Preliminary Estimation of the Atmospheric Nonlinear Frequency Dispersion and Absorption Effects on the Pulse SLR Accuracy

Yu. S. Galkin(1) and R. A. Tatevian(2)

(1) Moscow State Forest University, 1, 1st Institutskaia St., Mytischi-5, Moscow reg., 141005, Russia (galkin@mgul.ac.ru)

(2) Central Scientific Research Institute of Geodesy, Aerosurvey and Cartography, 26, Onezhskaia St., Moscow, 125413, Russia

Abstract It is known, that the in SLR, because of changes of the received ranging signal parameters are differed relatively of assumed in the standard calculation methods. The additional group delay, shift of carrier frequency and the change of the Gaussian laser pulse for SLR of the receiver are estimated. The estimation is made for "smooth" dispersion curve (only effective UV resonances) without the effects of local atmospheric resonances. The effects shown are most important for multi color SLR and the Marini-Murray atmospheric model correction because the effects depend on the elevation angle of the laser beam.

Introduction

The propagation of ranging signals through the earth's atmosphere has received much attention lately. In particular, efforts were made to improve the atmospheric refractive index determination /1/ and to get the best atmospheric model /2/ to correct the atmospheric effect on SLR. For single color measurements the distance is calculated as:

$$R=0.5(ct - □R_{ref})=0.5(ct - N_{gr}S)=0.5(ct - N_{gr0}(□)F(T,P,e,S)), \qquad (1)$$

where R – geometrical distance from SLR station to satellite, c – light velocity in vacuum, t – delay time of the received pulse relatively of transmitted one, $□R_{ref}$ – atmospheric correction, N_{gr} – integral group refractive index of the atmosphere, S – length of path of the laser pulse the atmosphere, N_{gr0} – group refractive index of air for standard conditions, □ - carrier frequency of the laser, F – model of distribution of the temperature T, pressure P and parcial water vapor pressure e along ray trajectory in the atmosphere.

The multicolor devices have been developed to decrease the atmospheric influence and to use the later to improve the atmospheric model for the single color measurements /3/.

For two color measurements the atmospheric correction may be obtained via the time delay □t the pulses of different color (let $\square_1 < \square_2$)

$$N_{gr}(\square_1)\, S = cN_{gr0}(\square_1)\, \square t\, (N_{gr0}(\square_2) - N_{gr0}(\square_1))^{-1} = cK\square t \tag{2}$$

It is assumed that the precision of □t measurement depends only on the device precision. The other components ($N_{gr0}(\square_1)$, $N_{gr0}(\square_2)$ therefore a factor K) may then be calculated by the known formulas /4,5/.

However, the known formulas were obtained under special conditions where the dispersion of the phase refractive index is linear. These conditions are given as Eqs (A5a), (A5b) in /5/, but not estimated for real situations. Since then, it was shown that the nonlinear effect is important for precise measurements /6/. It was supposed that this effect might be important near absorption lines and negligible within in transparent windows. Unfortunately it is not so.

Theoretical Basis

The transmitted form envelope A(t) of a signal at a carrier frequency $\square_0$ can be written as

$$E_0(t) = A(t)\exp(i\square_0 t), \tag{3}$$

After traveling a path length S through an absorptive and dispersive medium the received signal will be the following

$$E(t,S) = \frac{1}{2\square}\square\square A(\square)\exp\left[i(\square_0\square\square\square\square + \square t \square k(\square)S)\right]d\square d\square \tag{4}$$

where $k(\square)$ is a complex vector in a Taylor series in powers of $\square=(\square\square\square_0)$ for the absorption coefficient $\square(\square)$ and the phase delay $\square(\square)$

$$\begin{aligned} k(\square) &= k(\square_0) + k\square(\square_0)\square + 0.5k\square(\square_0)\square^2 + \ldots = \\ &= \square(\square_0) + \square\square(\square_0)\square + 0.5\square\square(\square_0)\square^2 + \ldots \\ &+ i\square(\square_0) + i\square\square(\square_0)\square + i0.5\square\square(\square_0)\square^2 + \ldots \end{aligned} \tag{5}$$

Usually it is assumed that

$$\square\square(\square_0)\square\ \square\square 0.5\square\square(\square_0)\square^{2}$$

$$\square(\square_0)\square\square\square\square(\square_0)\square$$

Then the known the group refractive index formula can be used to obtain the atmospheric refractivity correction /5/.

Unfortunately, the above-mentioned inequalities do not allow to calculate and to estimate the residual error for different conditions. Further more it was shown that the nonlinear effect stronlyg depends of the waveform envelope. The calculation methods for several forms were developed /7/.

Let us assume a Gaussian pulse envelope form with a width □(at 50% power) for SLR devices i.e.

$$A(t) = \exp(-2\ln 2t^2/\square^2). \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) and using /7,8/ the electrical field in the receiving point may be obtained

$$E(t,S) = A(S)\exp\left[\square 1.39a\square^{\square 2}(t\square\square\square S+\square)^2\right]\exp\left[\square i(\tilde{\square}(t\square\square\square S)+\square)\right], \tag{7}$$

where the received amplitude is

$$A(S) = \frac{1}{\sqrt[4]{(1+2.78\square^{\square 2}\square\square S)^2+(2.78\square^{\square 2}\square\square S)^2}}\exp\left[\square(\square S+\frac{1.39\square^{\square 2}(\square\square S)^2}{1+2.78\square^{\square 2}\square\square S})\right], \tag{8}$$

the additional extinction is

$$a = \frac{1+2.78\square^{\square 2}\square\square S}{\left(1+2.78\square^{\square 2}\square\square S\right)^2+(2.78\square^{\square 2}\square\square S)^2}, \tag{9}$$

the additional group delay is

$$\square = \frac{2.78\square^{\square 2}\square\square\square\square S^2}{1+2.78\square^{\square 2}\square\square S}, \tag{10}$$

the frequency change is

$$\tilde{□} = □_0 □ 2.78a□^{□2}□□\$ + \frac{3.86□^{□4}□□\$(t□□\$)}{\left(1+2.78□^{□2}□□\$\right)^2 + (2.78□^{□2}□□\$)^2} \ , \tag{11}$$

and the phase change is

$$□ = (□_0□□□□)S□\frac{3.86□^{□4}□□(□□\$)^2}{\left(1+2.78□^{□2}□□\$\right)^2 + (2.78□^{□2}□□\$)^2} □ \frac{1}{2} \mathrm{arctg} \frac{2.78□^{□2}□□\$}{1+2.78□^{□2}□□\$} , \tag{12}$$

Eqs. (7)-(12) are depending on the second frequency derivatives of the dispersion and the absorption of media.

Atmospheric Parameters

Edlen's dispersion formula /4/ as a Taylor's series in power of □ is

$$n-1= 272.613\cdot10^{-6}+4.309\cdot10^{-31}□^2+1.085\cdot10^{-66}□^4 \tag{13}$$

Therefore

$$□(□)=90.87\cdot10^{-20}□+14.36\cdot10^{-52}□^3+36.17\cdot10^{-86}□^5 \tag{14}$$

$$□□(□)=90.87\cdot10^{-20}+43.08\cdot10^{-52}□^2+18.08\cdot10^{-85}□^4 \tag{15}$$

$$□''(□)=86.16\cdot10^{-52}□+72.32\cdot10^{-85}□^3 \tag{16}$$

However, if a medium has a phase refractive index dispersion, it has an absorption dispersion too (by Kramers-Kronig relations). Calculations (as a large UV resonance) give the following expressions

$$□(□)=1.27\cdot10^{-34}□^2 +2.86\cdot10^{-66}□^4 \tag{17}$$

$$□□(□)=2.54\cdot10^{-34}□+11.44\cdot10^{-66}□^3 \tag{18}$$

$$□''(□)=2.54\cdot10^{-34}+34.32\cdot10^{-66}□^2 \tag{19}$$

Equations (14)-(19) are then substituted into Eqs. (7)-(12) to calculate the required values.

Results of Calculations

The additional group delay is the most important aspect for SLR. Its magnitude is calculated for a vertical ray path (double way through at sea level is assumed as 16.6 km) and a horizontal ray path (of 50 km).

ONE - COLOR RESULTS.

Table 1. Vertical ray path. Pulse width is 1.0 ns

□ (□m)	□ (ps)	□R (□m)
1.064	0.006	1.8
0.532	0.033	9.9
0.355	0.11	33.0

Table 2. Vertical ray path. Pulse width is 0.1 ns

□ (□m)	□ (ps)	□R (mm)
1.064	0.6	0.18
0.532	3.32	1.0
0.355	11.0	3.3

Table 3. Horizontal ray path. Pulse width is 1.0 ns

□ (□m)	□ (ps)	□R (□m)
1.064	0.054	16.2
0.532	0.3	89.6
0.355	0.99	297

Table 4. Horizontal ray path. Pulse width is 0.1 ns

□ (□m)	□ (ps)	□R (mm)
1.064	5.4	1.62
0.532	29.9	9.0
0.355	99	29.7

TWO - COLOR RESULTS.

Table 5. Vertical ray path. Pulse width is 1.0 ns

□□(□m)	□□ (ps)	K	□N_{gr}S=□R(mm)
1.064-0.532	0.027	22.6	0.18
0.532-0.355	0.077	14.06	0.32
1.064-0.355	0.10	8.44	0.26

Table 6. Vertical ray path. Pulse width is 0.1 ns

□□(□m)	□□ (ps)	K	□N_{gr}S=□R(mm)
1.064-0.532	2.72	22.6	18.44
0.532-0.355	7.68	14.06	33.64
1.064-0.355	10.4	8.44	26.33

Table 7. Horizontal ray path. Pulse width is 1.0 ns

□□(□m)	□□ (ps)	K	□N_{gr}S=□R(mm)
1.064-0.532	0.30	22.6	2.02
0.532-0.355	0.69	14.06	2.91
1.064-0.355	0.99	8.44	2.50

Table 8. Horizontal ray path. Pulse width is 0.1 ns=100 ps

□□(□m)	□□ (ps)	K	□N_{gr}S=□R(mm)
1.064-0.532	24.5	22.6	166.11
0.532-0.355	69.1	14.06	237.0
1.064-0.355	93.6	8.44	302.7

Conclusions

The effects of dispersion and absorption on the propagation of a laser pulse have been estimated for a transparent atmosphere with a “smooth” dispersion dependence from the “effective strong UV resonance” only. Spectral lines of the atmosphere were excluded from the calculations.

Certainly, these effects may be larger near spectral lines of absorption where the derivatives in Eqs. (14)-(19) have larger magnitudes. For this case, the absolute absorption may even be negligible.

It is evident that the effects increase strongly when the pulse width and the frequency difference between colors are decreased. Unfortunately, the tendency of the modern instrumental and technological is forwards shorter pulses and closer carrier waves of two-color systems.

It is recommended to carry out a more detailed research of the atmospheric influence on the pulse propagation in SLR before devices are manufactured.
By the way, the shown physical phenomena may be able in some optical details of the optical chains of devices.

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