightarrow 3 \rightarrow 1.5 \rightarrow 0.75 \ldots \) in successive factors of two
  – thus this result is approximately \( 4.5\sigma \) in significance
A more sophisticated approach

• Many intricacies brushed under the rug:
  – outgoing beam profile (not tophat)
  – theoretical corner cube diffraction pattern
  – manufacturing tolerance of corner cubes
  – shadowing of recessed cubes in palette
  – velocity aberration
  – thermal degradation of cubes in sunlight
• A second stage of analysis treats these deficient
Outgoing Beam Profile

- Confidence in our seeing-limited outgoing beam comes from:
  - shear plate on collimated beam allows tuning of divergence at a level corresponding to 0.04 arcsec outside the telescope
  - corner cubes at telescope exit aperture can test divergence: see no divergence at < 0.5 arcsec level of confidence
  - rastering the transmit/receive offset while keeping the receiver fixed (i.e., slewing beam on moon with receiver fixed) has signal disappearing if we move the beam by more than about one arcsecond

- Should we really use 0.8 arcseconds?
  - Our CCD measures seeing consistent with other instruments on the telescope (thus APOLLO optics are not bad)
  - In good seeing, we see starlight concentrated on central 4 pixels of APD array (2×2 box is 0.7 arcsec on a side)
  - The median seeing for this telescope is 1.1 arcsec
    - thus best APOLLO performance likely better than this
  - The 0.5 photon per pulse results were obtained in very good seeing
Walking the beam toward optimal signal gives idea of beam profile
In this case, less than or about 1 arcsec FWHM fits reasonably well
Correction for Gaussian Beam

- The simple form of the link equation assumes a “tophat” intensity distribution.
- Gaussian distribution with same FWHM (full-width at half-max) as tophat has central intensity $0.69 \times \ln(2)$ times that of tophat with same total intensity.
Proper treatment of C.C. diffraction

- The diffraction pattern from an uncoated total internal reflection (TIR) corner cube is far from the tophat pattern used in the link equation.
- The core follows the full-diameter Airy pattern.
- But the wings contain significant power.
- Peak is about 0.25 of perfect Airy pattern.
- 36% of energy inside first Airy ring (84% for perfect Airy).

Images courtesy David Arnold.
Airy vs. TIR

![Graph comparing Airy and TIR patterns with different cuts.](image-url)
Central Irradiance Compared to Tophat

• Compared to a tophat with diameter \( \lambda/D \), what is the central irradiance of an uncoated (total internal reflection) corner cube?

• Relative to perfect Airy pattern, central irradiance is:
  – 0.278 if no reflective loss at front surface
  – 0.248 if uncoated fused silica front surface

• Central Airy irradiance from diameter \( D \) is reduced from \( \lambda/D \) tophat by factor of 0.68

• Composite reduction of central irradiance is:
  – 0.182 if no reflective loss at front surface
  – 0.169 if uncoated fused silica front surface

• Recipe: treat return as tophat (\( \Phi=2.89 \) arcsec at 532 nm) de-rated by 0.182
  – will apply 0.93 reflection loss separately
Velocity Aberration and Recessed Cubes

- Velocity of lunar orbit is about 1000 m/s
- Earth rotation is 400 m/s
- Typical velocity offset is 600–900 m/s
  - $2v/c \rightarrow 4–6 \mu R = 0.8–1.2$ arcsec
- Results in de-rating irradiance:
  - typically factor of $0.64–0.86$ for 532 nm [avg = 0.75]
- Apollo 11 cubes recessed by half-diameter with 1.5° half-angle conical opening (6° for Apollo 14 and 15 arrays)
- Lunar libration (~7° in both longitude and latitude) presents angular offsets as high as 10°
  - typical angle is 6.5°
- Central irradiance down as much as 0.50 (at 10°)
Recessed Cube Influence

All recesses are half-diameter, and throughput is total geometrical flux. Central diffraction irradiance is reduced from this, but not much at first.

data courtesy Jim Williams
Manufacturing Tolerance

• Nominal angular tolerance on Apollo cubes is ±0.3 arcsec
• The cubes that were selected for flight all demonstrated at least 90% the theoretical central irradiance
  – Use factor of 0.93 to account for typical manufacture error
Thermal Impact

• Detailed thermal conductivity/radiation studies predict degradation of central irradiance at a range of sun angles
  – most of effect is thermal gradient of refractive index

FIGURE 14-4. Comparison of calculated thermal performance expected from Apollo 11, 14, and 15 LRRR arrays.
Putting it all together

- Shortfall from normal-incidence central irradiance due to:
  - velocity aberration: 0.64–0.86
  - angular offset: 0.5–1.0
  - thermal degradation: 0.7–1.0 (for Apollo 15)
  - manufacturing tolerance: 0.90–1.0
- amounts to 0.20–0.86
- Now using a tophat with angular diameter $\frac{\lambda}{D}$ ($\Phi=2.89$ arcsec at 532 nm) and associated TIR de-rating of 0.182, together with above detrimental effects, and 0.93 reflection loss from surface, we must de-rate the Apollo performance by a factor of 0.034–0.146
- Equivalent to tophat of 8–15 arcsec of uniform irradiance
Two cases

- Libration angle: 3.94° → 0.84
- Vel. aber.: 1.09 arcsec → 0.71
- Sun: −73° to normal → 0.85
- range: 371425 km
- expect 9.8 ± 4 photons/pulse
- 30 s peak was 0.5 photons/pulse
- ratio: 20

- Libration angle: 4.04° → 0.81
- Vel. aber.: 0.86 arcsec → 0.81
- Sun: +35° to normal → 0.70
- range: 404301 km
- expect 6.4 ± 2.7 photons/pulse
- 30 s peak was 0.5 photons/pulse
- ratio: 13
Scaling to Other LLR Stations

- A quick-and-dirty scaling of APOLLO to MLRS and OCA is interesting: assume similar detector/optical performance
- Use aperture, seeing (or image quality), and pulse energy alone
- MLRS: 
  \[
  \left( \frac{3.5 \text{ m}}{0.76 \text{ m}} \right)^2 \left( \frac{3 \text{ arcsec}}{1 \text{ arcsec}} \right)^2 \left( \frac{100 \text{ mJ}}{E} \right) \approx 180 \left( \frac{100 \text{ mJ}}{E} \right)
  \]

- So if APOLLO gets $1/4$ photons per pulse, MLRS $\rightarrow 1/720$
  - using $E = 100$ mJ
  - if we allow 2 arcsec for a “good” night, this goes to $1/320$
- OCA: 
  \[
  \left( \frac{3.5 \text{ m}}{1.5 \text{ m}} \right)^2 \left( \frac{2 \text{ arcsec}}{1 \text{ arcsec}} \right)^2 \left( \frac{100 \text{ mJ}}{E} \right) \approx 22 \left( \frac{100 \text{ mJ}}{E} \right)
  \]

- So if APOLLO gets $1/4$ photons per pulse, OCA $\rightarrow 1/40$
  - using $E = 200$ mJ
  - if we allow 1 arcsec for a “good” night, this goes to $1/10$
Source of Degradation

- To get a factor of 16 degradation at the array, we need a factor of 4 surface degradation (since light passes through twice)
- **Dust** is a very likely culprit
  - Apollo 17 astronauts saw glow & rays scattering at sunrise (from orbit)
  - Apollo 17 LEAM module saw tremendous dust activity at lunar sunrise/sunset, including horizontal transport
  - LEAM module began to overheat in lunar day: possibly albedo reduction due to dust plus thermal blanketing effect
  - Dynamic dust fountain model (Timothy Stubbs et al.) predicts many-kilometer ballistic lofting of dust due to charging (solar radiation and solar wind)
- **Micrometeorites** and meteoric ejecta can pit surface of glass
  - could have a frosted surface by now