A Summary of LLR Activity and Science Results

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Abstract

The Lunar Laser Ranging (LLR) experiment was originally a part of the NASA Apollo program. It has been active for more than 30 years. The data provide for varied, multi-disciplinary science results. Analyses exist in areas of solid Earth sciences, geodesy and geodynamics, solar system ephemerides, terrestrial and celestial reference frames, lunar physics, general relativity and gravitational physics. Combined with other observing techniques, LLR expands our understanding of the precession of the Earth’s axis in space, the induced lunar nutation, Earth orientation, the Earth’s obliquity to the ecliptic, the intersection of the celestial equator and the ecliptic, lunar and solar solid body tides, lunar tidal deceleration, lunar physical and free librations, the structure of the moon and energy dissipation in the lunar interior. LLR provides input into lunar surface cartography and surveying. It helps determine Earth station and lunar surface retroreflector locations and motions, mass of the Earth-Moon system, lunar and terrestrial gravity harmonics and Love numbers, relativistic geodesic precession, and the equivalence principle of general relativity. Due to the passive nature of the reflectors and the improvement of observing equipment and data analysis capabilities with time, LLR data will continue to provide for improved results. Gains are steady and as the data base expands we are now striving to learn even more by applying more clever observation strategies.

Background

Lunar Laser Ranging (LLR) is one of the most modern and exotic of the observational disciplines within astrometry, being used routinely for a host of fundamental astronomical and astrophysical studies. It consists of accurately measuring the round-trip travel time for a laser pulse emitted from an observing station on the Earth and returning after bouncing off of a retroreflector array on the surface of the Moon. The analysis of this constantly changing distance, using different observatories on the Earth and different reflectors on the Moon, provides for a wide spectrum of terrestrial, lunar, solar system, and relativistic science. See, for example, Bender et al, 1973; Mulholland, 1980; Dickey et al, 1994; Newhall et al, 1996; Nordtvedt, 1996; Shelus, 1996b; Chapront et al, 1999; Duncombe & Shelus, 1999; Shelus, 1999.

However, even after more than 30 years of routine observational operation, LLR remains a non-trivial, sophisticated, highly technical, and remarkably challenging task. Signal loss, mainly proportional to the inverse 4th power of the Earth-Moon distance, but also the result of optical and electronic inefficiencies in equipment, still requires that one observe mostly single photoelectron events. If the moon were just 25% farther from the Earth than it is, LLR could probably not be performed today. With a laser firing at approximately 10 hertz, less than 25 photoelectrons/minute prove to be true lunar reflections at the MLRS. Raw timing precision is some tens of picoseconds with the out-and-back range accuracy being approximately an order of
magnitude larger. Presently, we are down to sub-cm lunar ranging accuracies. In this day of routine artificial satellite laser ranging (SLR), it is a sobering fact to realize that the Moon is more than a trillion times harder to than the Topex-Poseidon satellite.

Existing LLR Programs

Today, with several tens of artificial satellite ranging stations around the world, only two of them are capable of routinely ranging to the Moon, and a third is in the final stages of installation and operation. One of the presently operating stations is in the United States, at the McDonald Observatory. The other is in the south of France, at the Observatoire de la Côte d’Azur. Both stations operate in a multiple target mode, observing targets other than the lunar surface retroreflectors. The Matera Laser Ranging Observatory (MLRO) is a joint SLR/LLR station, presently coming up to observational status in Matera, Italy. Full operation of the MLRO is to commence in early 2001. Although LLR data has been taken by the Wettzell SLR station in Germany, recent station upgrades and other operational matters have prevented LLR data from being obtained during recent times. It is expected that LLR data will be perhaps forthcoming from Wettzell at some time in the near future. The only other significant producer of LLR data, the LURE station on Mount Haleakala, in Hawaii, ceased its LLR operations during the late 1980’s because of budgetary considerations. Low levels of effort toward attaining LLR capabilities have taken place in China and Australia.

The present McDonald Laser Ranging Station (MLRS) [Shelus 1985; Shelus, 1987; Shelus, et al 1993a], located at McDonald Observatory in the Davis Mountains of west Texas, was designed for ranging to artificial satellites, as well as to the Moon. See Figure 1. It was constructed as a replacement for the original 2.7-m lunar-only system [Silverberg, 1974] that had been the sole lunar capable laser ranging station in the world until the mid-1980's. Several attempts had been made at other installations, but none attained routine operation. The MLRS is built around a computer controlled 0.76-m x-y mounted Cassegrain/Coudé reflecting telescope and a short pulse, frequency doubled, 532-nm, neodymium-YAG laser with appropriate computer, electronic, meteorological, and timing interfaces. The MLRS, initially placed in the saddle between Mt. Locke and Mt. Fowlkes at McDonald Observatory, near Fort Davis, Texas, became operational in 1983. Wind tunneling effects in and around that saddle site produced very serious problems with atmospheric seeing and the station was moved to the top of Mt. Fowlkes in early 1988.
The station at the Observatoire de la Côte d’Azur (OCA) is on the Plateau de Calern, located on the first chain of mountains north of Cannes, 30 km from the Mediterranean Sea [Veillet et al, 1993; Veillet et al, 1994; Mangin et al, 1996; Samain et al, 1998]. The transmitter/receiver is a 1.5m alt-az Ritchey-Chrétien reflecting telescope. The mount and control electronics insure blind tracking on a lunar feature at the 1 arcsec level for 10 minutes. The OCA station uses a neodymium-YAG laser, emitting a train of pulses, each with a width of several tens of picoseconds. Although originally built to operate as a lunar-only station, operation is now divided among the four retroreflectors on the Moon, the two LAGEOS targets, and the several high altitude artificial satellites (Glonass, Etalon, and GPS). The OCA station is the premiere station of the LLR network in both data quality and quantity.

The new and soon to be operational Matera Laser Ranging Observatory (MLRO), located in the south of Italy, employs a 1.5m astronomical quality reflector (Seldon & Bianco 1996). The laser is a hybrid that uses a Lightwave CW mode-locked laser to produce a 100 MHz pulse train from which a pulse may be selected. The resultant pulse length is less than 50 picosec with an energy of some 100 mJ. LLR observations had been performed successfully during test firings in 1998 when the station was at the Goddard Space Flight Center’s GGAO site in Greenbelt, MD. Those data files are presently under investigation. It is expected that the system will be ready for routine operations, including LLR sessions, by the summer of 2001.

In addition to their supplying the world with a steady supply of first-rate lunar laser ranging data, the LLR-capable laser stations play a vital role within the artificial satellite ranging (SLR) community as well. As seen in Figure 2, for the most recent 12-month period, January-December 2000, the MLRS ranked in the top 10 of world-wide laser ranging stations, in Total Data Volume.
Figure 3 shows “high satellite” results for the same period. The OCA station (Grasse LLR) ranked 3rd in the world with respect to the total number of High Satellite Passes observed; the MLRS ranked 6th. At an LLR-capable station, epoch timing systems make all targets virtually equivalent to an observer, and a crew will routinely range to many different targets, from the closest of artificial satellites to the Moon, during a single shift. The differences among available targets are their angular speed across the sky and return signal strength. In spite of LLR’s low signal strength, LLR activity has enjoyed phenomenal success, and, over the years, has produced a steadily increasing number of observations with increasingly better accuracy and precision.


![High Satellite Pass Total Chart]

Even though the vast majority of laser firings at the Moon do not result in true lunar returns, the sheer bulk of individual photons increases rapidly with time. Further, the nature of the observing process, that requires that a number of observations be distributed over only a few minutes of time, makes it natural to consider the compression of shot-by-shot returns into normal points. An example of the normal point formation process and the means by which accuracy and precision estimates can be assigned to these points is found in Abbot, et al (1973). LLR normal point formation is now automatic and performed in real-time at the stations. Data deposits from the stations are made automatically and all LLR data are made available to the entire scientific community. The Crustal Dynamics Data Information System (CDDIS), maintained at Goddard Space Flight Center, and an analogous site at the European Data Center, archive all LLR data going back to the first returns detected at McDonald Observatory in the summer of 1969. An on-
line, interactive menu system allows a user to browse information about the system and the contents of the data archive. The facility is available 24 hours per day, 7 days per week. Information on the Goddard and European data sites may be obtained at the ILRS web-site, “ilrs.gsfc.nasa.gov/ilrs_home.html”.

From Observation to Science

Observations are only the first step toward getting scientific results. By applying a procedure that can be called "dynamic parameter improvement" (Mulholland, 1976) to a set of LLR observations, one can obtain information about the site from which the observations are being made, the target itself, and the rest of the universe around us. The procedure requires that not only a set of observations is available but also, for each one of those observations, a prediction must exist. The prediction tells us what an observation would have been if the universe acted in conformity with some specific, well-defined model. One begins with a convenient definition of the topocentric distance, \( r \), between an observing site on the Earth and some target on the Moon. In vector form,

\[
r = R(\text{geocenter-selenocenter}) - R(\text{geocenter-observatory}) + R(\text{selenocenter-reflector}).
\]

This can be approximated, in scalar form, by dotting the station vector into the remaining part, i.e.,

\[
r = R_1 - \rho \cos (\delta) \cos (H) - z \sin (\delta),
\]

where \( R_1 \) is the geocentric distance to the target, \( \rho \) is the perpendicular distance of the station from the Earth's spin axis; \( z \) is the perpendicular distance of the station from the Earth's equatorial plane; \( \delta \) is the declination of the target; and \( H \) is the local hour angle of the target. It must be remembered that the LLR measurement is not simply one of distance, it is a measurement of an out-and-back time interval. Therefore, it is necessary to solve for a two-way transit time, \( \tau \), by using an iterative scheme, with the Earth and the Moon each being in motion. That is, we cannot just use an instantaneous distance.

The calculation of a predicted transit time for any given laser firing, requires that we assume a gravitational and relativistic theory, the body-fixed coordinates of the station and the target, and all of their motions, precession and nutation, Earth orientation, lunar libration, the elastic deformations of the Earth and the Moon, and a model of atmospheric refraction. The increase in our knowledge about the universe around us comes from comparing our predicted transit times with the ones we have measured. Were prediction and observation to agree, our model would be accurately representing the universe. It is the difference between prediction and observation, \( \Delta \tau \), that allows us to study the workings of the universe.

To evaluate the above equation and then to use it to predict a transit time, we need a priori values for a large number of physical parameters, e.g., the masses, radii, internal make-up, and gravitational harmonics of the Earth and the Moon, the value of the gravitational constant, the locations of the telescope on the Earth and the retroreflectors on the Moon, and many others. In addition, for each of these parameters, \( \kappa \), one must have a partial derivative \( \delta \tau / \delta \kappa \), i.e., the manner in which the transit time, \( \tau \), would vary, were we to change the value of only that particular parameter in our model. Each observation then provides an equation of condition of the form
\[ \Delta \tau = \Sigma (\delta \tau / \delta \kappa_i) \Delta \kappa_i \]

and this system of equations of condition can be reduced by a suitable regression algorithm to provide improvements to the estimated values of each of the various parameters, \( \kappa_i \). Let us now examine what science has resulted from the LLR observations and their analysis over the past 30 years or so.

Relativity and Gravitational Physics

LLR has contributed to several Solar System tests of general relativity and gravitational physics. Table 1 lists some of the more important relativistic and gravitational parameters that have been derived using LLR data, alone and in combination with other data. The Moon proves to be especially valuable for this because the ratio of non-gravitational to gravitational forces acting upon it is very small.

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Principle of Equivalence parameter, ( E )</td>
<td>((3.2 \pm 4.6) \times 10^{-13})</td>
</tr>
<tr>
<td>Parameterized Post-Newtonian (PPN) superposition parameter, ( \beta )</td>
<td>(1.003 \pm 0.005)</td>
</tr>
<tr>
<td>Parameterized Post-Newtonian (PPN) curvature parameter, ( \gamma )</td>
<td>(1.000 \pm 0.005)</td>
</tr>
<tr>
<td>Deviation from the expected geodetic precession, ( K_{GP} )</td>
<td>(-0.003 \pm 0.007)</td>
</tr>
<tr>
<td>Change in gravitational constant ((G\text{-dot})/G)</td>
<td>((1 \pm 8) \times 10^{-12}/\text{yr})</td>
</tr>
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</table>

LLR now establishes the definitive limit for both the strong and the weak equivalence principles (Williams, et al 1996), which require that the ratio of gravitational mass to inertial mass be exactly unity. Therefore, all bodies fall with the same acceleration in an external gravitational field, with the gravitational self-energy contributing equally to the gravitational and inertial masses. Although the Equivalence Principle was tested in a laboratory setting, until the coming of LLR, it has not been tested for bodies large enough to have a significant fraction of their masses coming from gravitational self-energy. The gravitational self-energy corresponds to \(4.6 \times 10^{-10}\) of the Earth’s mass; the corresponding fraction for the Moon is \(1.9 \times 10^{-11}\). Considering the orbit of the Moon around the Earth, a violation of the Equivalence Principle would cause the orbit of the Moon about the Earth-Moon barycenter to be polarized in the direction of the Sun, the so-called Nordtvedt effect (Nordtvedt, 1968). This gives rise to a signature with a synodic period of 29.53 days. Today, LLR analysis gives \((M_G/M_I - 1) = (2 \pm 5) \times 10^{-13}\), the best current test of the Strong Equivalence Principle available. With expected improvements in LLR data accuracy and with a longer span of data, further improvement is guaranteed.

Another important test of gravitational physics is the measurement of a relativistic precession of the lunar orbit, resulting from the motion of Earth and Moon about the Sun. This is now known as geodetic precession. This was first predicted by deSitter in 1916. This effect should cause a precession of the entire lunar orbit with respect to the inertial frame of the Solar System by some 19 milliseconds of arc per year. The LLR data are sensitive to this effect mainly through the precession of the lunar perigee and node after accounting for ordinary precession due to the Newtonian and other relativistic effects of the Sun, the Earth, and the other planets. Early LLR reductions agreed with the predictions of General Relativity to within 2%. More recent
solutions give a difference of $-0.3 \pm 0.9\%$ from the expected value. New, more, and better observations will be invaluable in reducing uncertainties.

LLR data also provide information concerning the possible change of the gravitational constant, $G$, with time. This is possible because the lunar orbit is sensitive to the longitude of the Sun. Adding cosmological interest to this situation is the suggestion that very large changes in $G$ may have occurred during an inflationary phase in the early history of the universe. The LLR value is presently $(0.1 \pm 0.8) \times 10^{-11}$ per year (Williams, et al 1996). Other independent determinations can be made from Viking lander tracking data and binary pulsar data. The best results on $G$-dot will undoubtedly depend upon the analysis of a combination of all of these data types.

Solar System Dynamics

The Moon’s orbit around the Earth is strongly perturbed by the Sun. This perturbation produces a spectrum of range signatures that are sensitive to many Solar System parameters. For example, the lunar orbit orientation is determined at least two orders of magnitude more accurately and the radial component is determined at least four orders of magnitude more accurately than it was possible with classical optical data. In fact, radial distance variations are determined slightly better than the present 1-3 cm LLR range accuracy and the angular rate uncertainty is less than 0.15 milliseconds of arc per year. The lunar orbital components that have the greatest uncertainties are the mean distance and the orientation of the lunar orbital plane with respect to the Earth’s equator. At the present, these are 0.4 m (due to correlation with the retroreflector coordinates in the mean Earth direction) and 1.5 milliseconds of arc (3 m at the Earth-Moon separation).

The strong influence of the Sun on the Moon’s orbit also permits LLR data to be used to determine the mass ratio $\frac{\text{Mass}_\text{Sun}}{\text{Mass}_\text{Earth} + \text{Mass}_\text{Moon}}$ as well as the relative orientation of the Earth-Moon system orbit around the Sun. The actual size of the Earth-Moon orbit is determined by the gravitational constant times the sum of the masses of the Earth and the Moon, with the Moon’s orbit being perturbed from a simple Keplerian ellipse by the Sun. The two largest solar perturbations for the Moon, the monthly and semimonthly variations in distance, are obtained from LLR data to a few cm. This corresponds to a $10^{-8}$ relative accuracy in the value of the mass ratio in question. Further, the use of LLR observations allows the relative geocentric positions of the Sun and the Moon to be determined to within 1 millisecond of arc. Since planetary positions are measured with respect to the Earth’s orbit around the Sun, the geocentric position of the Moon and the heliocentric positions of the planets can be made internally consistent in their relevant orientation. Because LLR stations are located on a spinning Earth, the orientation of the Earth’s equatorial plane is determined relative to both the lunar orbit plane and the ecliptic plane of the heliocentric Earth-Moon orbit. Thus, LLR data is sensitive to the mutual orientation of the planes of the Earth’s equator, the lunar orbit, and the ecliptic. Hence, it locates the intersection of the ecliptic and equatorial planes (the dynamical equinox) and determines the angle between them (the obliquity of the ecliptic). The orientation of the planetary ephemerides with respect to the fundamental astronomical planes is established at the millisecond of arc level. The International Celestial reference Frame (ICRF) is defined by a set of extragalactic radio sources, rather than fundamental planes.

Of course, all of these results are degraded when one extrapolates outside the span of observations. This means that a continual supply of high quality measurements and analysis are
required to maintain and enhance these results. Using LLR data in combination with the other modern observing techniques will provide for the very best results available.

Lunar Science

LLR data analysis provides important information about the dynamics and the internal structure of the Moon. Selenocentric reflector surface coordinates, moment of inertia ratios, and the second and third degree lunar gravity harmonics are determined with high accuracy using LLR data. The reflector coordinates, together with the ALSEP radio transmitter coordinates, serve as fundamental control points for lunar surface cartography (Davies et al, 1987). Monitored by LLR, the changing apparent distances between the several reflectors and the Earth provide information on the lunar physical librations and solid body tides. Values of the lunar gravity harmonics, the moments of inertia and their differences, the lunar Love number, $k_2$ (which measures the tidal change in the moments of inertia and gravity), and variations in the lunar physical librations are all related to the Moon’s structure, mass distribution, and internal dynamics. They all give us insight and better understanding of the lunar interior. Presently, the most accurate estimate of the lunar moment of inertia is obtained from a combination of moment of inertia differences determined by the LLR solutions and the lunar gravity field coefficients coming from lunar satellite Doppler observations (Konopliv et al, 1998).

The lunar mass distribution also perturb the lunar orbit that, in turn, produces a secular precession in the lunar node and perigee directions. Lunar seismic data suggest a crust and a mantle, but the evidence for a core is not definitive. The existence of a lunar core, as well as whether it is solid or liquid, are important questions. The lunar polar moment can help set limits and the lunar rotation provides further evidence. Information concerning the apparent tidal distortion of the Moon and the mean direction of its spin axis can be inferred from the lunar librations, measured by LLR. If the Moon were a solid elastic body, the mean direction of its spin axis would precess with the orbit plane. LLR data show that the true spin axis is actually displaced from this expected direction. Dissipation can be caused by both solid-body tides and liquid-core/solid-mantle interaction. The presence of a fluid core with a turbulent boundary layer appears to be a plausible interpretation. The direct separation of the competing dissipative terms is difficult. Recent LLR work (Williams et al, 2000) indicate a fluid core in the Moon of some 350km radius. Better accuracy and greater amounts of multi-corner data are needed, since key answers depend on very small signatures.

Analysis of the LLR data detects three modes of free libration (Newhall & Williams, 1997). One is an apparent rotational free libration in longitude for the Moon with a 2.9 year period. The free-plus-forced blend has a 1.8 arcsecond amplitude and a free amplitude of 1.4 arcseconds. Also strongly seen in the LLR data, is a 74 year elliptical wobble of the lunar pole with semiaxes 3 by 8 arcseconds. Separate from the lunar forced physical librations that are driven by the time-varying torques of the Earth, the Sun, and the other planets, three modes of free librations exist. One of these theoretical modes is a 2.9 year oscillation in lunar rotation speed. Without suitable recent exciting torques, and because of substantial dissipation, the amplitudes of all lunar free librations should be damped to zero. However, the case of the observed 2.9 year free libration is complicated because two very small forcing terms in the lunar orbit, close to the resonance frequency for the free libration, are amplified to mimic the observed free libration. Eckhardt has suggested that passes though weak resonances have occurred for the lunar rotation in the geologically recent past that can perhaps stimulate free libration in longitude. Numerically integrated lunar rotational motions have been compared to semi-analytic calculations of the forced angular motions in an attempt to understand the free libration. Seismic events on the
Moon are insufficient to explain the observed amplitude. Other studies have been carried out to investigate whether the apparent free libration might have been excited by recent impacts on the Moon. Such excitation would have required an impact in very recent times by an object large enough to leave a crater with a 10 km diameter. A third free libration mode, much smaller than the others, i.e., only 0.02 arcseconds in magnitude, has only been recently detected; its small value argues against impacts as being an important stimulating mechanism. Another plausible explanation appears to be core-boundary effects, similar to those that are believed to account for the decade time-scale fluctuations in the Earth’s rotation. More and better LLR data are needed, that can irrefutably define the third mode of libration and extend the span and accuracy of measurement of the 74 year wobble.

Geodynamics

The classic geodynamical results from LLR derive from the long term study of the variation of the Earth’s rotation, the determination of the constants of precession and nutation, LLR station coordinates and motions, and solid-body and ocean tides that accelerate the motion of the Moon. LLR observations supplement and complement the results being obtained from other space-based observing techniques, but with an more than three decade long span of data, LLR data exceeds that available from any of the other space geodetic techniques. The very accurate value of the LLR-derived Sun/(Earth + Moon) mass ratio can be combined with the solar GM and the lunar GM (from lunar-orbiting spacecraft) to give the Earth’s GM in an geocentric reference frame with an accuracy of 1 part in 10^8. Within the uncertainties, this value is quite compatible with that derived from SLR.

Tidal dissipation of energy on the Earth causes a misalignment of the Earth’s tidal bulge to the Earth-Moon line. This bulge exerts a secular torque that causes both the Moon to increase its distance from the Earth and the Earth’s rotation rate to decrease. The resulting change has been seen in the geological record. With respect to specific Earth tides, the span and accuracy of the LLR data is such that the diurnal and semi-diurnal tides can be resolved from the amplitude of the 18.6 year along-track tidal perturbation. Due to the gravitational attraction of the Sun, Moon, and other planets the Earth’s spin axis precesses and nutates in space. These motions depend on the flattening of the Earth, its moment of inertia, the flattening of the core-mantle interface, the Earth’s anelasticity, as well as ocean and solid body tides. Both LLR and VLBI analyses indicate that significant corrections are required to the standard precession and nutation models. The almost 30 year span of LLR data is a distinct advantage when attempting to separate precession and the 18.6 year nutation correction. Joint VLBI and LLR solutions [Charlot, et al 1995] combine the strength of the LLR data for long period terms and the high resolution of the VLBI data for shorter periods.

Real-Time Earth Orientation Parameters

The modelling of all of the Earth’s motions affects many scientific disciplines. Although precession and nutation were well observed by the 19th century, it was not until the turn of this century that the irregularities in the Earth’s rotation and the phenomenon of polar motion were clearly recognized. By the 1960’s, largely from optical observations, augmented by artificial satellite Doppler measurements, 5-day mean values for each component of the polar motion were believed accurate to ± 40 cm and angular position of the Earth accurate to ± 0.03 arcseconds. From its inception in the late 1960’s, LLR was a source for accurate Earth rotation and polar motion information. Project EROLD (Earth Rotation from Lunar Distances) was conceived in 1974 under the auspices of COSPAR. The first series of LLR results for Universal Time was
Among the present benefits of LLR data are their accuracy (presently, 1-3 cm) and their long time span (more than 3 decades). Some parameters separate out on relatively short time scales, while others separate on the 18.6 year period of the lunar nodal regression, or longer. Certainly, as long a span of data as possible is vital for quantities that separate on these longer time scales. Among these are geodetic precession, the obliquity of the ecliptic, the dynamical equinox, orientation of the lunar orbit, and free librations. Length and continuity of the data span will allow for the better determination of all of these quantities and also allow for the determination of the rate of change with time of the gravitational constant, G. Also, when analyzing any kind of data, it is important to consider the distribution of observations over a relevant parameter space. The histograms in Figure 4 show the distribution of LLR data with respect to the classical fundamental arguments of the lunar theory. With more than 30 years of
observations, we see that the distribution with respect to the lunar mean anomaly, \( l \), the solar mean anomaly, \( l' \), and the argument of the lunar latitude, \( F \), are reasonably flat. Although not as flat, it is very significant that there exists more than a full period of precession of the mean longitude of the ascending node of the lunar orbit, \( \Omega \). That distribution is getting better annually. The new and full moon effects are clearly shown by the distribution of observations with respect to the mean elongation of the Moon from the Sun, \( D \). This is a consequence of limited observation time and the preferential scheduling of lunar operations at the lunar 1st and 3rd quarter phases, when ranges are most easy to obtain. Illustrating the long span of McDonald Observatory and OCA LLR data and their precision and accuracy are Figures 5 and 6 where are shown, on an annual basis, the number of normal points and the weighted root-mean-square of post-fit LLR residuals.

![MLRS Lunar Laser Ranging Data](image)

Figure 5. Number of normal points and weighted RMS of post-fit residuals for McD LLR observations.

![OCA Lunar Laser Ranging Data](image)

Figure 6. Number of normal points and weighted RMS of post-fit residuals for OCA LLR observations.

Several additional comparisons in lunar capability of the MLRS to OCA, and to itself, appear in Table 2. In almost every statistic referenced, the smaller aperture MLRS looks excellent in its own right and compares very well with the French LLR station. It can be seen that, while being one of the most prolific producers of SLR data in the world, the MLRS also produces roughly the same number of normal points and minutes of LLR data. It is only in the count of individual photon returns that the MLRS lags. Perhaps an even more telling statistic is the fact that the
MLRS gathered lunar data on as many or more nights that OCA did. Thus, the multi-faceted MLRS has accomplished a great deal vis-à-vis LLR.

In Summary

A large amount of fundamental astronomical and astrophysical science has been the result of analyzing LLR data over the past more than 30 years. And, because of the passive nature of the lunar retroreflectors and the steady, continuous improvement in the observing technique, LLR data will continue to provide ever-improving, state-of-the-art scientific research. Similar to other astrometric disciplines, LLR is broad ranging in its results and gains are steady as the data base expands, and the data quality improves, in accuracy as well as in precision. As a basic scientific data type, LLR’s temporal coverage and continuity is becoming one of its more important assets. One should note that the experiment has passed the 30th anniversary of the original emplacement of a retroreflector on the Moon during the NASA Apollo Program, and LLR is the only remaining active Apollo experiment. During these times of severe pressures on all forms of scientific research, it is important to be able to maintain an efficient and cost-effective experiment. LLR, and the science that it is able to provide, will continue to be a source of long-lasting results to the entire scientific community.

Table 2

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</table>
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