Optimization of the Correlation Range Receiver Parameters in SLR2000

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Ranger Link Equation

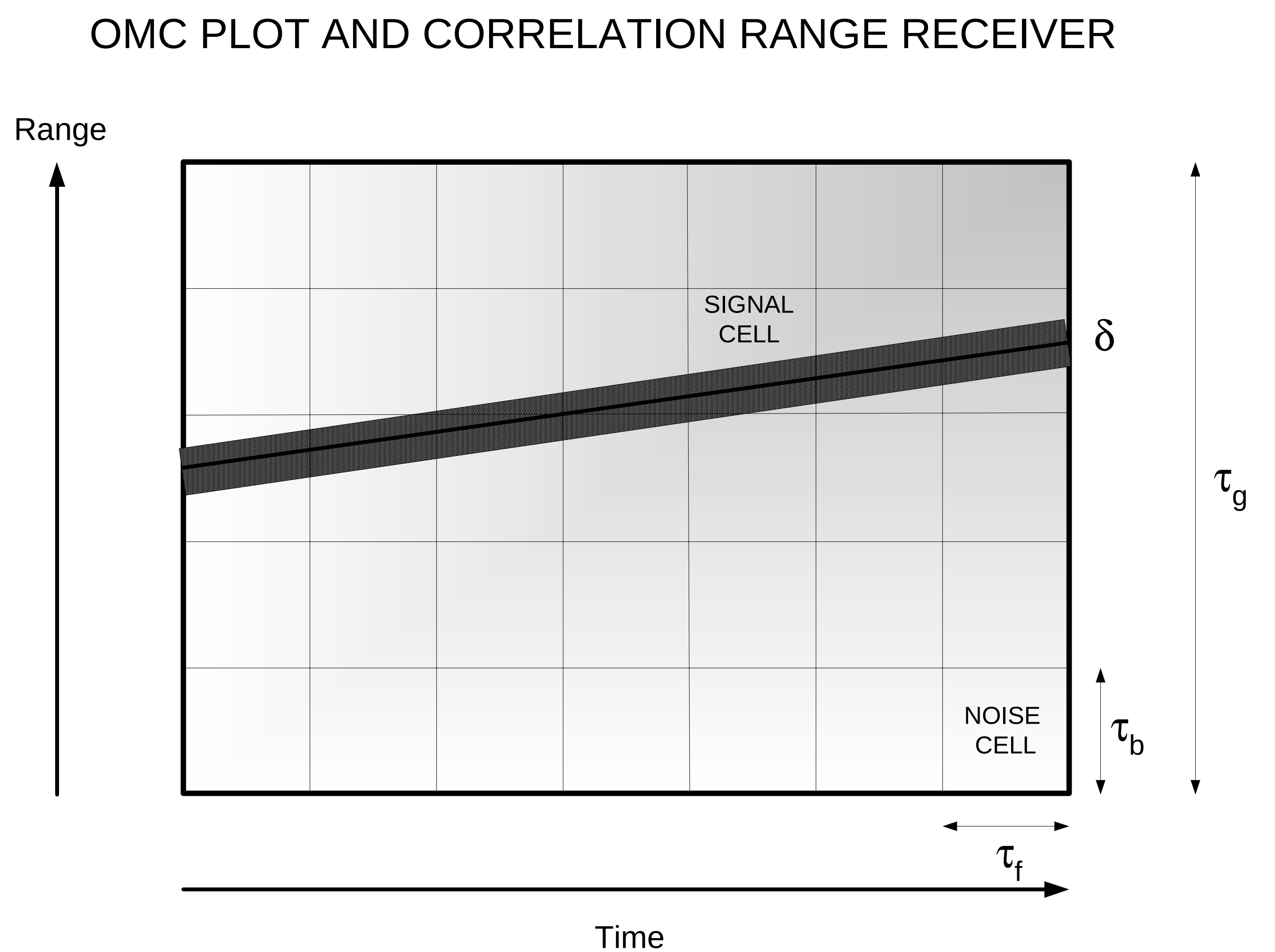
The mean received signal counts are given by the link equation :

$$\langle n_s \rangle = \frac{4\eta_q \sigma_t \eta_r T_0^{2\sec\vartheta_T}}{h\nu\theta_t^2 (4\pi)^2} \frac{E_t A_r}{R^4} \frac{1}{1+\left(\frac{\sigma_p}{\theta_t}\right)^2}$$

E_T= transmitted energy
f_{Qs} = laser fire rate
A_r = receive aperture
R = range to the satellite
σ_t = target array cross-section
$h\nu$ = laser photon energy

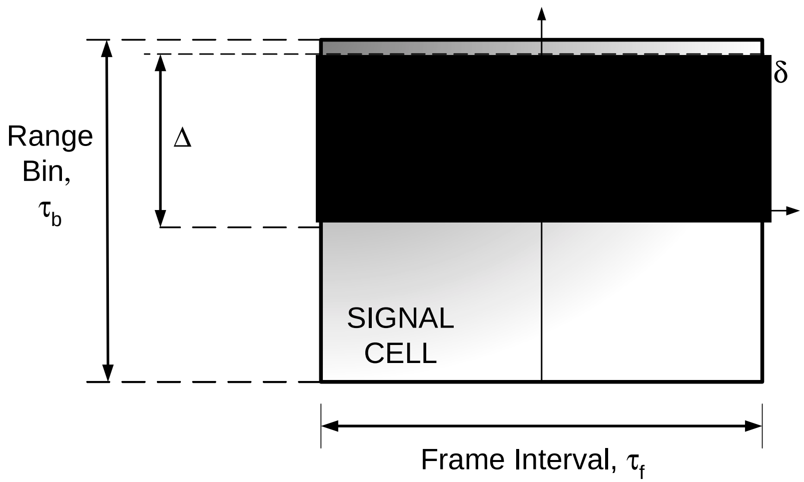
η_q = detector QE
η_r = receiver throughput efficiency
θ_t = transmitter solid angle
T_0 = atmospheric transmission at zenith
θ_T = target zenith angle
σ_p = RMS pointing error

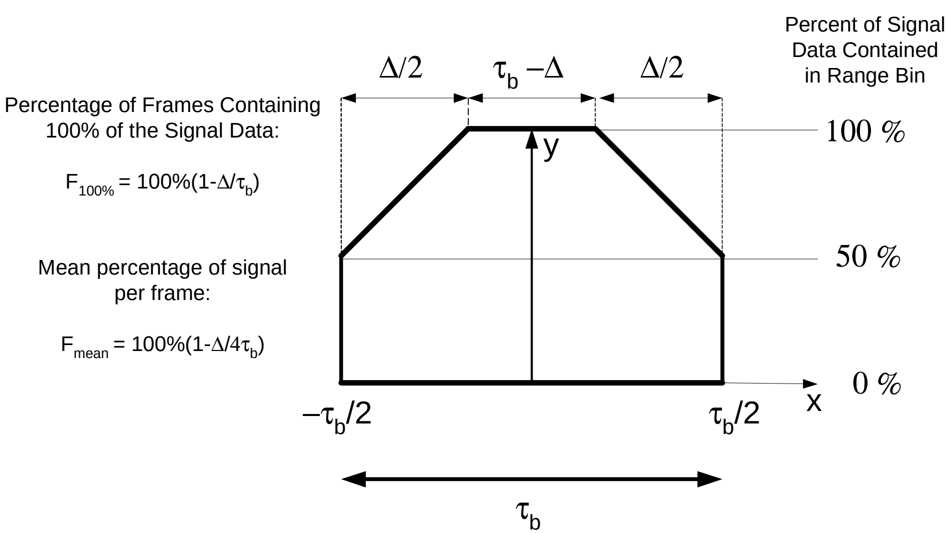
Correlation Range Receiver (CRR) Basics
Satellite Ranging Example

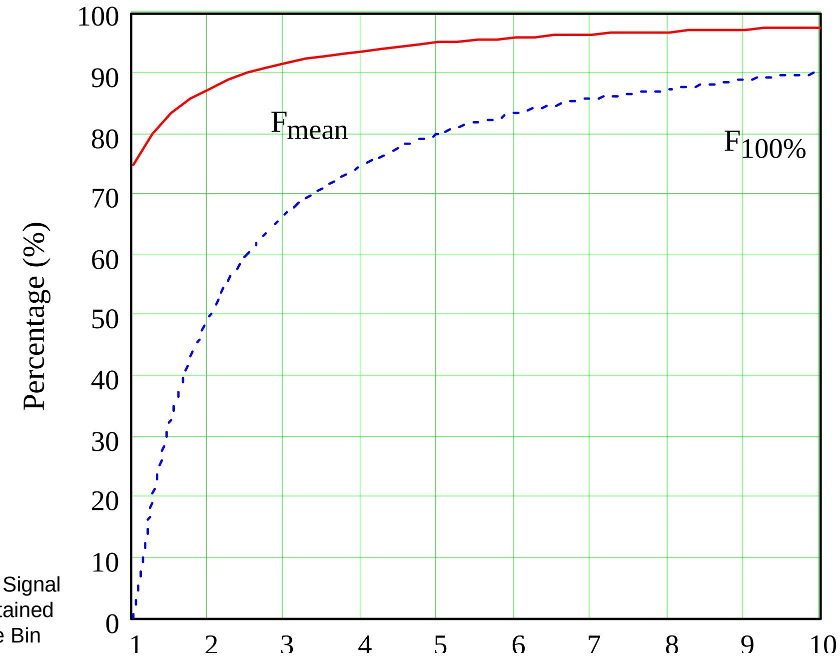


How do we choose the frame interval τ_f and range bin τ_b?

Choosing the Range Bin Size



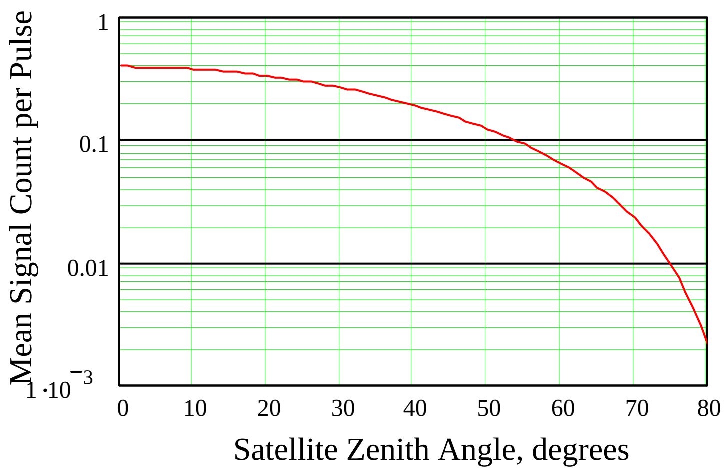




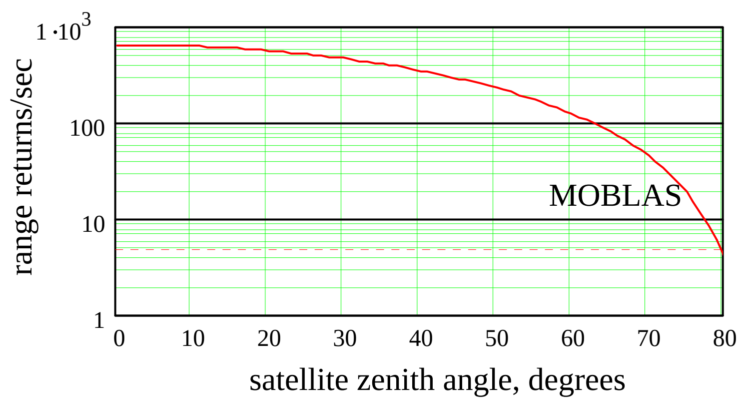
$$\beta = \frac{\tau_b}{\Delta}$$

A value $\underline{2} < \beta \underline{\leq} 4$ is a good compromise between the fraction of mean signal Counts captured (87% to 94%) in a single cell and signal contrast

Mean Signal Count per Pulse and Range Returns/sec for LAGEOS



(a)



(b)

Wavelength, λ	532 nm
LAGEOS Cross-section, σ_t	7×10^6 m^2
Laser Fire Rate, f_{QS}	2 kHz
Transmitted Laser Energy, E_t	133 μJ (eyesafe at exit aperture)
Average Transmitted Laser Power, P_t	266 mW (at exit aperture)
Height of station above sea level, h_s	0 m (worst case)
One-way Atmospheric Transmission at Zenith, T_0	0.7 (Standard Clear, 23 km visibility)
Exo-atmospheric solar irradiance, N_λ	0.2 W/m^2-ster-A^o
Exo-atmospheric lunar Irradiance (Full Moon)	4.8 x 10^{-7} W/m^2-ster-A^o
FWHM Bandwidth, spectral filter, $\Delta\lambda$	3 A^o
Transmitter Divergence Half-Angle, θ_t	25 μrad
RMS Transmitter Pointing Error, σ_p	15 μrad
Receiver FOV Half Angle, θ_r	50 μrad
Telescope Primary Diameter, D_r	40 cm
Atmospheric Scale Height, h_{sc}	1.2 km
Detector Quantum Efficiency, η_q	0.13 (Photek Quadrant MCP/PMT)
Detector Dark Count	50 kHz (Photek Quadrant MCP/PMT)
Receiver Throughput Efficiency, η_r	0.40
Range Gate, τ_g	200 nsec (McGarry et al)
Maximum Orbital Time Bias, t_{bias}	2 msec (McGarry et al)
Maximum Range Acceleration, R_{acc}	10 nsec/sec^2 (McGarry et al)
Maximum Data Slope, σ	0.02 nsec/sec
Range Bin, τ_b	2 nsec
Data Spread, δ	0.4 nsec ($\pm$ 3σ)

Table 1: Parameters used in LAGEOS 1 Link Analyses

PRINCIPAL NOISE SOURCES

Detector Dark Count :

$$\dot{n}_d < 50kHz$$

Solar/Lunar Light Scatter off Local Atmosphere:

$$\dot{n}_{ls} = \frac{\eta_q \eta_r}{h\nu} \frac{N_\lambda (\Delta\lambda) A_r \Omega_r}{4\pi} \left\{ \sec\theta_T T_0^{\sec\theta_T} \left[\frac{1 - T_0^{\sec\theta_S - \sec\theta_T}}{\sec\theta_S - \sec\theta_T} \right] \right\}$$

Laser Backscatter off Atmosphere:

$$\dot{n}_{bs}(\tau) = \frac{\eta_q \eta_r A_r}{2\pi h\nu} \frac{E_t}{h_{sc} c} \left[\ln\left(\frac{1}{T_0}\right) T_0^{2\sec\theta_T \left[1 - \exp\left(-\frac{c\tau}{2h_{sc}\sec\theta_T}\right)\right]} \right] \left[\frac{\exp\left(-\frac{c\tau}{2h_{sc}\sec\theta_T}\right)}{\tau^2} \right]$$

where the time after laser fire is given by

$$\tau = \frac{2s}{c} = \frac{2\sec\theta_T}{c}(z - h_s)$$

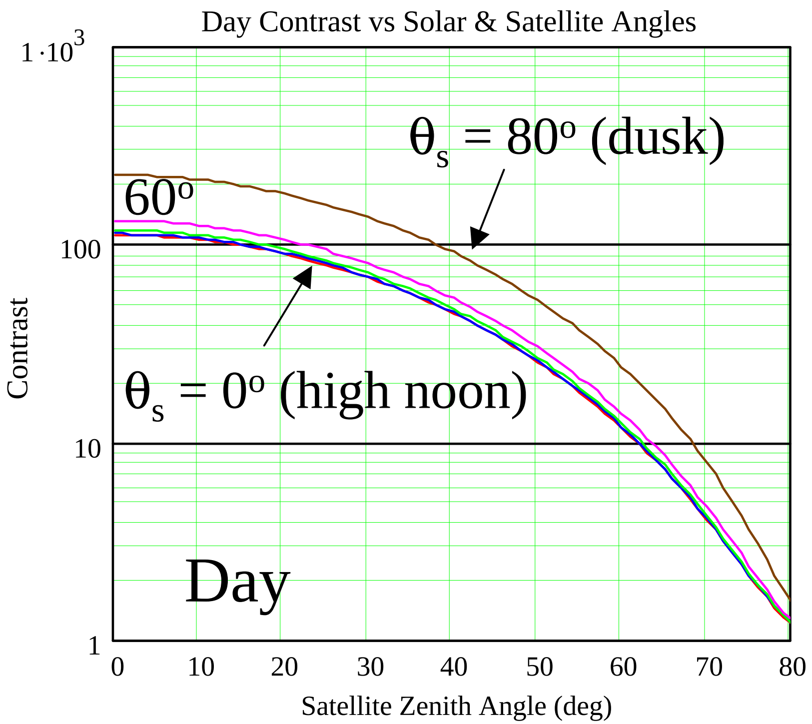
Signal Cell Contrast

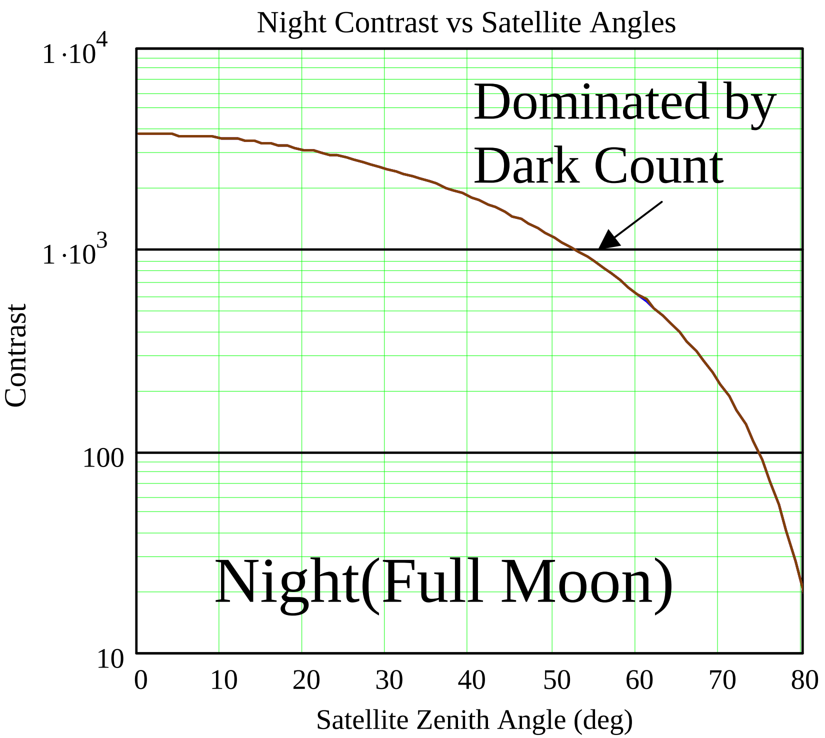
- **The *signal cell contrast* is defined by**

$$C(\tau)=\frac{N_s+N_b(\tau)}{N_b(\tau)}=1+\frac{\langle n_s\rangle F_{mean}}{\tau_b\left[\dot{n}_{ls}+\dot{n}_{bs}(\tau)+\dot{n}_d\right]}=1+\frac{\langle n_s\rangle}{\Delta\left[\dot{n}_{ls}+\dot{n}_{bs}(\tau)+\dot{n}_d\right]}\left[\frac{1}{\beta}\left(1-\frac{1}{4\beta}\right)\right]$$

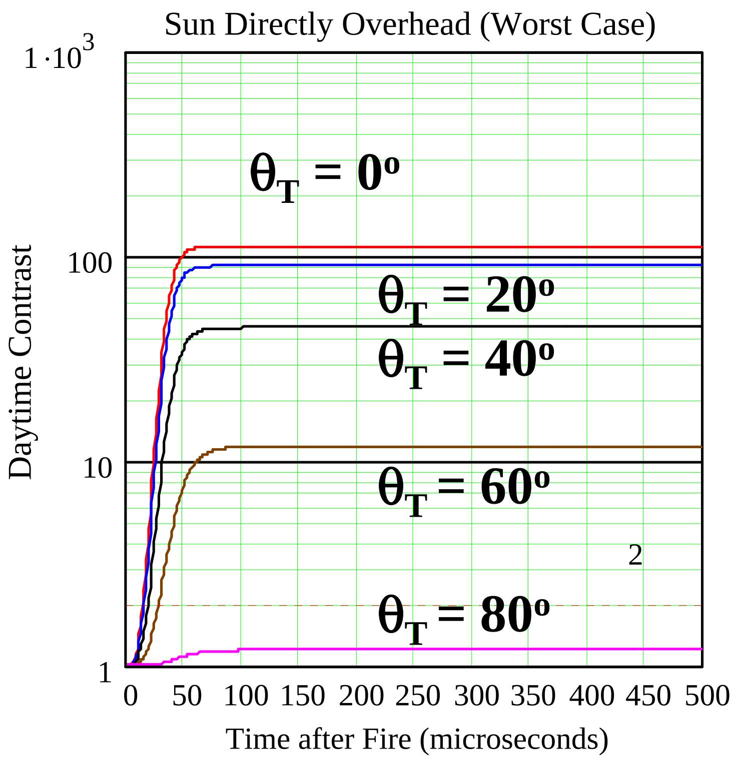
The product $CN_b = N_s + N_b$ gives the mean counts in the signal cell where N_s is the mean signal count in the signal cell and N_b is the mean noise count per cell. For small values of C more signal counts are required per frame (e.g. $N_s > 80$ for $C = 2$) for positive identification of the signal cell and low occurrence of false alarms and implies a longer frame interval. For high values of C, fewer signal counts are required (e.g. $N_s > 25$ for $C = 10$) for good discrimination. High values of C also usually imply a greater signal count rate and therefore much shorter frame intervals are necessary.

Signal Contrast for LAGEOS Satellite vs Solar (θ_s) and Satellite (θ_T) Zenith Angles

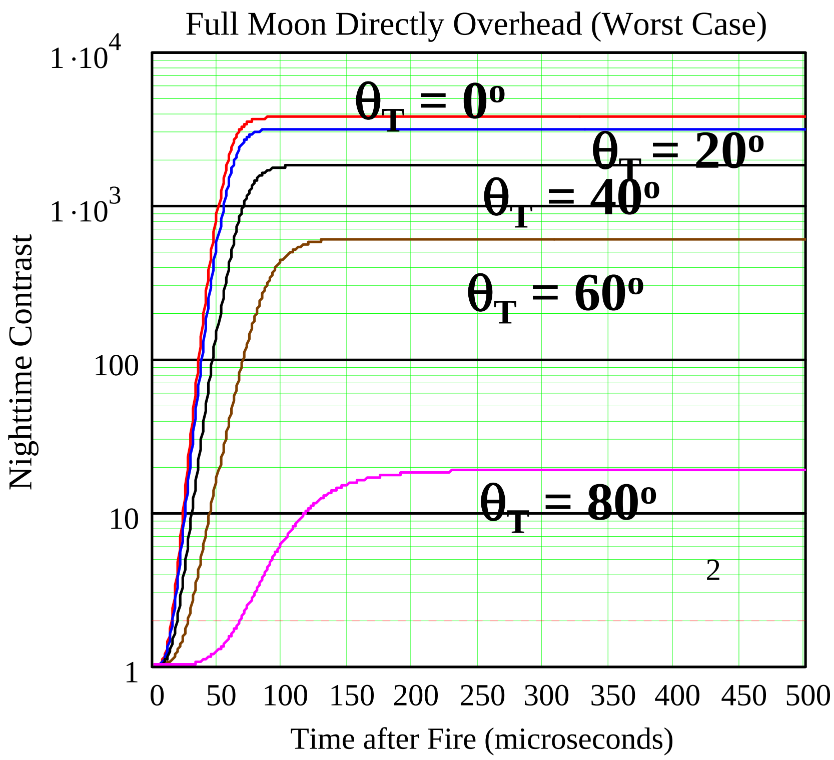




Effect of Laser Backscatter on Signal Contrast as a Function of Time after Laser Fire and Satellite Zenith Angle (θ_T)



HIGH NOON



NIGHT (FULL MOON)

Differential Cell Count

We want to maximize the probability of detecting the signal cell while minimizing the number of noise cells that trigger false alarms so we introduce the *Differential Cell Count (DCC)*

$$\delta N = P_d - N_{bin} P_{fa}$$

***defined as the mean number of correctly identified signal cells in a frame minus the number of incorrectly identified noise cells*, where**

P_d = the probability of correctly detecting the signal cell in a frame
P_{fa} = the probability of falsely identifying a given noise cell as signal
N_{bin} = the number of range bins in the range gate

The DCC has a maximum value of one when P_d = 1 and P_{fa} =0 !

Optimum Frame Threshold

We set the quasi-derivative of δN with respect to the frame threshold, K, equal to zero

$$\frac{\Delta(\delta N)}{\Delta K} = 0 = \delta N(K+1) - \delta N(K) = \frac{N_b^k e^{-N_b}}{K!}\left[N_{bin} - C^K e^{-(C-1)N_b}\right]$$

and solve for the ***optimum threshold, K_{opt}***

$$\boxed{K_{opt} = \frac{N_s + \ln(N_{bin})}{\ln C}}$$

and note that this is a maximum since the second quasi-derivative is less than zero for C>1

$$\frac{\Delta^2(\delta N)}{\Delta K^2} = -\frac{N_b^{K-1} C^K e^{-CN_b}}{(K-1)!}\left(1 - \frac{1}{C}\right) < 0$$

Probability of Signal Cell Detection and False Alarm Rates

Probability of detecting the signal cell:

$$P_d = \exp(-CN_b)\sum_{k=K_{opt}}^{\infty}\frac{(CN_b)^k}{k!}$$

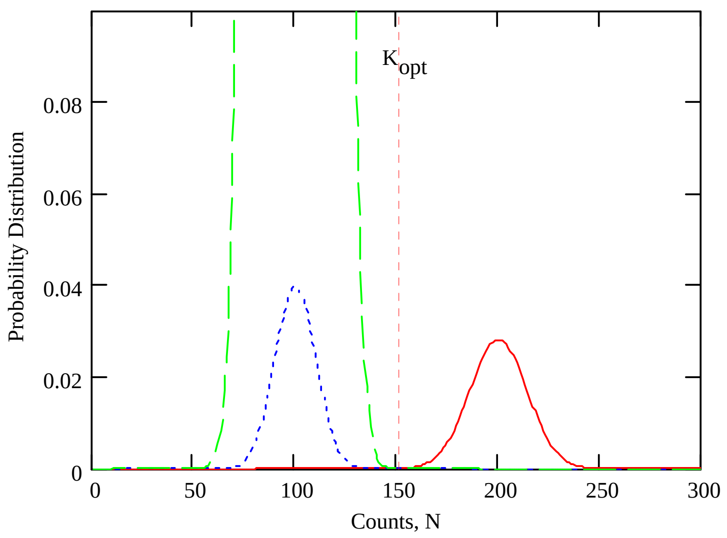
$$\cong \frac{1}{\sqrt{2\pi CN_b}}\int_{K_{opt}}^{\infty} dN \exp\left[-\frac{(N-CN_b)^2}{2CN_b}\right] = \frac{1}{2}\left[1+erf\left(\frac{CN_b-K_{opt}}{\sqrt{2CN_b}}\right)\right]$$

Probability of falsely identifying a given noise cell as signal:

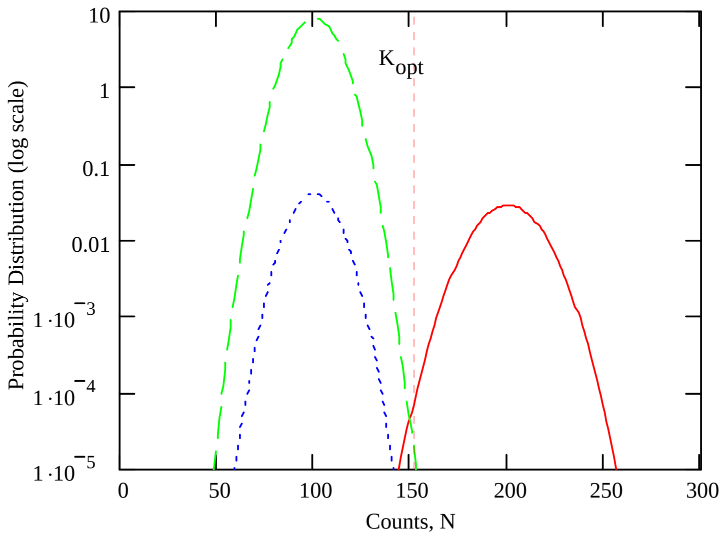
$$P_{fa} = \exp(-N_b)\sum_{k=K_{opt}}^{\infty}\frac{(N_b)^k}{k!} \cong \frac{1}{\sqrt{2\pi N_b}}\int_{K_{opt}}^{\infty} dN \exp\left[-\frac{(N-N_b)^2}{2N_b}\right] = \frac{1}{2}\left[1-erf\left(\frac{K_{opt}-N_b}{\sqrt{2N_b}}\right)\right]$$

Extracting Low Contrast Signals (e.g. $C = 2$, $N_s = N_b = 100$ pe)

Red Solid Curve = count distribution for signal cell (signal plus noise).
Blue Dotted Curve = count distribution for one noise cell (noise only).
Green Dashed Curve = total count distribution for all noise cells in a frame.
The optimum frame threshold, K_{opt}, is the vertical dashed line near the intersection of the red and green curves.



Linear Scale



Log Scale

Summary of Methodology for Choosing Correlation Range Receiver Parameters

- **Choose the frame interval, τ_f, based on the a priori signal contrast, *C*, and range return rate. For low contrast situations, we must collect on the order of 80 or more signal pe's for good discrimination.**
- **Compute the maximum data spread, Δ, for that frame interval using the instrumental data spread, δ, and the maximum expected data slope, σ.**
- **Range bins 2 to 4 times the total data spread per frame are a good compromise between the mean fraction of signal counts captured in a single cell and the reduction in signal contrast. Once the signal slope is taken out via an updated time bias, the mean fraction of signal counts increases**
- **Maximization of the Differential Cell Count is a very powerful algorithm for maximizing the probability of detection for the signal cell while minimizing the number of cell false alarms and leads to a simple formula for computing the optimum frame threshold, K_{opt}.**
- **Correlation Range Receiver parameters (frame interval, range bin, frame threshold) can be updated and optimized in real time using visibility and cloud data from the "smart meteorological station" plus actual noise and signal counts following satellite acquisition.**

LAGEOS Results Summary

- **During the day and with the assumption of a standard clear atmosphere and a 2 nsec range bin, LAGEOS signal contrast is degraded to values below 2 at very low elevation angles (10^o). Longer frame intervals (10 to 20 sec) are therefore necessary to accumulate the required number of signal photoelectrons (~100).**
- **The required frame interval decreases rapidly at higher elevation angles, e.g. 2 sec at 20^o elevation. Higher QE (40%) detectors are now available which can further reduce the frame intervals by factors of 4.**
- **For most solar and LAGEOS zenith angles, the link is expected to be very robust.**
- **The data slope does not have a significant effect on the choice of range bin except at very low elevation angles.**
- **For night operations, the noise is dominated by the detector dark count rate and signal contrasts are high ($C > 50$) for all LAGEOS elevation angles above 10^o.**
- **Laser backscatter in the atmosphere makes an additional noise contribution for up to 200 microseconds at low elevation angles following laser fire. However, the contrast drops below 2 for only 25 to 30 μsec following laser fire resulting in a 5% to 6% loss in data over the 500 μsec laser fire interval. However, the laser repetition rate can be varied slightly to avoid overlap of the outgoing and incoming pulses at the receiver.**