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RECENT CONTRIBUTIONS TO LLR ANALYSIS

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This presentation summarises recent results obtained by the Paris Observatory Lunar Analysis Center (POLAC) using the LLR observations made since 1972 until 2001. The team of the center is working on the subject since 5 years.

The main contributions are described in:
Chapront, J., Chapront-Touzé, M., Francou, G.: 2002, A new determination of lunar orbital parameters, precession constant and tidal acceleration from LLR measurements, Astron. & astrophys., 387, 700-709

Improvements of the LLR analysis

The last lunar solution which has been used for the analysis, on the basis of the semi-analytical theory of the Moon ELP and libration, is called S2001. Several improvements were introduced in S2001 with respect to the prior solutions:

- the libration model with numerical and analytical complements to the theory of Michelle Moons
- the program of reduction with an up-to-date nutation model and a realistic statistical treatment of the data (in particular an adequate distribution of weights among the various observing stations and periods of observations).

In parallel, the quality of the LLR observations has been noticeably improved during the last 15 years.

Table 1 shows the evolution of the residuals obtained for the distance between the LLR stations and the lunar reflectors expressed in centimeters. It gives the time distribution of the root mean square of post-fit residuals obtained with the different instruments:

- the McDonald Observatory 2.7m telescope which ceased operation in 1985,
- the McDonald Laser Ranging stations near Fort Davis, Texas, MLRS1 and MLRS2 (saddle site and Mt. Fowkles site),
- the Haleakala Observatory on Maui, Hawaii, which was operating a few years around 1990,
- the Observatoire de la Côte d'Azur in Grasse (CERGA), France, which changed its instrument in 1987.

We observe the gain in precision from several decimeters to a few centimeters between the earliest observations and the present ones.

Figure 1 put in evidence the evolution in the quality of the observations though the program of analysis (solution S2001). It gives the time evolution of the rms for the data provided by the 2 operational modern instruments, MLRS2 for McDonald and Yag for CERGA, since 1988. The post-fit residuals lie within 2 to 3 centimeters in the lunar distance for the more recent observations.

Figures 2 give an illustration of the distribution of the LLR data (i.e. normal points) during the time interval (1987-2001).

Figure 2a shows the time distribution of normal points for the 2 operating stations, MLRS2 (in dark) and CERGA (in light); the maximum of annual data is about 1000 observations in 2000, the average being around 670 per year.

Figure 2b shows that the reflector Apollo-15 contributes for the major part of the observations (80%) compared with the other reflectors Apollo-11, Apollo-14 and Lunakhod-2.

Figure 2c gives an example of the distribution of data with respect to the age of Moon (CERGA in 2000); the observations are more dense around the first and last quarters; note that the observations during the full moon (age 15) have been made during a lunar eclipse.

Determination of UT_0-UTC

Analysing the total set of LLR data between 1987 and 2001, we have estimated the earth orientation parameters UT_0-UTC and VOL (variation Of latitude).

Table 2 summarises the last determinations of UT_0-UTC and VOL (Variation Of Latitude).
With a total of 10079 normal points obtained at McDonald and Grasse between 1987 and 2001, we cover in fact 2774 nights of observation which represent an average of 3-4 observations per night/reflector. We have divided the nights where several reflectors are involved. Only 28% of the night/reflector have been retained (790) because we have disregarded the values obtained with less than 4 observations per station-reflector and those with just 4 observations covering a time span shorter than 1.5 hour. Hence, we have estimated 790 pairs of values for UT_0-UTC and VOL included in the whole time interval (1987-2001).

The value of UT_1-UTC can be deduced from UT_0-UTC by the relation:

$$UT_1\text{-UTC} = UT_0\text{-UTC} - (x \sin \lambda + y \cos \lambda) \tan \varphi / 1.002737909$$

where x and y are the pole coordinates and, λ and φ are the longitude and latitude of the station.

Figure 3 shows the comparison between the values of UT_1-UTC deduced from the values of UT_0-UTC obtained by LLR analysis using the IERS parameters (x,y) and the values of UT_1-UTC given by IERS (C04 series), for MLRS2 in red circles and for CERGA in blue diamonds. The root mean square residual obtained from this comparison with EOP(IERS) C04 is less than 1.4 ms.

Numerical experiences show that an annual fitting of the lunar solution (S2001 in the present case) is sufficient to maintain an accuracy better than 0.2 ms for UT0-UTC and 3 mas for VOL. Of course, the determination of UT_1-UTC obtained by VLBI observations are generally 10 times better. However, we note that sometimes the LLR determination of UT_0-UTC are still used by the EOP Product Centre of IERS, when the VLBI data are missing, for monitoring the evolution of UT_1-UTC series.

Relative positions of the mean inertial ecliptic

The lunar solution is referred to a dynamical frame and introduces the inertial mean ecliptic of J2000.0. Among the fitted parameters in LLR analysis there is the position of this plane with respect to one of the following 'equatorial frames' (R):

- ICRS, International Celestial Reference System,
- MCEP, frame linked to the Mean Celestial Ephemeris Pole (CEP),
- JPL, reference frames defined by a JPL numerical integration such as DE200, DE403 or DE405.

The position angles are decribed in Figure 4 with:

- γ_{2000}(R) the ascending nodes of inertial mean ecliptic J2000.0 on the equators of (R), i.e. the inertial dynamical equinox,
- ϵ(R) the inclination of the inertial mean ecliptic to the equator of (R),
- o(R) the origin of right ascensions on the equator of (R),
- ϕ(R) the arc between o(R) and γ_{2000}(R) on the equator of (R),
- ψ(R) the arc between γ_{2000}(ICRS) and γ_{2000}(R) on the mean ecliptic of J2000.0.

Table 3a gives the values of the different angles. Concerning the ICRS and MCEP systems two solutions have been investigated:

- S2001(ICRS) where the precession-nutation matrix is computed via the conventional set of values provided by IERS, in particular the nutation corrections $d\psi$ and $d\epsilon$ of the series EOP(C04),
- S2001(MCEP) where the precession-nutation is represented by analytical solutions: polynomial expression of the precession and theory of nutation.

In accordance with the two solutions, one gets the offsets of the Celestial Ephemeris Pole (CEP) with respect to ICRS. Table 3b is a comparison of the offsets computed with LLR measurements with those obtained with VLBI measurements.

Correction to the IAU76 precession constant

In the solution S2001(MCEP), which is linked to the Celestial Ephemeris Pole (CEP), the precession constant has been fit and the correction $\Delta_1 p$ to the IAU76 constant is:

$$\Delta_1 p = -0.3364 \pm 0.0027 \text{ ''/cy.}$$

We observed that this correction was noticeably divergent from the values of this correction obtained in our previous solution such as S1998 or S2000.

Figure 5 illustrates the evolution of $\Delta_1 p$ with the variation of the upper limit of the time span covered by the fit. In other words, the graph represents the different values of $\Delta_1 p$ for intermediate solutions in which the characteristics of the fit are the same as in S2001(MCEP), except the time interval covered by the LLR observations that we have successively limited to equidistant dates between 1996 and 2001.
The 'accidental jump' that we observe for 1997 corresponds to an offset of 0.7 nanosecond in the CERGA measurements which has been partly corrected in our analysis.
In fact, the relevance of this graph is to put in evidence a residual which also arises when we use the solution S2001(ICRS) linked to ICRS:

$$\Delta_2 p = -0.0316 \pm 0.0027 \text{ ''/cy.}$$

We note that the difference:

$$\Delta_1 p - \Delta_2 p = -0.3048\text{''/cy}$$

is almost constant on the whole time interval [1996-2001].

S2001(MCEP) and S2001(ICRS) use the same observations (the main source of errors) and the same models, except for the motion of the reference frame due to precession and nutation. The series EOP(C04) $\delta\psi$ and $\delta\epsilon$ used in S2001(ICRS) are based on VLBI observations. If we assume that these series contribute ideally to the precession-nutation matrix, the errors taken into account in $\Delta_2 p$ exist also in $\Delta_1 p$, such as the effects of an improper motion of the stations, or local bias produced by the observations themselves.

Hence, the difference Δp between the corrections $\Delta_1 p$ and $\Delta_2 p$ gives an better estimate of the corrections to the IAU76 precession constant than $\Delta_1 p$. Over the interval [1996-2001], Δp remains constant around the value:

$$\Delta p = -0.302 \pm 0.003 \text{ ''/cy.}$$

Table 4 shows that now the value of the correction to the IAU76 precession constant Δp obtained by LLR is very close to recent determinations obtained by VLBI: value presented in the IAU General Assembly in 2000 (Fukushima) and value introduced in the theory of nutation MHB2000 (Herring et al.). LLR and VLBI determinations converge nicely with a separation smaller than 0.03 mas/year.

Tidal acceleration of the Moon

Among the various parameters of the analysis, we concentrate ourselves on the tidal acceleration of the Moon which is a fundamental parameter in the evolution of the Earth-Moon system.

The expression of the lunar mean longitude of the Moon W_1 has the following secular expansion:

$$W_1 = W_1^{(0)} + W_1^{(1)}\, t + W_1^{(2)}\, t^2 \ldots,$$

where t is the time in century reckoned from J2000.0. $W_1^{(0)}$ is the constant term (mean longitude at J2000.0), $W_1^{(1)}$ is the sidereal mean motion and $W_1^{(2)}$ is the total half-secular acceleration of the Moon.

Figure 6 shows the evolution of the correction to the 3 components of W_1 with the variation of the upper limit of the time span covered by the fit (with the same division in 'slice of time' as we did previously for the correction to the precession constant). We can observe in particular the evolution of the acceleration. The convergence is mainly explained by the evolution of the fitted value of the tidal part of the acceleration when using more and more recent and precise LLR observations.

The tidal component of the secular acceleration of the Moon's longitude is a fundamental parameter which expresses the dissipation of energy in the Earth-Moon system. It is due to a misalignment of the bulge of the Earth relative to the Earth-Moon direction, which exerts a secular torque, and most of the effect comes from the ocean tides.
It produces a secular negative acceleration in the lunar longitude of approximately -25.8"/cy^2 and correspondingly a decrease in the Earth's rotation rate (or an increase of the length of day). Another consequence is the displacement of the Moon that corresponds to an increase of the Earth-Moon distance of 3.8 cm/year.

Table 5 a gives a list of determination of the tidal secular acceleration of the lunar longitude since 1939 (Spencer Jones) and provided by several types of observations: occultations, eclipses and LLR. The most recent values have been obtained with LLR observations. We note for this type of determination a significant improvement of the precision when increasing the number of observations and their accuracy.

We illustrate also in Table 5 the intrinsic values of the tidal acceleration in various JPL numerical integrations. The difference between the last JPL value (DE405) and our determination in S2001 gives an idea of the present uncertainty and allows to ensure nowadays a realistic precision of better than 0.03"/ cy^2 in the knowledge of this parameter.

In conclusion, we remind that the complete set of LLR observations covers a time interval longer than 30 years and during the last 15 years the precision in the measurements has been improved noticeably. Hence, the quality in the determination of several parameters has been also improved, such as the precession constant and the position angles of the inertial mean ecliptic. It is also the case for various orbital parameters of the Moon and the secular tidal acceleration in the Moon's longitude.

A analytical theory of the lunar motion including new planetary perturbations will replace in 2002 the above solution. In this version the constants will be fitted to LLR data taking into account the results presented here.

Table 1: LLR RESISUALS
TIME DISTRIBUTION OF THE POST-FIT RESIDUALS (RMS)

OBSERVATORY *and instrument*	Time Interval	S2001 rms	N
McDONALD	1972-1975	43.5	1487
Telescope 2.70 m	1976-1979	27.7	1035
and MLRS1	1980-1986	29.1	990
CERGA *Rubis*	1984-1986	18.7	1165
HALEAKALA	1987-1990	6.3	451
McDONALD	1987-1991	5.8	232
MLRS1	1991-1995	4.6	586
and MLRS2	1995-2001	3.3	1669
CERGA	1987-1991	5.3	1574
Yag	1991-1995	3.9	2044
	1995-2001	3.0	3273

N is the number of LLR normal points involved

Figure 1: TIME EVOLUTION OF THE QUALITY OF THE LLR OBSERVATIONS

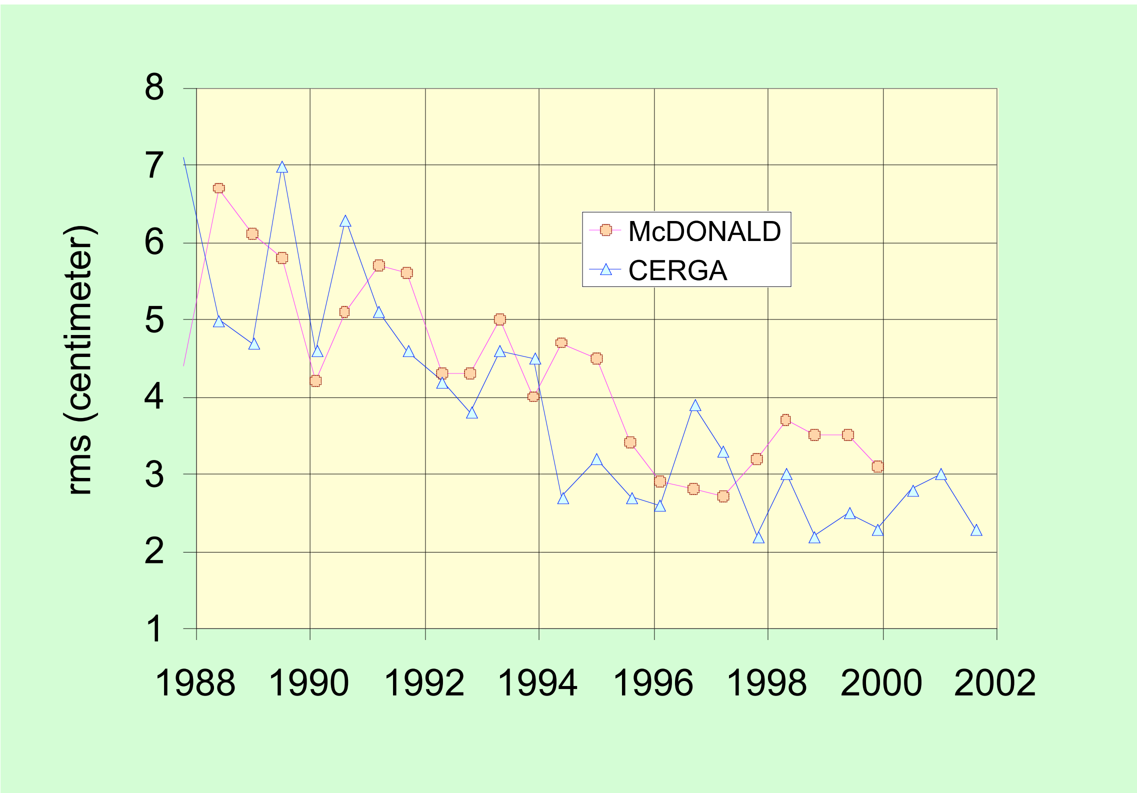


Figure 2a: NORMAL POINTS / STATION / YEAR 1987-2001

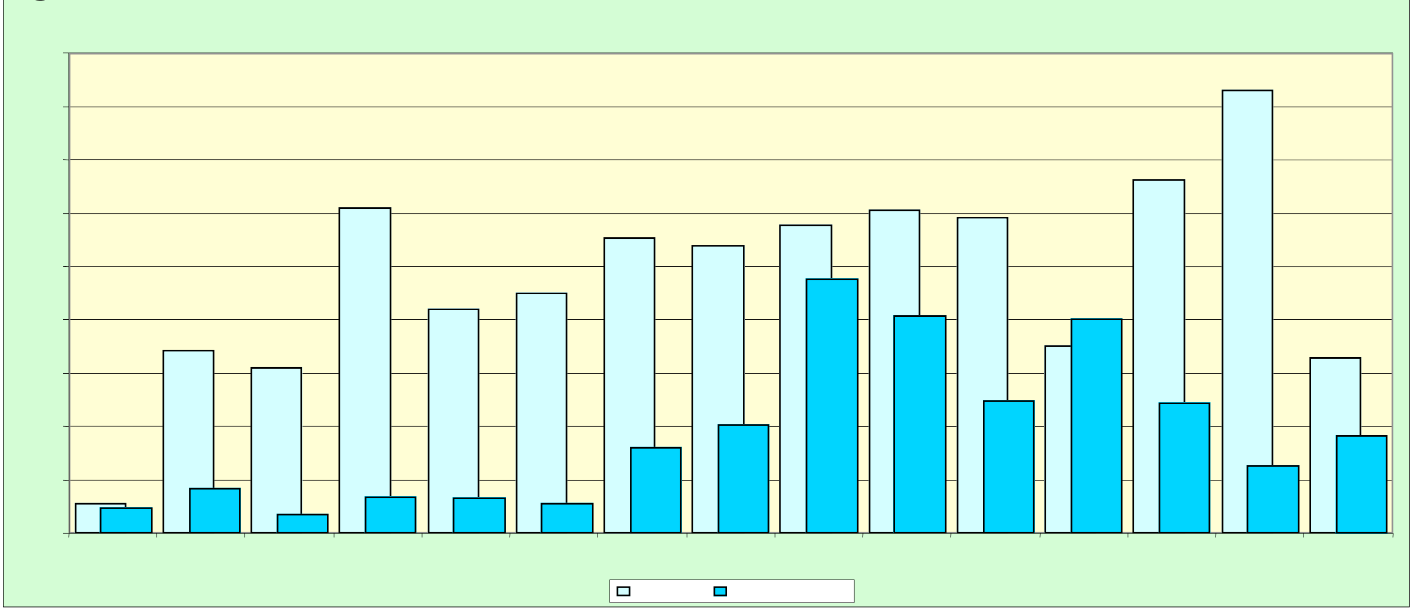


Figure 2b: LLR OBSERVATIONS / REFLECTOR198

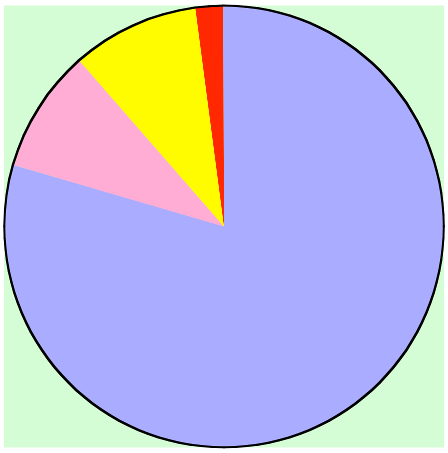


Figure 2c: Normal Points with the age of the Moon (200

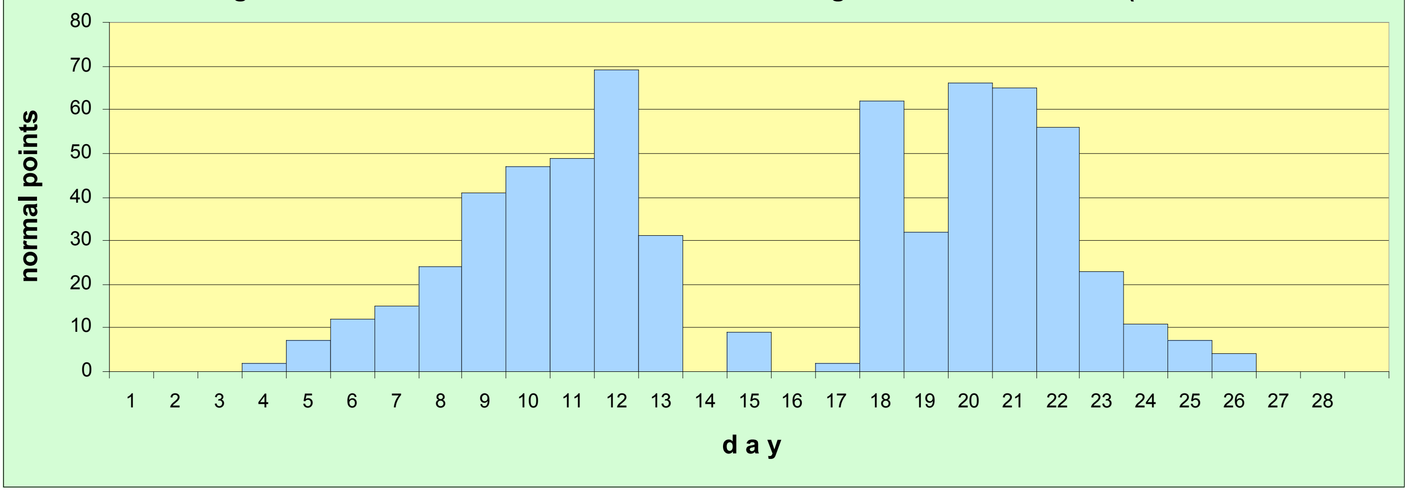


Table 2: DETERMINATION OF UT_0-UTC & VOL
1987 - 2001

Number of:	CERGA	MLRS2	Total
Normal points	7249	2830	10079
Night/reflector	1587	1187	2774
UT_0-UTC & VOL	571	219	790

$$UT_1\text{-}UTC = UT_0\text{-}UTC - (x \sin \lambda + y \cos \lambda) \tan \varphi / 1.002737909$$

x and y : pole coordinates

λ and φ : longitude and latitude of the station

Figure 3: COMPARISON OF UT1-UTC (LLR) WITH UT1-UTC (EOP C04)

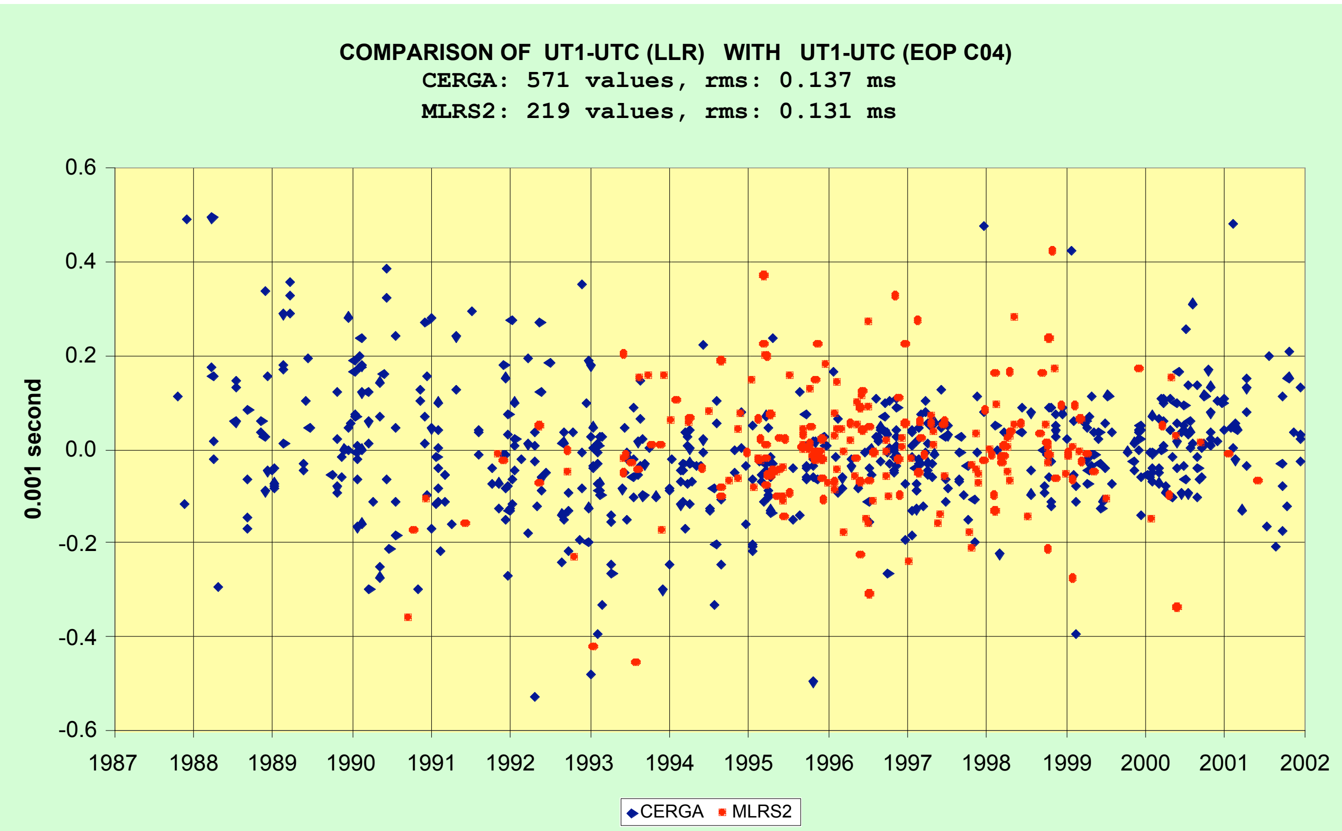
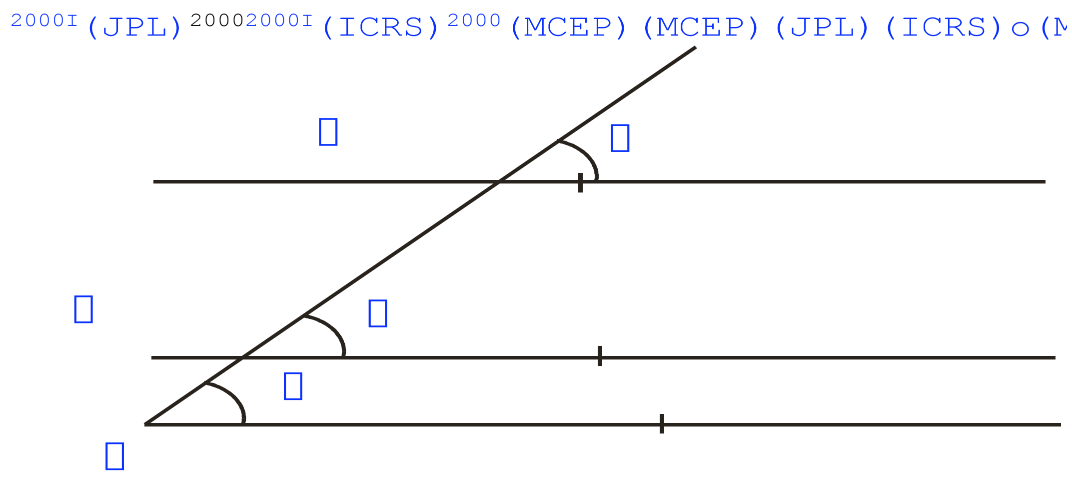


Figure 4: POSITION ANGLES OF THE INERTIAL MEAN ECLIPTIC OF J2000.0



R	Equatorial reference frame (ICRS, MCEP, JPL)
γ_{2000}(R)	Ascending node of inertial mean ecliptic J2000.0 on the equator of (R)
o(R)	Origin of right ascensions on the equator of (R)
ϵ(R)	Inclination of the inertial mean ecliptic to the equator of (R)
ϕ(R)	Arc between o(R) and γ_{2000}(R) on the equator of (R)
E(R)	Arc between γ_{2000}(ICRS) and γ_{2000}(R) on the mean ecliptic of J2000.0

Table 3a: RELATIVE POSITIONS OF THE MEAN ECLIPTIC OF J2000.0 WITH RESPECT TO ICRS, MCEP AND JPL REFERENCE FRAME

R	ϵ − 23°26'21"	ϕ	E	Mean Epoch
ICRS	**0.41100** ± 0.00005	**-0.05542** ± 0.00011		Dec.1994
MCEP	**0.40564** ± 0.00009	**-0.01460** ± 0.00015	**0.0445** ± 0.0003	Dec.1994
DE403	**0.40928** ± 0.00000	**-0.05294** ± 0.00001	**0.0048** ± 0.0004	Jan.1985
DE405	**0.40960** ± 0.00001	**-0.05028** ± 0.00001	**0.0064** ± 0.0003	Jan.1990

Table 3b: OFFSETS OF CELESTIAL EPHEMERIS POLE AT J2000.0

Method	Source	$\delta\epsilon$	$\delta\psi$ sinϵ
LLR	POLAC 2001	**-0.0054** ± 0.0002	**-0.0177** ± 0.0004
VLBI	IAU 2000	**-0.0049** ± 0.0003	**-0.0167** ± 0.0005

Unit : arcsecond
The uncertainties are formal errors

Figure 5:
EVOLUTION OF THE CORRECTION TO THE IAU76 PRECESSION CONSTANT WITH THE TIME SPAN COVERED BY THE FIT
Δ_1P for S2001(MCEP) and Δ_2P for S2001(ICRS)

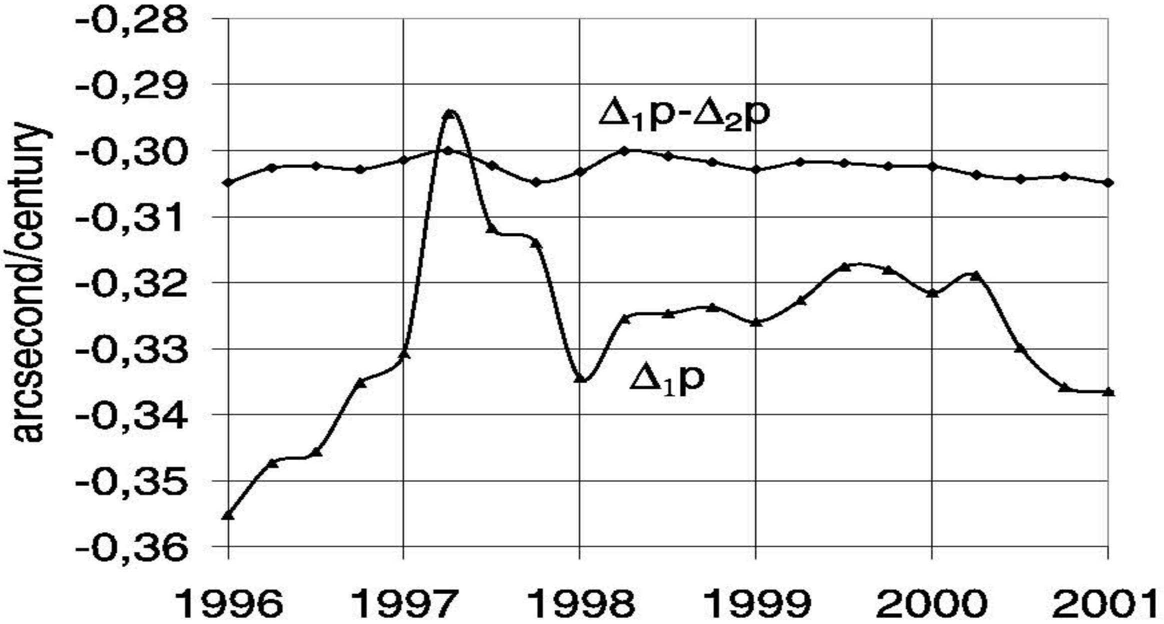


Table 4: CORRECTION TO THE IAU 1976 PRECESSION CONSTANT Δp

Method	Source	Year	Δp
LLR	POLAC	1998	**-0.344** ± 0.004
LLR	POLAC	2000	**-0.316** ± 0.003
LLR	POLAC	2001	**-0.302** ± 0.003
VLBI	IAU	2000	**-0.297** ± 0.004
VLBI	MHB2000	2002	**-0.2997** ± 0.0008

unit : arcsecond/cy; the uncertainties are formal errors.

POLAC : Chapront J. et al., Astronomy & Astrophysics, 387, 700 (2002)

IAU : Fukushima T., Report on Astronomical Constants, IAU General Assembly, Manchester, (2000)

MHB2000 : Herring T.A., Mathews P.M., Buffet B.A., Modeling of nutation-precession: very long baseline interferometry results Journal of Geophyscal Research, vol 107 (2002)

Figure 6:
TIME EVOLUTION OF THE CORRECTIONS □
TO THE SECULAR COMPONENTS OF THE LONGITUDE OF THE MOON
$$W_1 = W_1^{(0)} + W_1^{(1)}\, t + W_1^{(2)}\, t^2 + \ldots$$

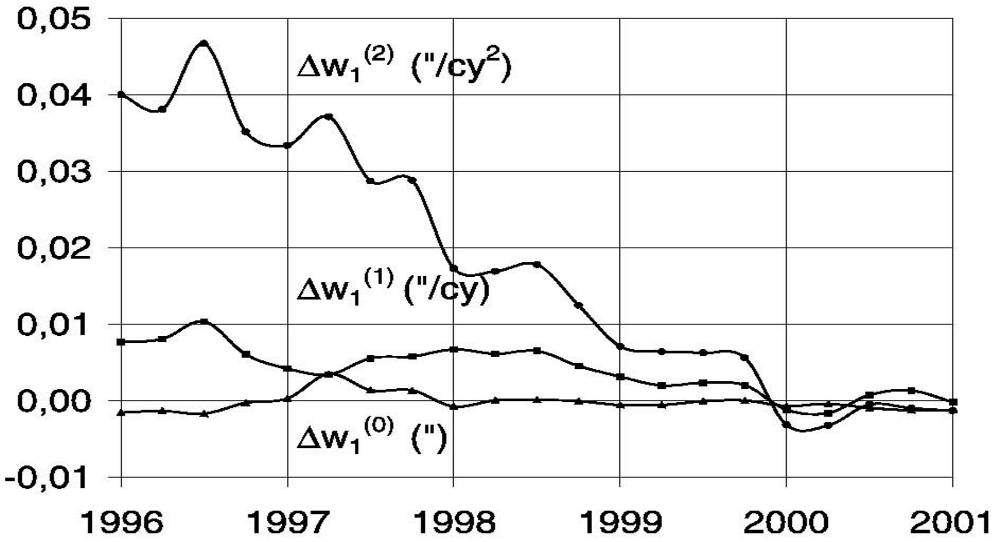


Table 5: LIST OF DETERMINATIONS OF THE TIDAL SECULAR ACCELERATION OF THE LUNAR MEAN LONGITUDE (IN ARCSECOND/CY^2)

	Authors	Value	Publication
(a)	Spencer Jones	-22	1939
(a)	Oesterwinter & Cohen	-38	1975
(a)	Morrisson & Ward	-26	1975
(b)	Muller	-30	1976
(c)	Calame & Mulholland	-24.6	1978
(d)	Ferrari et al	-23.8	1980
(c)	Dickey et al	-23.8	1982
(c)	Dickey & Willliams	-25.10	1982
(c)	Newhall et al	-24.90	1988
(c)	Chapront et al	-25.62	1997
(c)	Chapront et al	-25.78	1999
(c)	Chapront et al	-25.836	2000
(c)	Chapront et al	-25.858	2001
	Jet Propulsion laboratory Numerical Integratiions		
	JPL DE405	-25.826	1998
	JPL DE403	-25.580	1995
	JPL DE200	-23.895	1982

TYPES OF OBSERVATIONS

(a) Occultations (c) LLR

(b) Eclipses (d) LLR and Lunar Orbiter