# SLR SYSTEMS CHARACTERISTICS IN WESTPAC, LAGEOS-1 AND LAGEOS-2 OBSERVATIONS

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**ABSTRACT.** This paper concerns a multi-satellite analysis of three high (h = 6000 km) and low (h = 835 km) flying satellites: LAGEOS-1, LAGEOS-2 and WESTPAC (the abbreviations stand for LAser GEOdynamics Satellite and Western Pacific Laser Tracking Network Satellite, respectively).
The orbits of these satellites can be computed from SLR measurements with an rms-of-fit of about 3 cm or better. This study is based on SLR measurements taken by the global network of ground stations during the period from August 1, 1998, until March 30, 1999. The primary focus of this investigation is on assessing the characteristics of the SLR observations, in particular in terms of possible range biases. The latter will be computed and analysed for each satellite individually, for a number of the different scenarios.

## INTRODUCTION

At present, most SLR systems are claimed to have an accuracy equal to or better than 1 cm [ILRS, 2000]. This allows this technique to be applied to estimate satellite orbits, station positions and other geophysical parameters with high absolute accuracy. However, in spite of this, different kinds of systematic effects or periodic variations at the level of millimeters, centimeters or larger may be observed in the post-fit residuals. These systematics can typically be interpreted as range biases, which may be caused by different reasons (laser construction, registration errors, satellite signature or other), which often are very difficult to find and explain. Reliable estimates for range bias values and other characteristics are very important information for the staff of laser stations.
The goal of this paper is to study the characteristics of SLR observations on LAGEOS-1, LAGEOS-2 and WESTPAC, in particular in terms of range biases. The strong point of LAGEOS-1 and LAGEOS-2 is the high orbit accuracy, which is in the order of about 2 cm (in radial direction); a weak point is, however, that the return signal originates from a variety of reflectors with an uncertainty on the exact center-of-mass correction as a consequence. For WESTPAC, the strong point is its construction (a limited set of baffled reflectors, which allow a reflection from 1 reflector at a time only; the center of mass offset can be modelled with sub-mm accuracy (Shargorodsky, 1997), the weak point is its limited orbit accuracy by virtue of its orbital altitude and the tracking coverage (Rutkowska and Noomen, 2000). The study is based on normal points taken by the global network of laser stations during the period from August 1, 1998, until March 30, 1999. An overview of the passes for LAGEOS-1, LAGEOS-2 and WESTPAC is shown in Table 1. The total number of passes for LAGEOS-1 and LAGEOS-2 is about three times larger than that for WESTPAC. This can be expected to have a big influence on the accuracy of range bias estimates for stations with a small number of passes (cf. Table 1).

Table 1. Overview of passes observed by the global network of SLR stations on LAGEOS-1, LAGEOS-2 and WESTPAC in the period August 1, 1998, until March 31, 1999, respectively.

| Station | location | LAG-1 | LAG-2 | WESTPAC |
|---|---|---|---|---|
| 1864 | Maidanak, Uzbekistan | 31 | 26 | 21 |
| 1868 | Komsomolsk-na-Amure, Russia | 17 | 32 | 0 |
| 1870 | Mendeleevo, Russia | 0 | 0 | 21 |
| 1873 | Simeiz, Crimea, Ukraine | 10 | 19 | 0 |
| 1884 | Riga, Latvia | 42 | 54 | 28 |
| 1893 | Kazivili, Crimea, Ukraine | 15 | 18 | 11 |
| 7080 | McDonald Obs., Fort Davis, TX | 133 | 146 | 14 |
| 7090 | Yarragadee, Australia | 285 | 226 | 134 |
| 7105 | Greenbelt, Maryland | 181 | 173 | 44 |
| 7110 | Monument Peak, California | 261 | 302 | 86 |
| 7124 | Papeete, Tahiti | 44 | 43 | 12 |
| 7210 | Haleakala Obs., Maui, Hawaii | 104 | 129 | 41 |
| 7236 | Wuhan, China | 5 | 7 | 0 |
| 7237 | Changchun, China | 64 | 92 | 10 |
| 7249 | Beijing, China | 0 | 5 | 0 |
| 7328 | Koganei, Japan | 18 | 19 | 1 |
| 7335 | Kashima, Japan | 2 | 1 | 0 |
| 7337 | Miura, Japan | 10 | 10 | 0 |
| 7339 | Tateyama, Japan | 26 | 27 | 0 |
| 7403 | Arequipa, Peru | 60 | 36 | 8 |
| 7548 | Cagliari, Italy | 24 | 41 | 0 |
| 7594 | Wettzell, Germany | 9 | 15 | 4 |
| 7806 | Metsahovi, Finland | 4 | 9 | 1 |
| 7810 | Zimmerwald, Switzerland | 87 | 96 | 35 |
| 7811 | Borowiec, Poland | 44 | 64 | 8 |
| 7820 | Kunming, China | 22 | 42 | 0 |
| 7824 | San Fernando, Spain | 17 | 23 | 9 |
| 7831 | Helwan, Egypt | 3 | 8 | 0 |
| 7835 | Grasse, France | 102 | 108 | 120 |
| 7836 | Potsdam, Germany | 59 | 68 | 25 |
| 7837 | Shanghai Observatory, China | 19 | 55 | 6 |
| 7838 | Simosato Observatory, Japan | 37 | 56 | 6 |
| 7839 | Graz, Austria | 267 | 197 | 84 |
| 7840 | Herstmonceux, United Kingdom | 315 | 246 | 110 |
| 7843 | Orroral Valley, Australia | 94 | 87 | 51 |
| 7845 | Grasse, France | 43 | 35 | 0 |
| 7849 | Mount Stromlo, Australia | 292 | 224 | 72 |
| 7939 | Matera, Italy | 59 | 83 | 1 |
| 8834 | Wettzell, Germany | 94 | 81 | 2 |
| | total | 2899 | 2903 | 965 |

In the analysis, the observations have been processed in data arcs of 7 days (34 in total). The number of normal points, total and separately for each satellite and each arc, are shown in Figure 1. The number of normal points for LAGEOS-1 and LAGEOS-2 is approximately similar, but for WESTPAC this is about 3 times smaller. For 3 arcs, no normal points are available for WESTPAC.

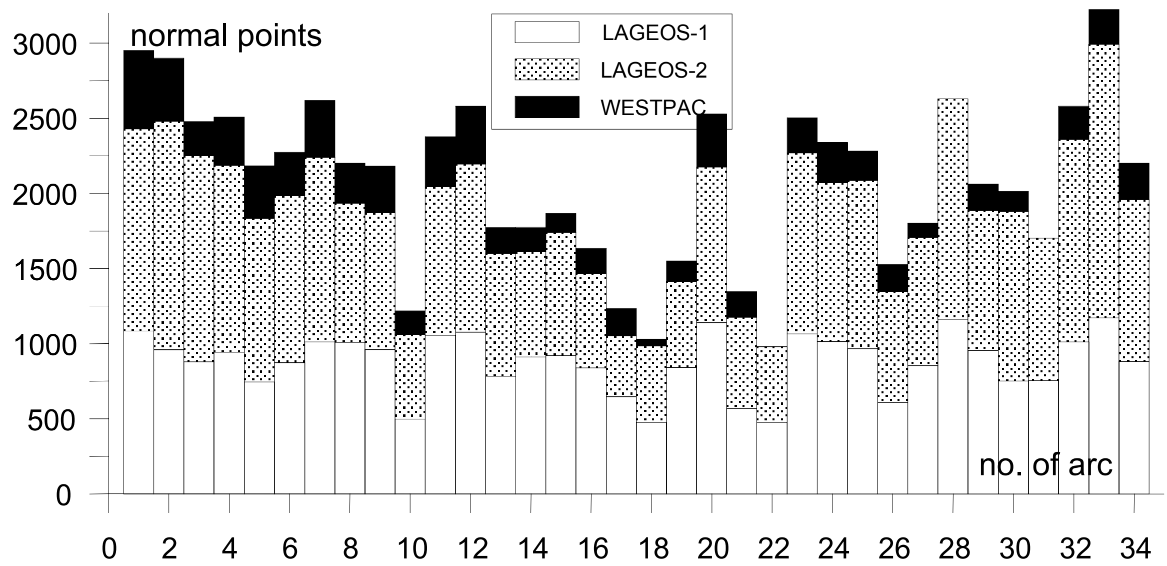


Fig. 1. Number of normal points for LAGEOS-1, LAGEOS-2 and WESTPAC used in the analysis.

## ANALYSIS METHOD

The analysis described here investigates the existence of range biases in the observations of the three satellites mentioned before. This is done by analysing the measurements using a variety of assumptions on stability of such range biases and the treatment of the SLR station positions. Table 2 shows the main characteristics of the different scenarios that have been used in this preliminary study (station position models and 8-month or 4-week interval descriptions). The full computation model was taken after a detailed assessment of various elements (Rutkowska and Noomen, 2000). Table 2 also shows the values for the weighted rms-of-fit obtained for the various scenarios; this corresponds with orbit solutions for LAGEOS-1, LAGEOS-2 and WESTPAC of 2.5, 2.5 and 3.7 cm, respectively. All observing stations and all normal points were accepted in the solutions.

The results of the computations are used to assess the quality of each individual scenario. Theoretically, values for the range bias solutions for LAGEOS-1, LAGEOS-2 and WESTPAC equal to 0.0 m should be expected. In the real situation, a more or less random distribution of range bias around zero should be expected for the scenarios under investigation.

Table 2. Overview of scenarios.

| Scenario | A | B | C | D |
|---|---|---|---|---|
| Analysis interval | 8 months | 8 months | 4 weeks | 4 weeks |
| Station positions | Solved | SSC(DEOS) 98C01 | Solved | SSC(DEOS) 98C01 |
| Range biases | Solved | Solved | Solved | Solved |
| Weighted rms-of-fit LAGEOS-1 | 0.97 | 1.07 | 0.92 | 1.04 |
| Weighted rms-of-fit LAGEOS-2 | 1.00 | 1.13 | 0.95 | 1.07 |
| Weighted rms-of-fit WESTPAC | 0.72 | 0.76 | 0.64 | 0.73 |

**RESULTS**

In principle, various parameters exist to assess the quality of each scenario. One of these is the rms-of-fit, which is included in Table 2. However, it is clear that the outcome of this parameter is more-or-less identical for all scenarios. In the remainder of the text, the emphasis will be on the outcome of the bias estimates. Tables 3 and 4 show the range bias solutions in mm, estimated for scenario A and B respectively, for all stations and all satellites taking part in the analyses (LAGEOS-1, LAGEOS-2 and WESTPAC).

Inspection of the tables yields a wide variety of bias solutions, in some cases going up to values of several decimeters. Many of them, fortunately, are accompanied by large a posteriori standard deviations (the two tables give the 1-sigma values), which indicates that the solutions may be weak because of insufficient amounts of data (cf. Table 1). In particular, and in accordance with the impression already given in Table 1, the WESTPAC results appear rather poor in (statistical) quality.

A better impression on the quality of the bias estimates can be obtained by plotting the solutions against each other: Figures 2-4 for scenario A and Figures 5-7 for scenario B. Not accounting for weak stations (datasets), Figure 2 clearly shows that the reliable bias solutions for LAGEOS-1 and LAGEOS-2 show a variation of about ± 50 mm (with a significant amount around 0 mm). Equally important, the LAGEOS results for these stations show a consistency (deviation from the solid 1:1 line) of about 10-15 mm. These numbers (absolute values and internal discrepancies) may be attributed to a variety of causes: satellite signature, actual station bias, errors or uncertainties in the computation model, etcetera. To get a better explanation, more detailed analysis is needed; because of time constraints this had to be deferred to a follow-up of this preliminary analysis.

The agreement between the results for WESTPAC and those for LAGEOS is slightly worse: it is clearly visible that the bias solutions for WESTPAC are weaker from a statistical point of view, which then also reflects in a slightly larger scatter w.r.t. the line denoting 100% correlation (Figures 3, 4). Clearly, more fine-tuning of the computations is needed here too.

Similar conclusions hold for the results of scenario B (Figures 5-7). The internal consistencies are a little bit better than those obtained for scenario A, which is to be expected since the degrees of freedom are less (coordinates are kept fixed for most of the stations). On the other hand, the overall pattern of the two computations is more or less identical, certainly where it concerns the outliers (Figure 2 vs. Figure 5).

The results for scenarios C and D are summarized in Table 2 and Figures 8-10. Table 2 shows the weighted rms-of-fit for both cases, obtained for all 3 satellites independently. These fit values are slightly better than the values obtained for comparable cases for scenarios A and B. This is to be expected, since the observations now have been treated in batches of 4 weeks instead of the full 8-month period at once; the increment of the amount of solve-for parameters must always lead to a reduction in the factor which is minimized. The bias estimates obtained for scenario C are shown in Figures 8-10, for each satellite and each station separately. It will be clear that the bias solutions obtained here are by definition statistically weaker than those obtained for the corresponding scenario A, since the amount of observations is less for each solution. This is also reflected in the bias values themselves,

Table 3. Range bias solutions and sigmas (estimated simultaneously with station positions) for scenario A.

| | SCENARIO A | | | | | |
|---|---|---|---|---|---|---|
| | LAGEOS-1 | | LAGEOS-2 | | WESTPAC | |
| Station | Bias (mm) | sigma (mm) | Bias (mm) | sigma (mm) | Bias (mm) | sigma (mm) |
| 1864 | 87.0 | 83.5 | 125.5 | 99.5 | 135.7 | 138.1 |
| 1868 | 20.4 | 121.5 | -46.3 | 95.6 | | |
| 1870 | | | | | -164.4 | 131.8 |
| 1873 | -383.9 | 339.5 | -454.3 | 187.9 | | |
| 1884 | -31.5 | 29.0 | -32.9 | 23.9 | -37.7 | 29.4 |
| 1893 | 11.8 | 61.1 | 51.7 | 51.9 | 23.6 | 84.9 |
| 7080 | -3.4 | 3.4 | -4.0 | 3.4 | -47.6 | 39.7 |
| 7090 | 7.2 | 1.7 | 2.9 | 1.6 | -64.9 | 6.8 |
| 7105 | -10.3 | 2.5 | 4.4 | 2.4 | -8.4 | 13.8 |
| 7110 | 0.8 | 1.6 | -1.2 | 1.5 | -25.6 | 10.4 |
| 7124 | -12.2 | 4.8 | 4.7 | 5.2 | -14.2 | 33.1 |
| 7210 | -5.7 | 3.2 | -3.3 | 2.8 | -18.7 | 11.2 |
| 7236 | -326.2 | 186.5 | -210.2 | 164.7 | | |
| 7237 | -45.7 | 11.6 | -36.1 | 10.8 | -28.2 | 47.4 |
| 7249 | | | 127.3 | 95.3 | | |
| 7328 | 3.3 | 16.5 | 16.1 | 12.9 | -1.3 | 99.5 |
| 7335 | 62.0 | 85.4 | 0.4 | 99.4 | | |
| 7337 | 21.9 | 20.8 | 14.7 | 29.4 | | |
| 7339 | -7.4 | 12.8 | 0.8 | 14.3 | | |
| 7403 | 4.7 | 4.0 | -40.5 | 6.3 | -4.6 | 30.9 |
| 7548 | 101.8 | 51.1 | 104.5 | 39.2 | | |
| 7594 | 25.9 | 30.5 | 35.2 | 19.3 | -45.2 | 70.6 |
| 7806 | 63.2 | 55.5 | -30.4 | 27.2 | | |
| 7810 | -11.9 | 7.5 | -5.5 | 7.4 | 17.4 | 20.7 |
| 7811 | -19.5 | 20.6 | -24.9 | 17.3 | -58.0 | 71.1 |
| 7820 | 210.3 | 41.8 | 256.6 | 26.8 | | |
| 7824 | 81.1 | 18.2 | 46.3 | 23.1 | 33.1 | 71.1 |
| 7831 | 163.5 | 79.6 | 541.5 | 38.9 | | |
| 7835 | 8.6 | 5.3 | 20.4 | 4.4 | -10.2 | 7.9 |
| 7836 | 23.2 | 10.2 | 25.5 | 10.6 | -33.8 | 31.7 |
| 7837 | -45.1 | 25.6 | 20.2 | 12.1 | 45.9 | 55.9 |
| 7838 | 30.6 | 7.1 | 57.4 | 5.4 | 91.6 | 47.0 |
| 7839 | 3.3 | 1.5 | 15.1 | 1.5 | -7.4 | 6.1 |
| 7840 | 1.8 | 2.6 | 9.5 | 2.7 | -6.1 | 11.7 |
| 7843 | 37.9 | 3.6 | 46.9 | 4.2 | -.2 | 12.3 |
| 7845 | - 1.4 | 4.7 | 12.7 | 5.1 | | |
| 7849 | - 9.6 | 1.9 | 4.1 | 2.3 | 2.1 | 13.8 |
| 7939 | -38.9 | 19.8 | -28.4 | 16.1 | -130.0 | 242.6 |
| 8834 | 1.5 | 3.5 | -7.1 | 3.7 | 56.8 | 43.9 |

which shows a scatter (min-max) of about 50 mm for LAGEOS-1 and LAGEOS-2 (3rd generation systems) and about 70 mm for WESTPAC (idem). The results have also been plotted against each other as was done in Figures 2-7, but such plots did not yield much useful information.

Other options, like studying the correlations between bias solutions obtained for different stations in identical periods, have not been pursued yet, but will be done in subsequent analyses. It will be clear that the computation technique and models will have to be refined further to provide more reliable and useful bias estimates.

Table 4. Range bias solutions and sigmas estimated for scenario B, where half of the stations (responsible for the far majority of the tracking data) were kept fixed at the coordinates of SSC(DEOS)98C01. The positions of stations labeled with an "E" were estimated.

| SCENARIO B | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | LAGEOS-1 | | LAGEOS-2 | | WESTPAC | |
| Station | | Bias (mm) | sigma (mm) | Bias (mm) | sigma (mm) | Bias (mm) | sigma (mm) |
| 1864 | E | 89.2 | 84.6 | 132.8 | 101.5 | 346.8 | 156.9 |
| 1868 | E | 14.8 | 131.2 | -51.2 | 107.5 | | |
| 1870 | E | | | | | -163.9 | 133.4 |
| 1873 | E | -358.4 | 343.4 | -297.7 | 198.1 | | |
| 1884 | | -27.4 | 4.6 | -21.1 | 3.7 | -31.2 | 8.3 |
| 1893 | E | 4.3 | 61.3 | 44.0 | 52.0 | 19.1 | 85.4 |
| 7080 | | -21.8 | 0.8 | -13.0 | 0.8 | -17.5 | 9.6 |
| 7090 | | 30.4 | 0.5 | 36.3 | 0.5 | 19.6 | 2.4 |
| 7105 | | -17.0 | 0.6 | 0.8 | 0.6 | -17.5 | 3.9 |
| 7110 | | 7.9 | 0.5 | 14.0 | 0.4 | -6.2 | 3.0 |
| 7124 | E | -7.4 | 4.8 | 4.2 | 5.2 | -15.9 | 33.1 |
| 7210 | | -60.8 | 0.9 | -52.7 | 0.8 | -77.6 | 3.5 |
| 7236 | E | -356.2 | 191.1 | -223.0 | 167.7 | | |
| 7237 | E | -49.0 | 11.6 | -37.0 | 10.8 | 10.5 | 48.6 |
| 7249 | E | | | 111.8 | 95.8 | | |
| 7328 | E | 0.9 | 16.5 | 14.1 | 12.9 | 0.1 | 100.0 |
| 7335 | E | 62.2 | 85.8 | 0.6 | 100.0 | | |
| 7337 | E | 34.5 | 20.8 | 16.8 | 29.4 | | |
| 7339 | E | -9.2 | 12.8 | 12.7 | 14.3 | | |
| 7403 | | -32.9 | 1.0 | -17.8 | 01.3 | -53.2 | 8.8 |
| 7548 | E | 96.7 | 51.2 | 109.8 | 39.3 | | |
| 7594 | E | 23.4 | 30.5 | 33.9 | 19.4 | 22.7 | 70.8 |
| 7806 | E | 66.2 | 55.6 | -36.1 | 27.2 | | |
| 7810 | | -25.8 | 02.0 | -17.7 | 1.8 | 13.2 | 5.8 |
| 7811 | | -21.0 | 3.1 | -11.9 | 2.3 | -59.4 | 13.4 |
| 7820 | E | 215.8 | 41.9 | 263.6 | 26.8 | | |
| 7824 | E | 79.7 | 18.8 | 74.0 | 24.1 | 35.3 | 73.0 |
| 7831 | | 583.0 | 8.2 | 178.2 | 5.1 | | |
| 7835 | | -44.9 | 1.2 | -39.6 | 1.0 | -40.7 | 2.6 |
| 7836 | | -6.5 | 1.6 | -1.6 | 1.5 | -13.9 | 5.9 |
| 7837 | | 24.5 | 4.5 | 35.5 | 2.5 | 15.5 | 12.1 |
| 7838 | | -2.0 | 1.8 | 6.7 | 1.3 | 41.4 | 12.7 |
| 7839 | | 7.5 | 0.4 | 11.5 | 0.5 | 1.0 | 2.3 |
| 7840 | | -7.7 | 0.5 | -0.6 | 0.6 | -6.6 | 2.7 |
| 7843 | | 8.8 | 0.9 | 4.9 | 1.0 | 4.3 | 3.7 |
| 7845 | E | -9.6 | 4.7 | 9.4 | 5.1 | | |
| 7849 | E | -5.3 | 1.9 | -0.1 | 2.3 | 5.4 | 13.6 |
| 7939 | | -8.1 | 4.7 | -1.5 | 3.7 | 0.5 | 36.4 |
| 8834 | | -17.7 | 0.9 | -18.5 | 0.9 | 31.7 | 16.8 |

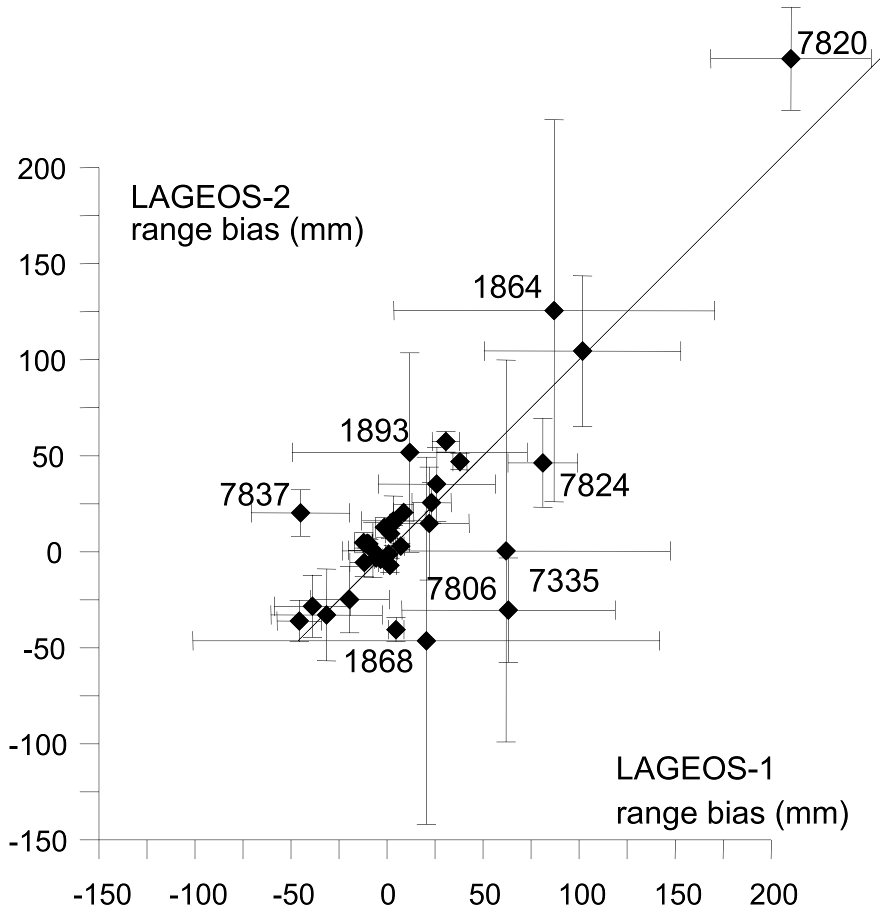


Fig. 2. Estimated range bias values for LAGEOS-1 and LAGEOS-2 and their errors (Sc. A).

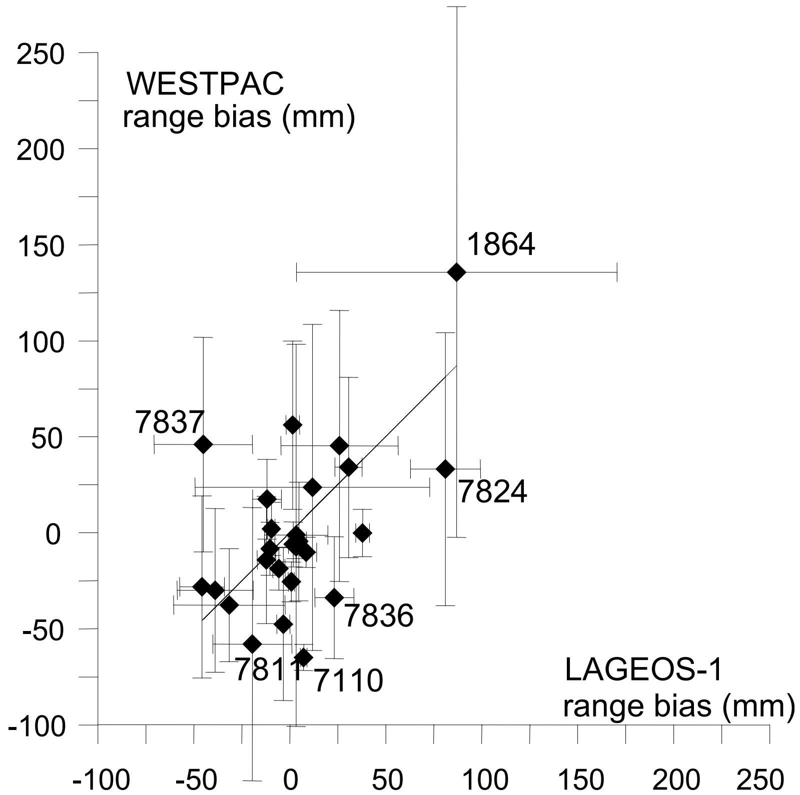


Fig. 3. Estimated range bias values for LAGEOS-1 and WESTPAC and their errors (Sc. A).

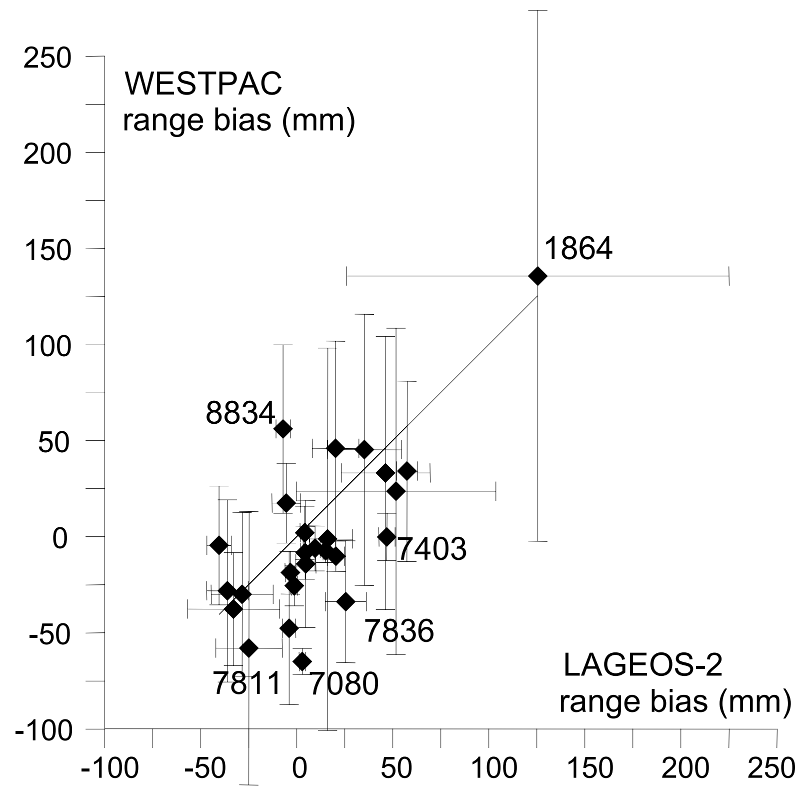

Fig. 4. Estimated range bias values for LAGEOS-2 and WESTPAC and their errors (Sc. A).

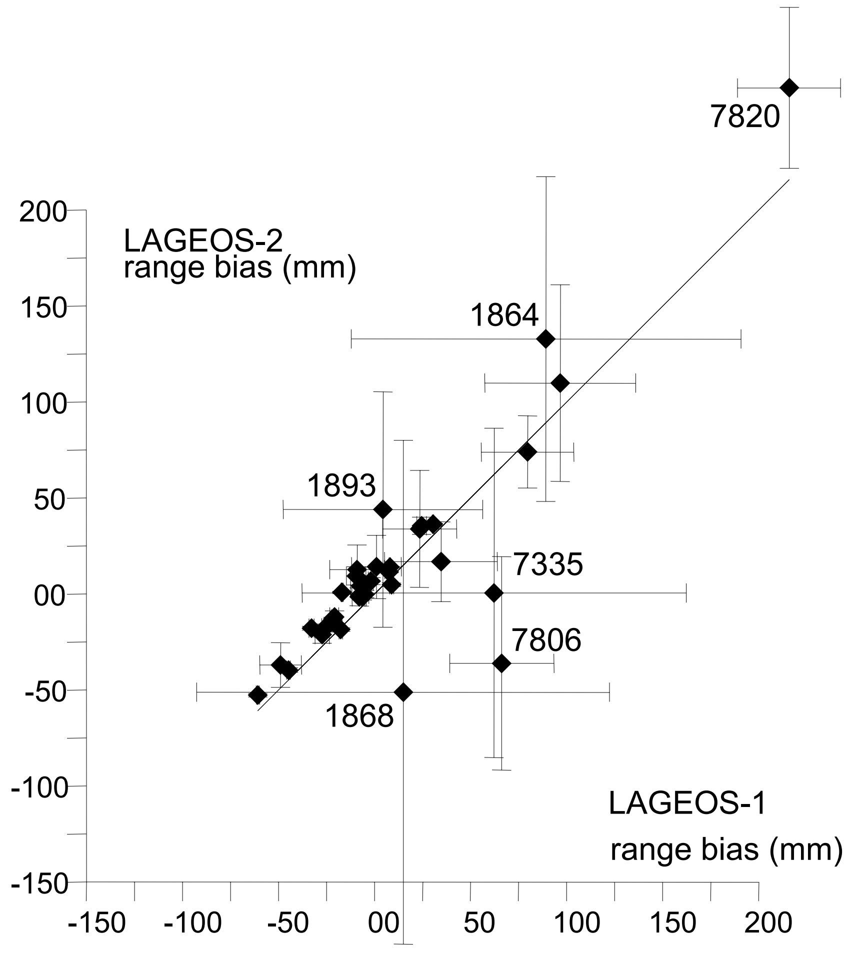

Fig. 5. Estimated range bias values for LAGEOS-1 and LAGEOS-2 and their errors (Sc. B).

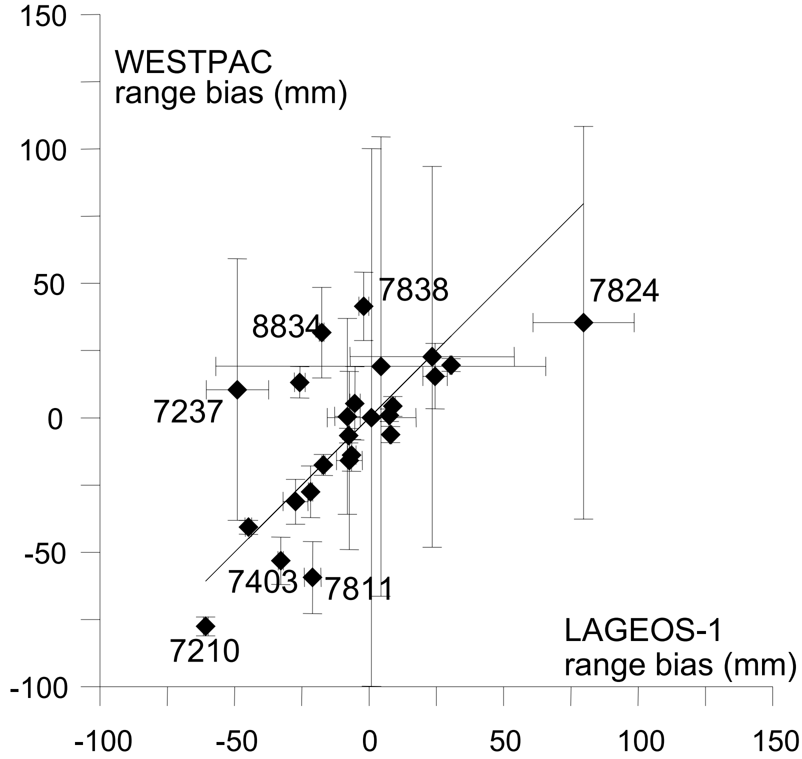

Fig. 6. Estimated range bias values for LAGEOS-1 and WESTPAC and their errors (Sc. B).

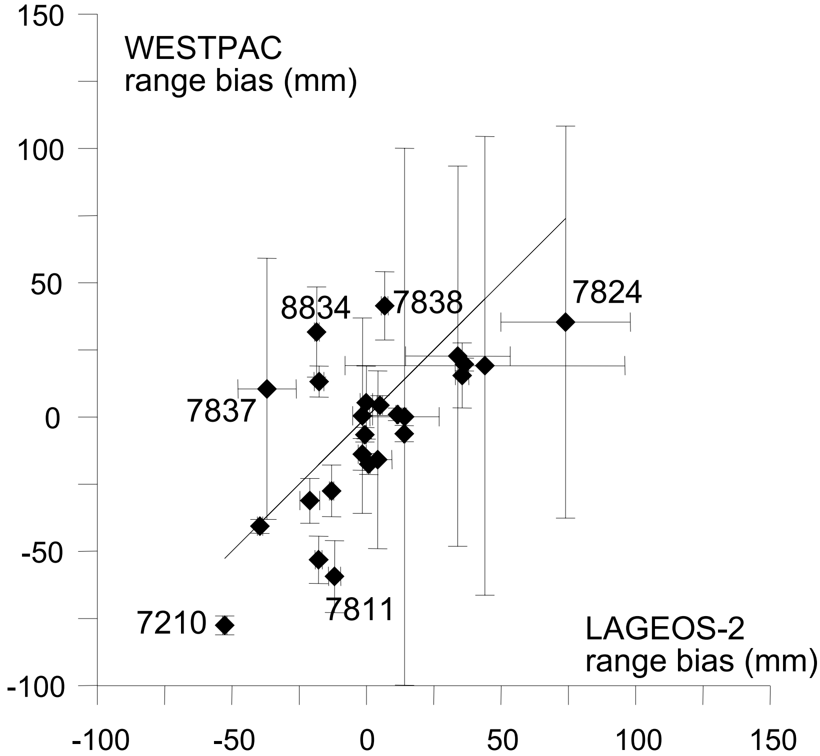

Fig. 7. Estimated range bias values for LAGEOS-2 and WESTPAC and their errors (Sc. B).

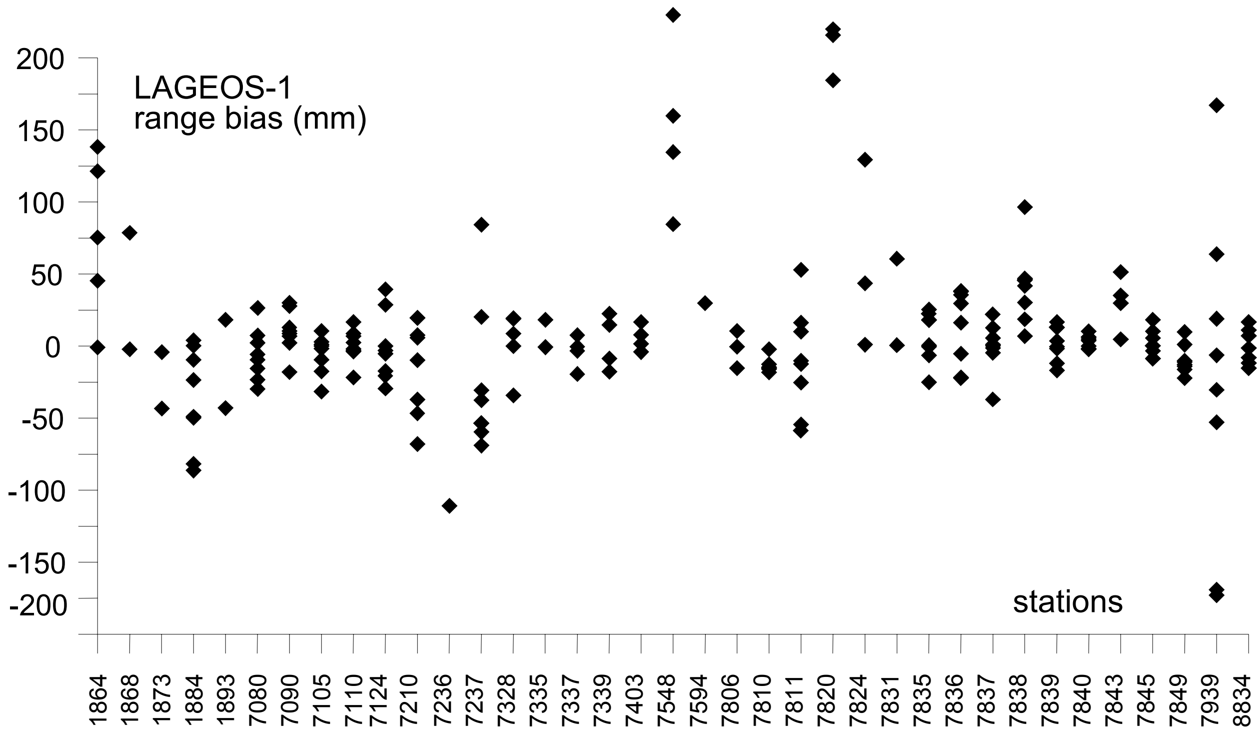


Fig. 8. Range bias solutions obtained for LAGEOS-1, in batches of 4 weeks of data (Scenario C).

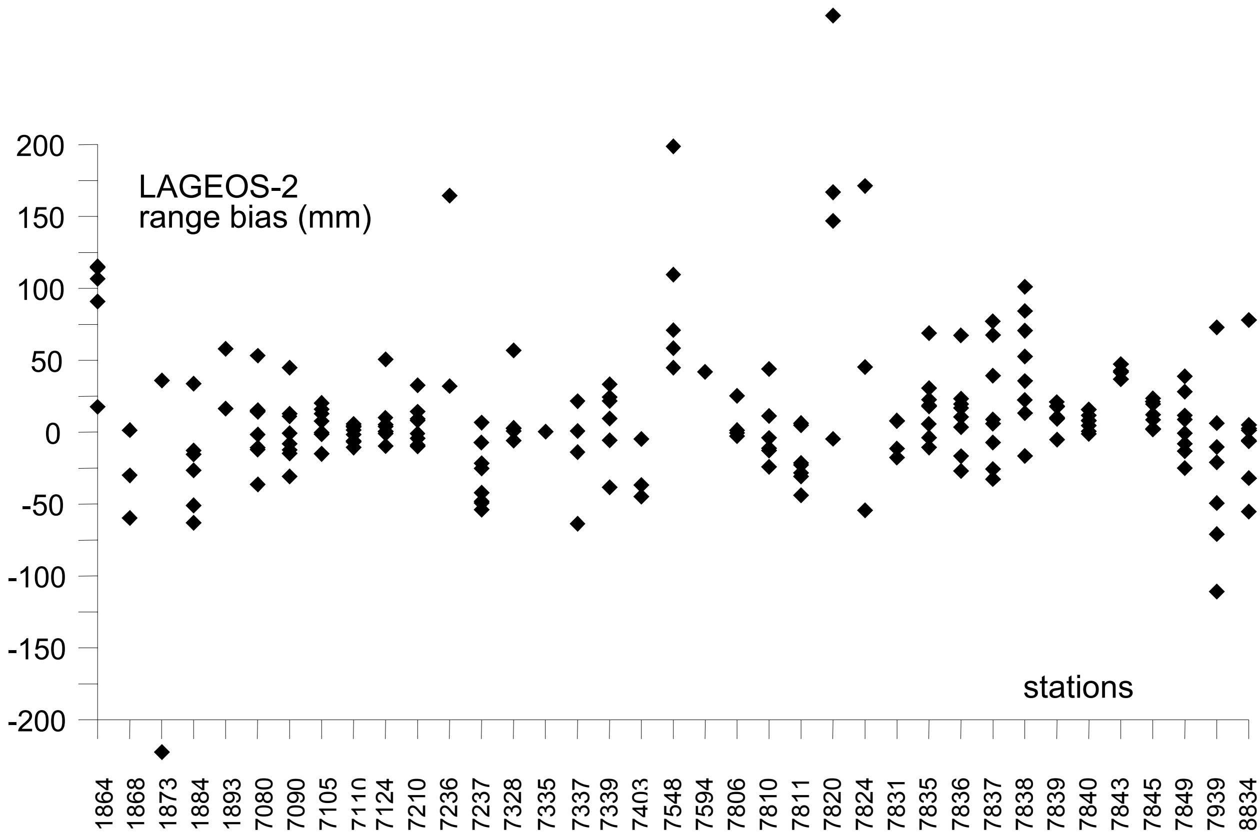


Fig. 9. Range bias solutions obtained for LAGEOS-2, in batches of 4 weeks of data (Scenario C).

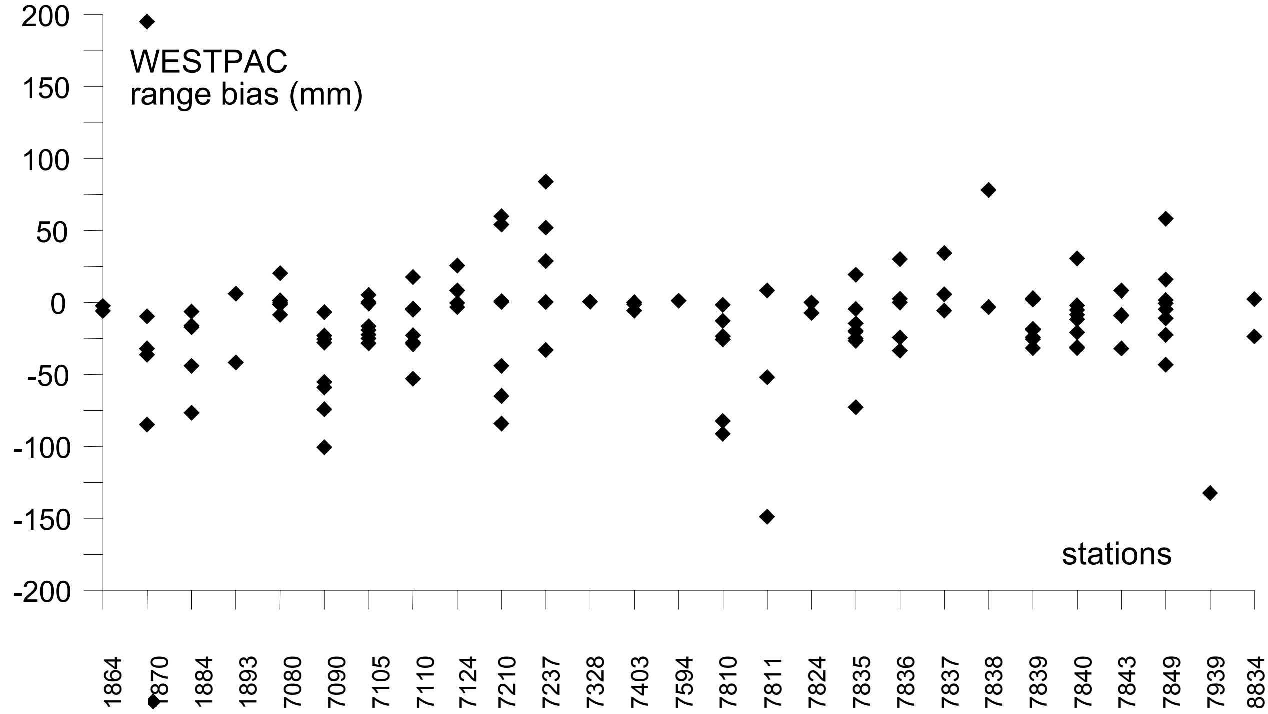


Fig. 10. Range bias solutions obtained for WESTPAC, in batches of 4 weeks of data (Scenario C).

For the reasons mentioned above, the presentation of results for scenario D is limited to the weighted rms-of-fit (Table 2) only; plots of the range bias solutions have not been made since their quality is comparable to that of scenario C. As for the rms-of-fit (Table 2), the conclusions and comments are similar to those of scenario C.

## CONCLUSIONS

On the basis of the current, preliminary analysis the following conclusions can be drawn:

- The number of measurements and passes for WESTPAC is significantly smaller than for LAGEOS-1 and LAGEOS-2.
- The range bias estimates for WESTPAC are relatively poor because of this small amount of observations; in addition, the limited orbit quality (a fit of 3.7 cm vs. 2 cm for LAGEOS-1 and LAGEOS-2) also plays a role.
- LAGEOS-1 and LAGEOS-2 bias solutions are consistent at the level of about 1 cm, but w.r.t. WESTPAC consistencies at the level of 2 cm are observed. The range biases themselves are between –4 and +4 cm and –4 and +6 cm, for LAGEOS-1/2 and WESTPAC, respectively.
- Model assumptions influence the results at the same level as that of the consistencies reported here.
- To fully exploit the unique accuracy of the center-of-mass modelling for WESTPAC and the good orbital quality that can be obtained for the LAGEOS satellites, a fine-tuning of the current analysis is needed.

- The range bias solutions that have been obtained in an analysis of the LAGEOS and WESTPAC observations in batches of 4 weeks yields values in range of 50 and 70 mm, respectively.
- No useful correlations between bias solutions for different satellites can be obtained for data intervals of 4 weeks, at least not with the current analysis technique and computation model.
- A further development of the analysis technique and computation model is required to be able to drive information on the characteristics of the SLR observations at mm level. This applies to all scenarios investigated here.

## ACKNOWLEDGEMENTS

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