Experimental Verification of the MARINI-MURRAY Model by Two Colour SLR

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Abstract

Two colour laser ranging to artificial satellites is an attractive technique, which is capable to provide refraction corrected ranges without the need of an atmospheric model by measuring the dispersive delay of laser pulses of different wavelength. Although the required accuracy of the detection scheme is stringent, the technique has matured so far, that routine two colour observations became feasible. The present paper describes the verification of the MARINI-MURRAY model by using two colour laser range observations reduced with a normal point procedure, exploiting the knowledge of satellite response signatures in conjunction with detector characteristics and the appropriate center of mass correction models. Moreover the dispersion model of the atmosphere is briefly reviewed, paying attention to the wavelength domains provided by modern two colour ranging lasers, e.g. the Ti:SAP laser. Preliminary data is presented and compared to both, normal point data reduced with a standard procedure and zenith path equivalent meteorological parameters.

1 Introduction

There have been several experiments [Schreiber et al.(1994)], [Lucchini(1995)], [Zagwodzki et al.(1997)], [Riepl and Schreiber (1997)] aiming at the determination of the atmospheric correction by two colour satellite laser ranging (SLR). Although the state of the art experiments, employing streak cameras, were found to be impractical for routine observations due to their complex design, there are nowadays a few SLR systems being capable to deliver ranging data at two different laser frequencies. In order to exploit this data, the present paper proposes a normal point procedure following a philosophy introduced by [Greene and Herring,(1986)], which was termed "difference average range" (DAR), i.e. the range difference is obtained from an ensemble average of range measuremnts (normal points) of two laser wavelengths. The other opportunity, "average range differences", was found to be impractical [Riepl and Schreiber (1997)], [Zagwodzki et al.(1997)] due to the statistical jitter of the received pulse amplitude caused by atmospheric turbulence. Moreover the DAR technique has several advantages:

- Conventional Detectors (APD, MCP) can be used
- Detector characteristics can be included
- Satellite signatures allow for deconvolution

Although most recent satellite response function measurements allow for modelling of the center of mass correction at or below the millimeter level, which is encouraging for two colour SLR, the sampling intervals to obtain those accurate response functions are quite large. So one ends up with a trade off between the achievable accuracy of a two colour range reduction and the sampling interval length, limiting the resolution in elevation. The latter fact leads to the determination of the zenith value range correction by two colour measurements on a pass by pass basis as a first approach.

2 Review of the MARINI-MURRAY model

Originally developed for use with the ruby laser wavelength, the MARINI-MURRAY model still serves as a standard for reducing SLR measurements. For two colour SLR, the requirements of the dispersion model for dry air are quite stringent.

Figure 1 gives a comparison between the dispersion model used by MARINI-MURRAY, developed by [Barrel and Sears(1939)] and the nowadays recommended model of [Ciddor (1966)] normalized to the ruby wavelength of $0.6943 \mu m$. The discrepancy between these models is neglegible for wavelengths larger than $0.5 \mu m$. In the UV region the differences are increasing presumably due to the lack of experimental data below $0.44 \mu m$, from which the dispersion model of BARREL and SEARS was deduced. When scaled to a zenith path delay of $2.5m$, the difference in the models indicates a systematic error of about $1mm$ for a system ranging at $427.5nm$ like the Zimmerwald and TIGO SLR system, if we assume the CIDDOR formula is more accurate.
Due to the insensitivity of the two colour range differences with respect to the water vapour content of the atmosphere, the MARINI-MURRAY model was reinvestigated in terms of the approximations associated with this constituent in the atmospheric reduction formula. Moreover a dispersion formula was included for wavelngths smaller than $0.5 \mu m$ due to the above mentioned discrepancies. The resulting formulas along with the involved constants are shown in table 1. The range correction ΔR is obtained through the atmospheric constants g_1, g_2 and a separate term g_3 for the water vapour.

The two colour range correction is obtained through the formulas given in table 2. The atmospheric parameters g_1 and g_2 can be derived from the measured differential range $\Delta R(\lambda_2, \lambda_1, \theta_w)$ given by equation 11 and correspond to the zenith path delay. Moreover they can be compared with the values obtained

$$\Delta R(\lambda, \theta_w) = f_{Gr}(\lambda)\left[\frac{g_1}{\sin(\theta_w)} + \frac{g_2}{\sin^3(\theta_w)}\right] + \frac{g_3}{\sin(\theta_w)} \tag{1}$$

$$g_1 = 80.343 \times 10^{-6}\left[\frac{1}{g(\phi, H)}\frac{\mathcal{R}}{M_d\bar{g}}P(0) + \left(1 - \frac{M_w}{M_d}\right)\frac{\mathcal{R}}{4M_d\bar{g}}P_w(0)\right] \tag{2}$$

$$g_2 = 10^{-6}\frac{80.343\mathcal{R}^2}{R_E M_d^2\bar{g}^2}P(0)T(0)K + 10^{-12}\frac{80.343^2 2\mathcal{R}P(0)^2}{4M_d\bar{g}T(0)\left(3 - \frac{1}{K}\right)} \tag{3}$$

$$g_3 = -10^{-6}\frac{11.3\mathcal{R}}{g(\phi, H)4M_d\bar{g}}P_w(0) \tag{4}$$

$$g(\phi, H) = 1 - 0.0026\cos(2\phi) - 0.00031H \tag{5}$$

$$K = 1.163 - 0.00968\cos(2\phi) - 0.00104T(0) + 0.00001435P(0) \tag{6}$$

$$f_{Gr}(\lambda) = \begin{cases} f_{\text{Barrel\&Sears}} & : \ 0.44\mu m < \lambda < 0.64\mu m \\ f_{\text{Ciddor}} & : \ 0.23\mu m < \lambda < 2.06\mu m \end{cases} \tag{7}$$

$$f_{\text{Barrel\&Sears}} = 0.9650 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4} \tag{8}$$

$$f_{\text{Ciddor}} = \frac{k_1(k_0 + \sigma^2)}{(k_0 - \sigma^2)^2} + \frac{k_3(k_2 + \sigma^2)}{(k_2 - \sigma^2)^2} \tag{9}$$

where

$$\begin{array}{rcl rcl}
\mathcal{R} & := & 8314.36\ mJ/K/mol & \bar{g} & := & 9.784\ m/s^2 \\
R_E & := & 6378\ km & M_d & := & 28.966\ g/mol \\
M_w & := & 18.016\ g/mol & k_0 & := & 238.0185 \\
k_1 & := & 210.7486 & k_2 & := & 57.362 \\
k_3 & := & 6.109744 & \sigma & := & 1/\lambda
\end{array}$$

Table 1: *Reviewed MARINI-MURRAY atmospheric refraction correction formulas giving the range correction ΔR in meters for a site at latitude ϕ and height H above the reference ellipsoid in kilometers. θ_w denotes the true angle of elevation. The parameters $P(0)$,$T(0)$ and $P_w(0)$ are the total pressure in millibar, temperature in Kelvin and the partial pressure of water vapour in millibar. The wavelength λ is input in microns. For wavelengths smaller than $0.5\mu m$ it is recommended to use the dispersion formula derived by [Ciddor (1966)].*

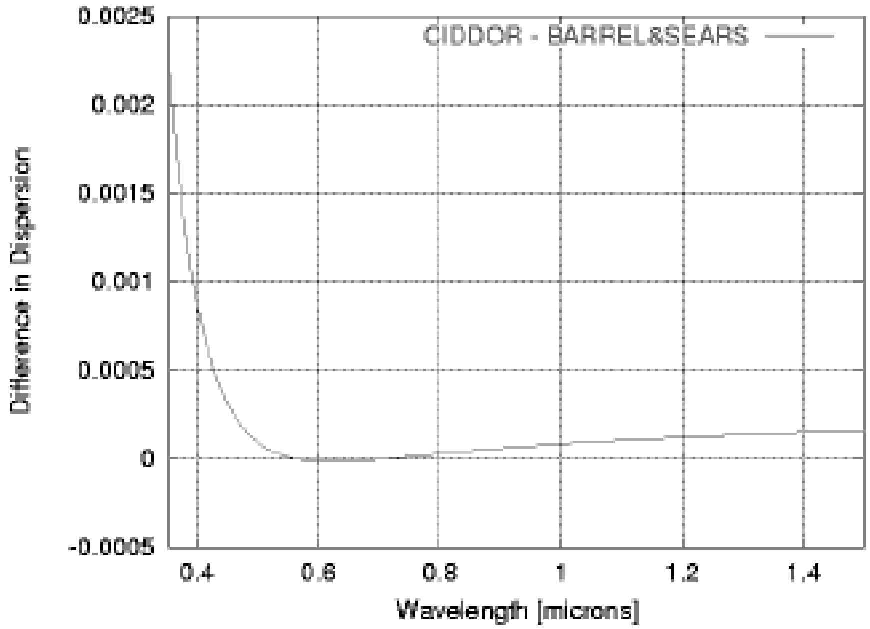

Figure 1: *Differences between the dispersion formulas of CIDDOR and BARREL & SEARS.*

from meteorological parameters at the observing station through equations 2 and 3. The term g_3 may be calculated from the water vapour partial pressure at the ground $P_w(0)$. The latter quantity can be even remote sensed by a water vapour radiometer, which measures the zenith value of the wet path delay ΔR_{wp} for microwaves. Equation 12 relates this value to an equivalent water vapour partial pressure at the ground.

3 Normal Point Procedure

The two colour normal point procedure can be cast into the following steps:

- Calculate residuals of each wavelength using the RGO normal point procedure SOLVE giving a residual distribution with respect to the mean of the detrended measurements.
- Calculate the satellite transfer function STF for each wavelength.

$$\Delta R(\lambda_1, \theta_w) = \frac{f_{Gr}(\lambda_1)\Delta R(\lambda_2, \lambda_1, \theta_w)}{f_{Gr}(\lambda_2) - f_{Gr}(\lambda_1)} + \frac{g_3}{\sin(\theta_w)} \tag{10}$$

$$\Delta R(\lambda_2, \lambda_1, \theta_w) = \Big[f_{Gr}(\lambda_2) - f_{Gr}(\lambda_1)\Big]\left[\frac{g_1}{\sin(\theta_w)} + \frac{g_2}{\sin^3(\theta_w)}\right] \tag{11}$$

$$P_w(0) = -\frac{\Delta R_{wp}}{M_w k_{wp}} \mathcal{R}\beta T(0)(\delta - 1) \times 10^{-5} [mbar] \tag{12}$$

$$\begin{array}{lcl lcl} k_{wp} & := & 1.723 \times 10^{-3} m^3 K/g & \delta & := & 4\gamma \\ \gamma & := & -\frac{\bar{g} M_d}{\mathcal{R}\beta} & \beta & := & -6.5 \times 10^{-3} K/m \end{array}$$

Table 2: *Two colour range correction formulas including the contribution g_3 from water vapour. g_3 can be obtained from ground based measurements of the water vapour partial pressure, as well as from the wet path delay ΔR_{wp} measured by a water vapour radiometer through equation 12.*

- Form the incoherent satellite respnse function (ISR) by convolution of the STF and the flat target response (FTR) in order to take the individual detector characteristics into account by use of a satellite response model [Fitzmaurice et al.(1977)].
- Convolve the ISR with the residual distribution leading to a smoothed residual distribution with a mean value corresponding to the center of mass correction for each wavelength.
- Form the differential ranges $\Delta R(\lambda_2, \lambda_1, \theta_w)$ using the obtained orbit approximations from SOLVE including the center of mass correction obtained in the previous step. Obtain the parameters g_1 and g_2 by a least squares fit of equation 11.
- Form refraction reduced and center of mass corrected normal points by use of equation 10 taking into account the water vapour contribution by deriving g_3 either from water vapour radiometer data or from ground based measurements.

Figure 2 gives an illustration of the tasks performed to obtain the center of mass correction from the residuals in each wavelength.

4 Preliminary Results

The procedure discussed in the previous section is applied to a two colour streak camera measurement carried out to STARLETTE (see also [Riepl and Schreiber (1997)]). Residuals are formed from range detections at the first and second harmonic Nd:YAG wavelengths. Further the residuals are convolved with the ISR giving the center of mass offset with respect to the approximate ranges obtained from the orbit fit. From the data sets of each wavelength the range differences

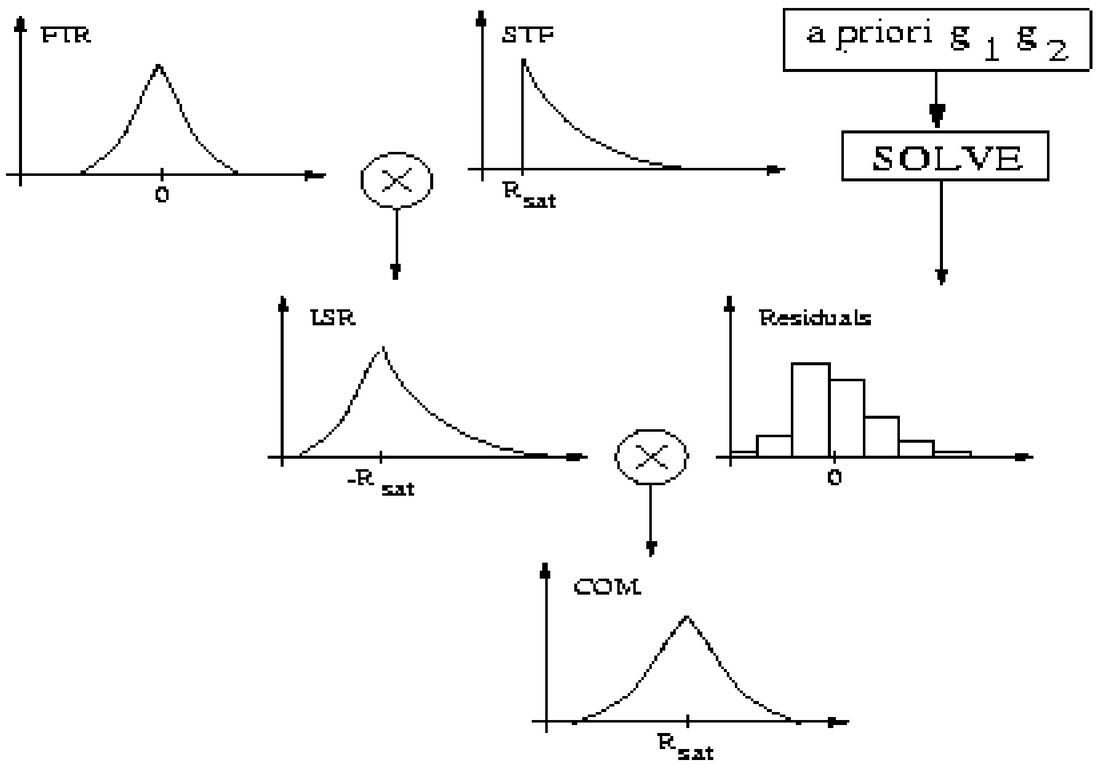

Figure 2: *Signal processing applied to residuals from SOLVE to obtain center of mass corrected range measurements.*

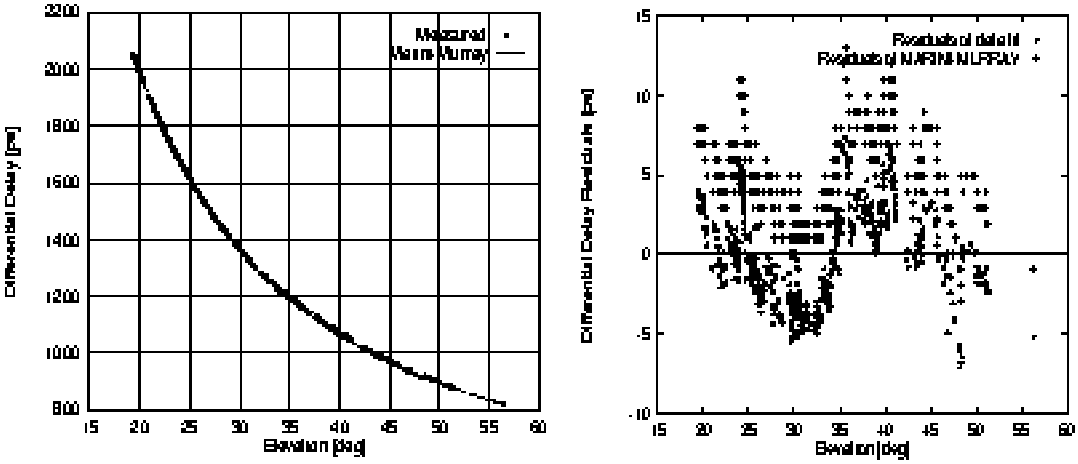

Figure 3: *Differential delay (left) obtained from a streak camera measurement to STARLETTE. The line indicates the differential delay as expected from the model of MARINI and MURRAY. The right plot shows the residuals with respect to both, a regression of parameters g_1 and g_2 and the conventional model of MARINI and MURRAY.*

$\Delta R(\lambda_2, \lambda_1, \theta_w)$ are calculated as a function of the true angle of elevation, which is shown in figure 3. The right plot of figure 3 indicates an agreement between the measured differential delay and those derived from the model of MARINI and MURRAY at the level of 10ps (round trip time), which translates into a range correction accuracy of about 3cm. There seems to be also a constant offset of 5ps between the measured and modeled differential delay.

The result of the regression for g_1 and g_2 obtained from $\Delta R(\lambda_2, \lambda_1, \theta_w)$ as well as a comparison to these parameters derived from ground based meteorological data is given in table 3. The standard deviation for the coefficient with major influence g_1 can be measured with submillimeter accuracy and differs from the value derived from meteorological data by 19mm. The parameter g_2 derived from the measurement differs in sign, with respect to the meteorological derived one. This indicates that the applied mapping function is not adequate over the elevation range the measurement was caried out. To overcome this discrepancy the parameter estimation would have to be carried out over smaller elevation intervalls, keeping an eye on the achievable accuracy of statistical means.

Coefficient	derived from meteorological data $[mm]$	derived by differential delay $[mm]$	standard deviation $[mm]$
g_1	2217.96	2236.97	0.12
g_2	2.56	-3.54	0.03

Table 3: *The coefficients g_1 and g_2 calculated from meteorological data in contrast to those obtained from the regression of the differential delay.*

5 Conclusion and Future Work

A summary of formulas have been given herein which enable for

- accurate refraction correction for wavelengths smaller than $0.5\mu m$,
- determination of zenith range correction values from two colour SLR measurements, which can be compared to the model of MARINI and MURRAY,
- a two colour normal point procedure accurate enough to supply refraction reduced and center of mass corrected normal points.

It is planned to supply time series of the parameters g_1 and g_2 derived from two colour measurements to test the applied mapping function. Moreover a time series of g_3 derived from water vapour radiometer data will be compared to conventional water vapour measurements.

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