SLR Contribution to the ITRF

- Review of the SLR Contribution to the ITRF
- TRF Implementation: Theoretical Aspects
- Reality of the Current SLR Network
- Reality of the Current Collocation Sites
- Some Analysis of the ILRS Pilot Project Solutions
- Conclusion

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http://lareg.ensg.ign.fr/ITRF/
SLR Contribution to ITRF

- Origin: Center of Mass
- Scale: Together with VLBI
- Unconstrained Solutions for ITRF2000
ITRF2000: Translation Variations (mm)
ITRF2000: Translation Variations (mm)

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ITRF2000: Scale Variations (ppb = $10^{-9}$)

(1 ppb $\approx$ 6 mm shift in station heights)
TRS & TRF in Space Geodesy

**TRS**: Mathematical model of the physical world:  
No physical existence  
Observations provide ”Network Shape”

- **TRF & Space Geodesy techniques:**
  - **Origin**: Dynamical techniques provide the CoM  
  - **Scale**: Same for all techniques ?  
  - **Orientation**: Unobservable by any technique

- Specific constraints are needed to complete the TRF datum definition, leaving the shape undistorted

- **Rank Deficiency** in terms of Normal Equation System

- Separate the variance (noise) of the observations (having a stochastic character) & the (deterministic) frame parameters: **Use of minimum constraints**
Datum Definition: ITRF combination

\[ X_2 = X_1 + T + DX_1 + R.X_1 \]  \hspace{1cm} (1)

Assuming constant velocities and differentiating Eq. (1) w.r.t time:

\[ \dot{X}_2 = \dot{X}_1 + \dot{T} + D\dot{X}_1 + \dot{DX}_1 + R\dot{X}_1 + \dot{R}.X_1 \]  \hspace{1cm} (2)

\[
\dot{T} = \begin{pmatrix}
\dot{T}_1 \\
\dot{T}_2 \\
\dot{T}_3
\end{pmatrix}
\]
\[
\dot{R} = \begin{pmatrix}
0 & -\dot{R}_3 & \dot{R}_2 \\
\dot{R}_3 & 0 & -\dot{R}_1 \\
-\dot{R}_2 & \dot{R}_1 & 0
\end{pmatrix}
\]

=> 14 degrees of freedom to define a TRF.
Datum Definition / Minimum Constraints (1/4)

Application of Minimum Constraints (MC) approach based on theoretical works by many authors, since the 70’s on, e.g.:

- Free Network Adjustment
- S-transformation
- Minimum/Inner Constraints

Main Goal: The ”best” TRF datum definition preserving both the actual quality of space geodesy observations and the ”Network Shape”
Datum Definition / Minimum Constraints (2/4)

The starting point is the standard relation between two TRF’s:

\[ X_2 = X_1 + A\theta \]  \hspace{1cm} (3)

\[ \theta = (T_1, T_2, T_3, D, R_1, R_2, R_3, \dot{T}_1, \dot{T}_2, \dot{T}_3, \dot{D}, \dot{R}_1, \dot{R}_2, \dot{R}_3)^T \]

\[ A = \begin{pmatrix}
\dot{x}_i & 0 & \dot{z}_i & -\dot{y}_i & 1 & 0 & 0 & \dot{x}_i & 0 & \dot{z}_i & -\dot{y}_i & \approx 0 \\
0 & 1 & 0 & -\dot{z}_i & 0 & \dot{x}_i & 0 & \approx 0 \\
0 & 0 & 1 & \dot{y}_i & -\dot{x}_i & 0 & 0 & \approx 0 \\
0 & 0 & 1 & \dot{z}_i & \dot{y}_i & -\dot{x}_i & 0 & \approx 0 \\
\end{pmatrix} \]
Datum Definition / Minimum Constraints (3/4)

\[ \theta = (A^T A)^{-1} A^T (X_2 - X_1) \]

L.S. of eq. (3) yields:

Using \( B = (A^T A)^{-1} A^T \), containing all the necessary info. to define a TRF, a ”datum definition” equation at \( \Sigma_\theta \) level could be written as:

\[ B(X_2 - X_1) = 0 \] \( (\Sigma_\theta) \) \( (4) \)

and in terms of normal equation:

\[ B^T \Sigma^{-1}_\theta B(X_2 - X_1) = 0 \]
The initial NEQ system of space geodesy observations could be written as:

\[ N_{unc}(\Delta X) = K \] (5)

where \( \Delta X = X - X_{apr} \) (Linearized Unknowns)

Selecting a Reference TRF \((X_R)\), MC equation is:

\[ B^T \Sigma_\theta^{-1} B(\Delta X) = B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \] (6)

Cumulating (5) and (6) yields:

\[ (N_{unc} + B^T \Sigma_\theta^{-1} B)(\Delta X) = K + B^T \Sigma_\theta^{-1} B(X_R - X_{apr}) \]
TRF + EOP Simultaneous Combination

CATREF Software upgraded:

- inclusion of EOP’s
- implementation of minimum constraint equations

Some Analysis Tests follow using:

- **SLR**: ILRS Pilot Project Monthly Solutions

- **GPS**: IGS weekly combined solutions

- **VLBI**: GSFC session sinex files

- **DORIS**: IGN monthly solutions

Focus on Origin, Scale, EOP consistency

\[
\begin{align*}
x_s^p &= x^p + R2_k \\
y_s^p &= y^p + R1_k \\
UT_s &= UT - \frac{1}{f} R3_k \\
\dot{x}_s^p &= \dot{x}^p + \dot{R}2_k \\
\dot{y}_s^p &= \dot{y}^p + \dot{R}1_k \\
LOD_s &= LOD + \frac{\Lambda_0}{f} \dot{R}3_k 
\end{align*}
\]
ILRS Origin and Scale Consistency

Data: Monthly solutions over 1-3 years: A-type Solutions

Chart showing TX, TZ, TY, and Scale (mm) data for ASI, CSR, DGFI, and JCET from 1999 to 2001.
ILRS EOP Consistency: One year solutions (1999) X & Y Pole Residuals
ILRS EOP Consistency: One month solutions:
(APR 2001) X-Pole Residuals
ILRS EOP Consistency: One month solutions:
(APR 2001) Y-Pole Residuals

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<td>jceB3v3 YPO</td>
<td>jceB4v3 YPO</td>
<td>nerB4v3 YPO</td>
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ILRS EOP Consistency: One month solutions: X-Pole, Y-Pole Rates and LOD Residuals
Current SLR-VLBI Collocations

Current SLR-GPS Collocations

Current SLR-DORIS Collocations

Current VLBI-GPS Collocations

Current VLBI-DORIS Collocations

Current DORIS-GPS Collocations
One year multi-technique combination
X & Y_pole Residuals

ILRS XPO

IGS XPO

VLBI/GSFC XPO

DORIS/IGN XPO

ILRS YPO

IGS YPO

VLBI/GSFC YPO

DORIS/IGN YPO
One year multi-technique combination
X & Y_pole Rate & LOD Residuals
One Month multi-technique combination: X & Y pole Residuals
One Month multi-technique combination: X & Y pole Rate & LOD Residuals

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Conclusion

- ILRS TRF Origin and Scale:
  - Some small Tz and Scale differences exist between AC’s
  - More refinement is needed for the TRF origin and Scale maintenance?
  - SLR and VLBI current networks/collocations are very poor: Scale Comparaison?

- ILRS EOP:
  - Good estimates of X-pole, Y-pole and LOD
  - The rate estimates of X-pole & Y-pole degrade the overall results
  - Good agreement of X-pole, Y-pole and LOD with other techniques