Challenges to Achieving Millimeter Accuracy Normal Points in Conventional Multiphoton and kHz Single Photon SLR Systems

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Current Sources of Range Bias

- Pulse time-of-flight measurements in legacy multiphoton and kHz single photon systems are currently all based on threshold detection of the start and stop pulses, where changes in signal amplitude can impact the measured range.
- In legacy multiphoton systems, amplitude variations in the start channel are generally small and large variations in the stop channel can be greatly reduced through the use of Constant Fraction Discriminators (CFDs).
- True single photon sensitive kHz systems are subject to "first photon range bias" as the received signal strength increases. As a result, they are often run with return rates of 10% or less to greatly favor single photon returns. The result is a "bias free" range measurement but with a large variance Probability Distribution Function (PDF). The large variance, in turn, requires long time intervals to achieve a 1 mm precision Normal Point (NP), thereby greatly reducing the number of satellites tracked and greatly extending the length of the satellite arc that the NP represents.
- Nevertheless, in both types of system, the PDFs for photon events in the start and stop channels are different since the satellite signature affects only the PDF of the stop channel. Thus, a range bias in the normal point can result unless the start and stop times are determined by the <u>centroids</u> of the distributions rather than a simple threshold crossing.
- The PDF of the start channel/instrument is well-mapped by ranging to a single cube calibration target which has a delta-function impulse response.

Central Limit Theorem (CLT)

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If two or more photons are sampled per pulse, the CLT tells us that, on average, the mean of n samples is equal to the mean of the parent distribution, t_c. Thus, no bias is introduced by the multiphoton measurement. As the mean number of photons per pulse increases (n>15), the PDF distribution becomes highly Gaussian in shape, i.e. a “normal” distribution. For an arbitrary distribution of single photons, one can prove the following for the mean and variance of the n-photon distribution:

<u>n-photon mean</u>

$$\left\langle \frac{1}{n}\sum_{i=1}^{n} t_i \right\rangle = \frac{1}{n}\sum_{i=1}^{n}\langle t_i\rangle = \frac{1}{n} n t_c = t_c$$

Therefore the n-photon mean is unbiased

<u>n-photon variance</u>

$$\sigma_n^2 \equiv \left\langle \left(\frac{1}{n}\sum_{i=1}^{n} t_i\right)\left(\frac{1}{n}\sum_{i'=1}^{n} t_{i'}\right)\right\rangle - t_c^2 = \frac{1}{n^2}\left\langle \sum_{i=1}^{n} t_i^2 + \sum_{i=1}^{n} t_i \sum_{i'\neq i}^{n} t_{i'} \right\rangle - t_c^2$$

$$= \frac{1}{n^2}\left[n\langle t^2\rangle + n(n-1)t_c^2\right] - t_c^2 = \frac{\langle t^2\rangle - t_c^2}{n} = \frac{\sigma_1^2}{n}$$

So the n-photon mean has a variance n times smaller than the single photon distribution

Detection Probability and Normal Point Precision

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For a SLR system with a single photon detection threshold, the probability of detecting the satellite signal is

$$P_d(n) = 1 - \exp(-n)$$

and the number of range measurements contributing to a satellite "normal point" is

$$N = P_d(n) f_L \tau_{np} = \left(1 - e^{-n}\right) f_L \tau_{np}$$

where
f_L = the laser repetition rate = 2 kHz for SGSLR
τ_{np} = the normal point time interval
and the desired normal point precision is equal to

$$\sigma_{np} = \frac{\sigma_n}{\sqrt{N}} = \frac{\sigma_1}{\sqrt{nN}} = \frac{1}{\sqrt{nN}} \sqrt{\sigma_L^2 + \sigma_D^2 + \sigma_{ET}^2 + \sigma_S^2} \approx 1mm$$

where σ_1 is the satellite-dependent , single pulse, single photon range precision obtained from the contributions of the laser (L), detector (D), Event Timer (ET), and Satellite (S). Thus, the integration time required to generate a normal point with precision σ_{np} is given by

$$\tau_{np} = \frac{N}{\left(1 - e^{-n}\right) f_L} = \frac{1}{n\left(1 - e^{-n}\right) f_L} \left(\frac{\sigma_1}{\sigma_{np}}\right)^2 \qquad n \geq 1$$

$$\frac{1}{\left(1 - e^{-n}\right) f_L} \left(\frac{\sigma_1}{\sigma_{np}}\right)^2 \qquad n < 1$$

SGSLR LAGEOS 1mm NP Integration Time vs n

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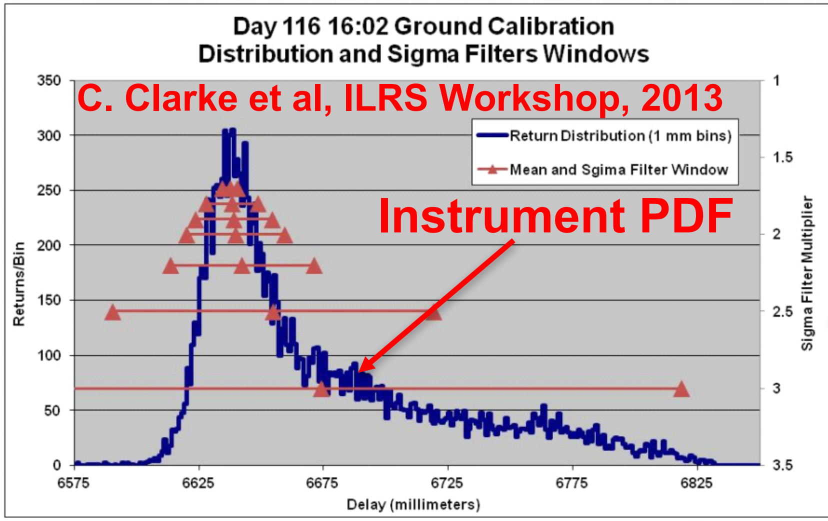
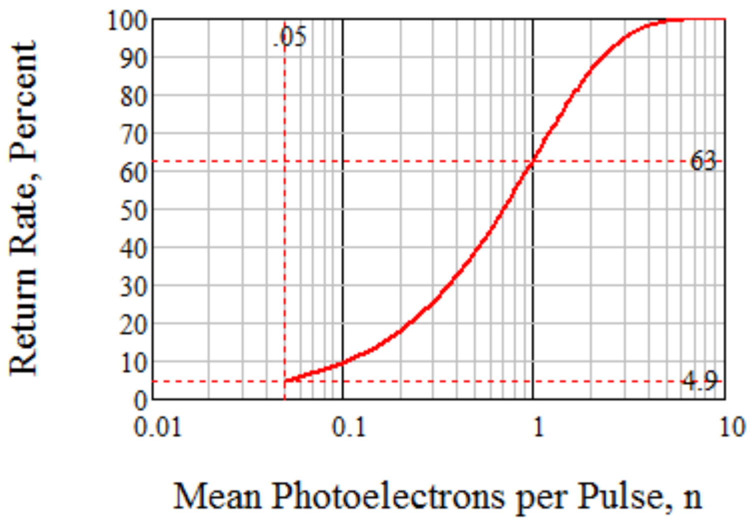
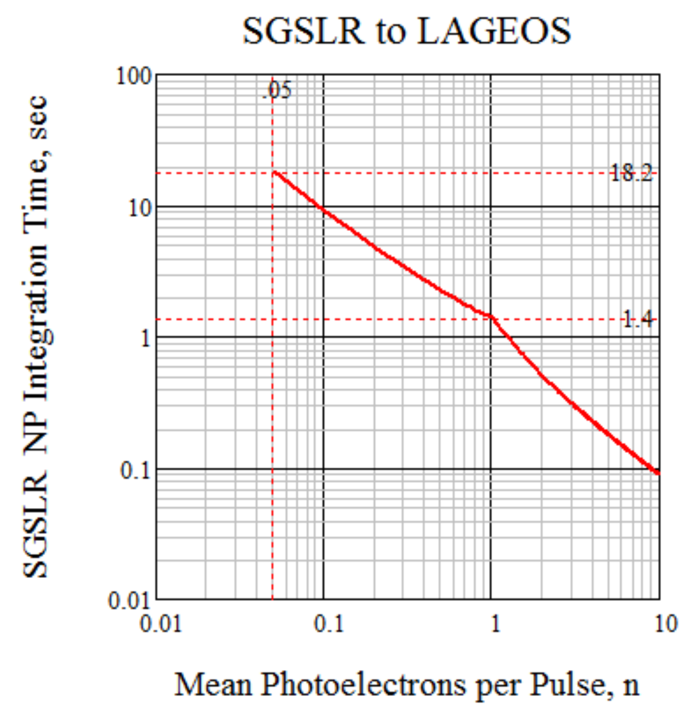


$$\sigma_1 = \sqrt{\sigma_L^2 + \sigma_D^2 + \sigma_{ET}^2 + \sigma_S^2} = \sqrt{\sigma_{inst}{}^2 + \sigma_S^2} = 42mm$$

σ_L = 6.3mm for 50 psec FWHM Laser
σ_D = 39.9 mm for MCP/PMT
σ_{ET} = 3.4 mm for baseline Sigma ET
σ_S = 11.5 mm for LAGEOS signature
σ_{inst} = instrument RMS ~ 40.5mm

~225 mm = 5.6 σ_{inst}





Suitable Centroid Detectors

- The detector must be sensitive to single photons and have the ability to record multiple single photon events on a single anode.
- MicroChannel Plate PhotoMultiplier Tubes (MCP/PMTs) and Silicon Photomultipliers (SIPMTs) meet this requirement, but single stop detectors like C-SPADs do not.
- MCP/PMTs have thousands of microchannels and SiPMTs have hundreds to thousands of individual APDS, separated spatially by a few microns, which are capable of recording multiple single photon events and combining the outputs onto a single anode.
- The finite size of the satellite image in the telescope focal plane ensures that multiple microchannels or SiAPDs are illuminated and can be further blurred if required.
- Since the photons coming back from the satellite are grouped too closely together in time to permit the measurement of individual photon arrival times, the individual photons will create an irregular and complex single pulse of varying amplitude out of the anode whose centroid must be determined.
- Fortunately, radar engineers have developed simple circuits to measure the centroid of an irregularly shaped pulse.

Centroid Detector for Radar

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US Patent #3,906,377 (Sept. 16, 1975)

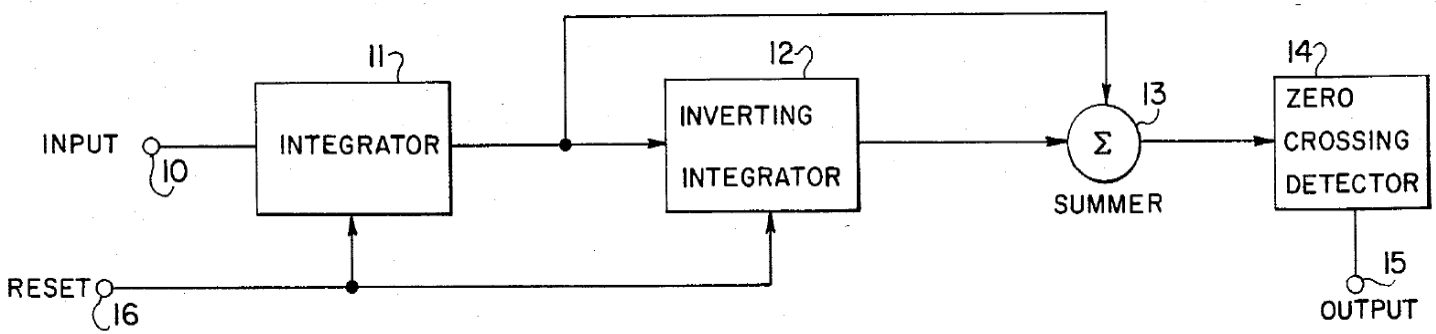
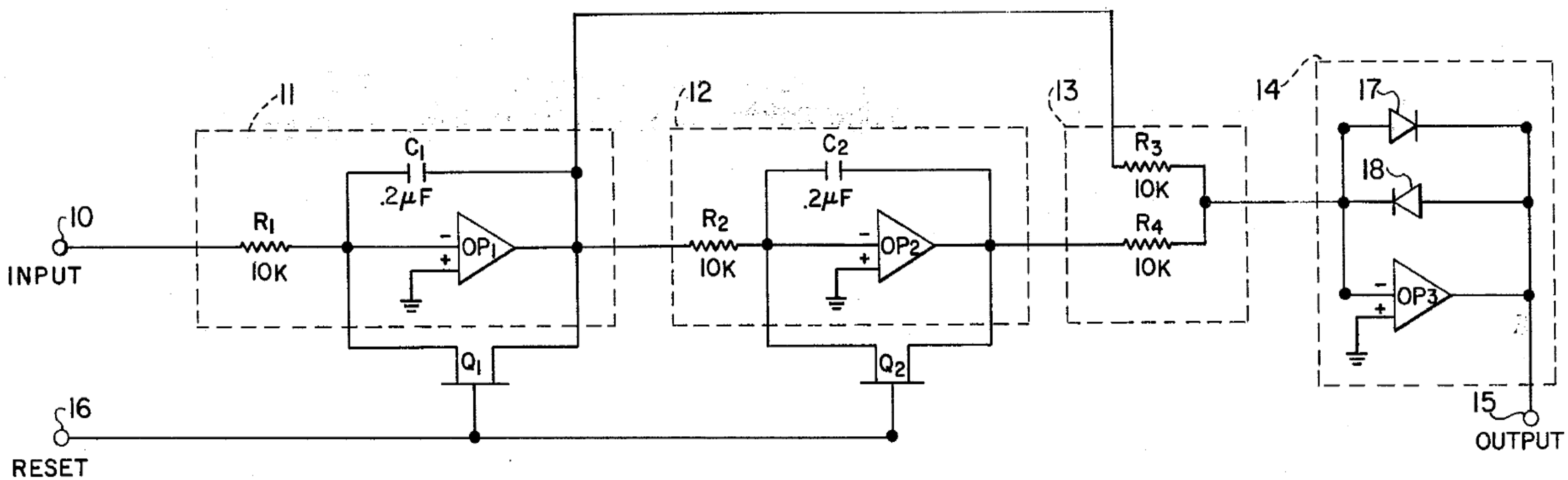
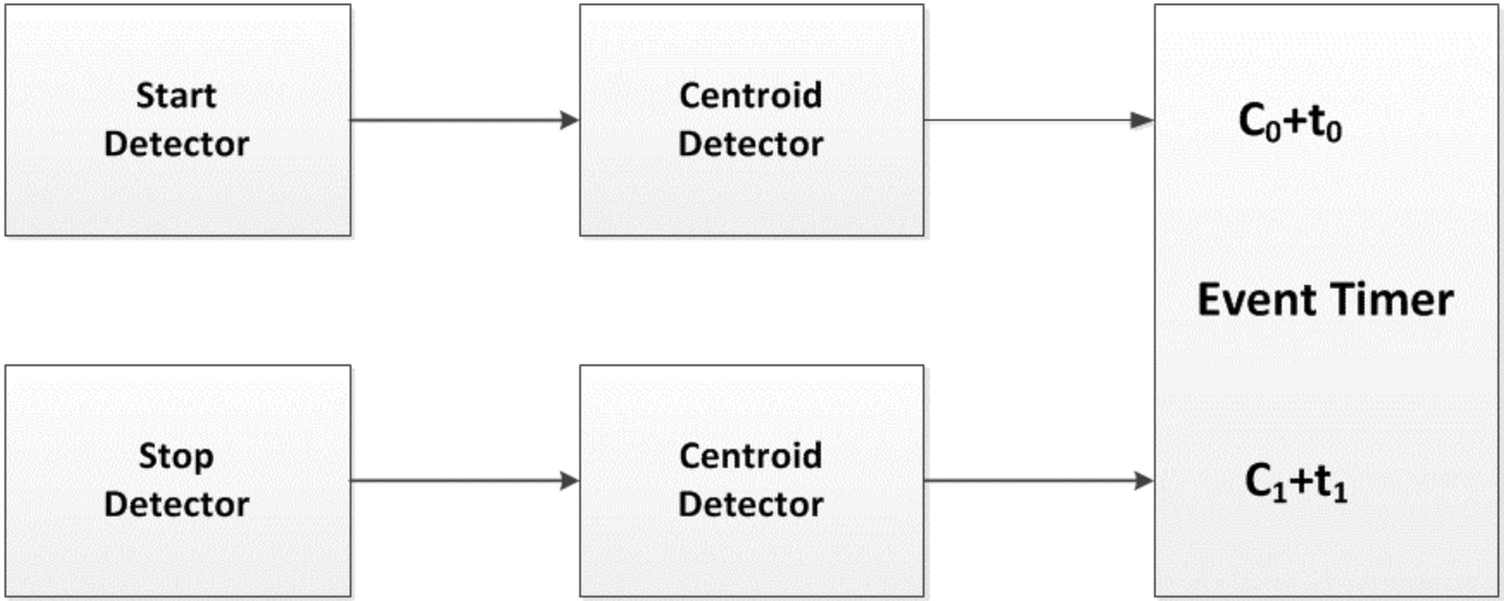


FIG. 1



Field Calibration



The measured TOF to the calibration target is given by

$$TOF = (C_1 - C_0) + (t_1 - t_0)$$

where t_1 and t_0 are the centroids of the stop and start pulses respectively and ($t_1 - t_0$) is the actual pulse TOF. The constant (C_1-C_0) is determined by ranging to a single cube corner at a known distance R_{cal} via the equation

$$(C_1 - C_0) = TOF - 2R_{cal} / c$$

and subtracted from all satellite TOF measurements to obtain a bias free range.

CD Configuration for SiPM Array Detector

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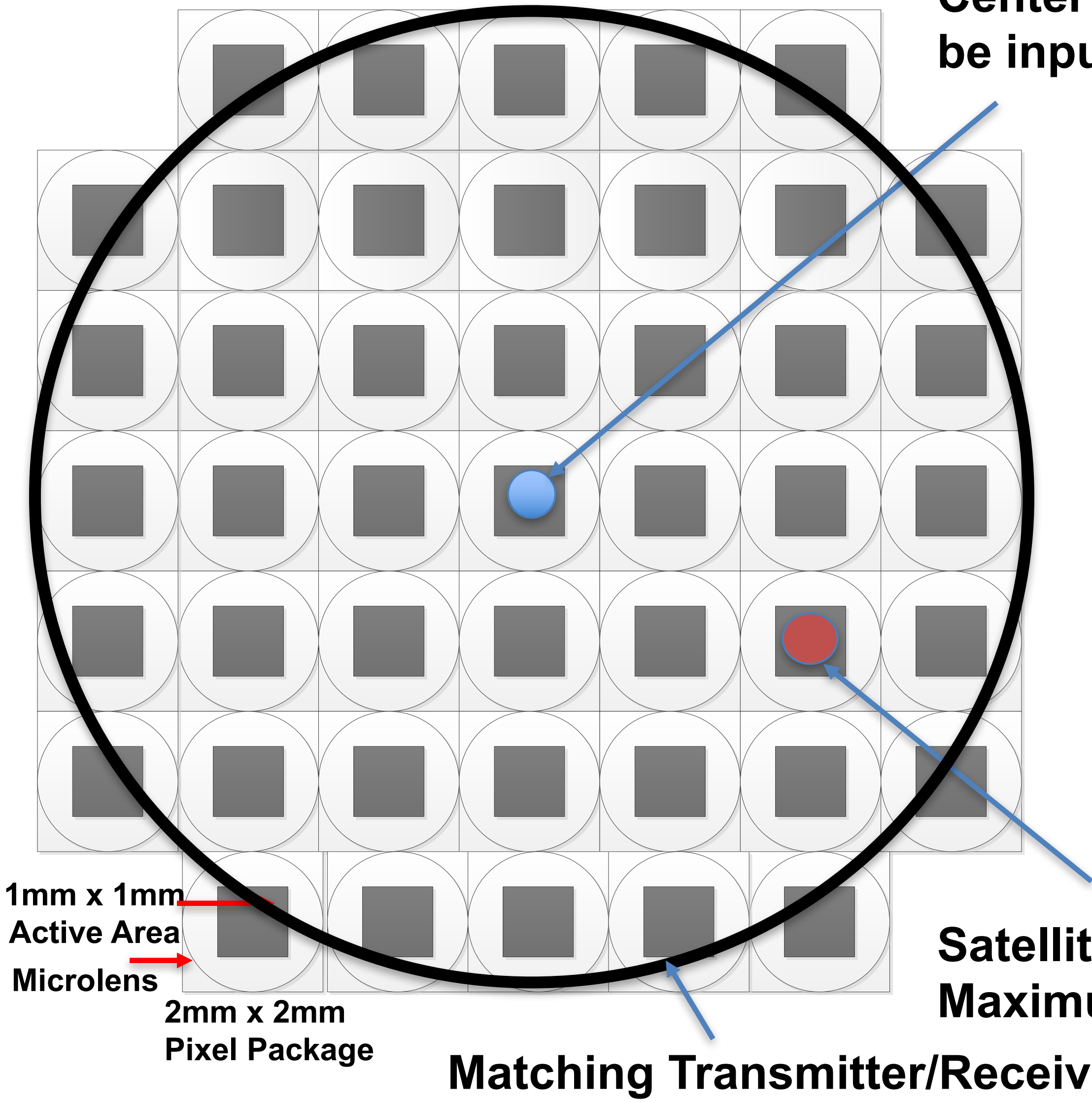
Center Pixel(s) of Telescope FOV can be input to a single Centroid Detector



<u>Quasi 7x7 Array (45 pixels)</u>

- Single TOF card can handle all 45 pixels plus additional time events with few psec resolution.
- Can monitor large FOV with adequate angular resolution (FOV/7)
- Central pixel corresponds to center of telescope FOV.
- Each SiPM pixel has a 1mmx1mm active area but the current mechanical packaging limits the spacing between pixels to 2mm.

<u>Laser-Ablated Microlens Array (LAMA)</u>

- The LAMA gathers most of the light falling within the 2mm x 2mm and distributes it within the active area.

Summary

- According to the Central Limit Theorem (CLT), use of a Centroid Detector (CD) on the start and stop channels results in a bias-free range measurement which is independent of signal strength, thereby eliminating range bias inherent in current threshold based timing systems.
- Centroid detection can remove bias in both heritage multiphoton systems as well as the newer kHz single photon sensitive systems.
- Operating at higher signal strengths in single photon sensitive kHz lidars can result in 1 to 2 orders of magnitude shorter integration times necessary to achieve 1 mm precision normal points. This in turn results in greatly reduced satellite arc lengths per normal point and greatly increased station productivity in terms of number of satellites tracked.
- For large receiver arrays, such as SGSLR's Multifunctional Range Receiver designed to maximize received signal strength by monitoring and automatically correcting for telescope pointing errors, it is adequate to tie the central pixel (or a small number of central pixels) to a single CD stop channel since the other pixels are primarily used during acquisition or reacquisition of the satellite. This has the added advantage of greatly reducing (by up to 98%) the number of detector dark counts and solar counts not associated with the pixel containing the satellite return and therefore seen by the CD. Nevertheless, the performance of CDs under low SNR conditions should be studied.