Challenges to Achieving Millimeter Accuracy Normal Points in Conventional Multiphoton and kHz Single Photon SLR Systems

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Abstract. *The concept of "centroid detection" is introduced as a bias-free alternative to conventional "threshold detection" used in both legacy multiphoton and the newer kHz single photon sensitive SLR stations, to achieve more accurate range measurements and shorter normal point integration times.*

1. Introduction

Current GGOS goals of achieving 1 mm accuracy and RMS precision present the biggest challenges to the ILRS network ,which currently consists of both legacy multiphoton systems as well as the newer kHz single photon sensitive systems. The latter include NASA's planned Space Geodesy Satellite Laser Ranging (SGSLR) network [McGarry et al, 2017] derived from earlier SLR2000/NGSLR system development. Collocation experiments carried out in 2013 between NASA's network standard MOBLAS-7 and NGSLR demonstrated good agreement between LAGEOS normal points (within 2 to 3 mm) provided the centroid of the NGSLR normal point distribution was used to compute the satellite range rather than the usual 2.5 σ filter centered on the peak of the distribution, which resulted in range differences on the order of 12 to 13 mm [Clarke et al, 2013]. This behavior can be explained by treating the transition between no detection and detection as a two stage Markov process [Degnan, 1994] which causes the Probability Distribution Function (PDF) to be highly skewed in favor of shorter ranges as the signal strength increases or the detection threshold decreases and vice versa.

Pulse time-of-flight measurements in legacy multiphoton and kHz single photon systems are currently all based on threshold detection of both the start and stop pulses, where changes in signal amplitude can bias the measured range. In legacy multiphoton systems, amplitude variations in the start channel are generally small and common large variations in the stop channel can be greatly reduced through the use of Constant Fraction Discriminators (CFDs). Similarly, single photon sensitive kHz systems are subject to "first photon range bias" as the received signal strength increases. As a result, the kHz systems are currently operated with return rates of 10% or less in order to strongly favor single photon returns. The result is a "bias free" range measurement but with a large variance Probability Distribution Function (PDF) for the return signal.

2. The Central Limit Theorem (CLT)

If two or more photons are sampled per pulse, the CLT tells us that, on average, the mean of n samples is equal to the mean of the parent distribution, t_c. Thus, no bias is introduced by the multiphoton measurement. As the mean number of photons per pulse increases (n>15), the PDF distribution becomes highly Gaussian in shape, i.e. a "normal" distribution. For an arbitrary distribution of n single photons, one obtains the following expressions for the mean

$$\left\langle \frac{1}{n}\sum_{i=1}^{n} t_i \right\rangle = \frac{1}{n}\sum_{i=1}^{n}\langle t_i\rangle = \frac{1}{n} n t_c = t_c \qquad (1)$$

and variance of the n-photon distribution

$$\sigma_n^2 \equiv \left\langle \left(\frac{1}{n} \sum_{i=1}^{n} t_i \right) \left(\frac{1}{n} \sum_{i'=1}^{n} t_{i'} \right) \right\rangle - t_c^2 = \frac{1}{n^2} \left\langle \sum_{i=1}^{n} t_i^2 + \sum_{i=1}^{n} t_i \sum_{i' \neq i}^{n} t_{i'} \right\rangle - t_c^2$$
$$= \frac{1}{n^2} \left[n \langle t^2 \rangle + n(n-1) t_c^2 \right] - t_c^2 = \frac{\langle t^2 \rangle - t_c^2}{n} = \frac{\sigma_1^2}{n} \qquad (2)$$

Eqs. (1) and (2) respectively tell us that the mean of the *n*-photon distribution is the same as that of the parent single photon distribution (t_c) but the *n*-photon variance is a factor of n smaller than the single photon variance, σ_1.

3. Dependence of Normal Point Integration Times on the Photon Number n

For a SLR system with a single photon detection threshold, the probability of detecting the satellite signal is

$$P_d(n) = 1 - \exp(-n) \qquad (3)$$

where n is the mean number of photoelectrons detected per pulse and the number of range measurements contributing to a satellite "normal point" is

$$N = P_d(n) f_L \tau_{np} = \left(1 - e^{-n}\right) f_L \tau_{np} \qquad (4)$$

where the laser repetition rate f_L = 2 kHz for SGSLR, τ_{np} is the normal point integration time, and the desired normal point precision is equal to

$$\sigma_{np} = \frac{\sigma_n}{\sqrt{N}} = \frac{\sigma_1}{\sqrt{nN}} = \frac{1}{\sqrt{nN}} \sqrt{\sigma_L^2 + \sigma_D^2 + \sigma_{ET}^2 + \sigma_S^2} = \frac{1}{\sqrt{nN}} \sqrt{\sigma_{inst}^2 + \sigma_S^2} \approx 1mm \qquad (5)$$

where the single photon variance of the instrument PDF, σ_{inst}^2, is equal to the sum of the variances contributed by the laser pulse shape (σ_L^2), the detector (σ_D^2), and the Event Timer (σ_{ET}^2). In the case of the stop pulse, the satellite impulse response introduces yet another variance (σ_S^2) not shared by the start pulse. Thus, the integration time required to generate a normal point with n-photons per pulse is given by

$$\tau_{np} = \frac{N}{\left(1 - e^{-n}\right) f_L} = \frac{1}{n\left(1 - e^{-n}\right) f_L} \left(\frac{\sigma_1}{\sigma_{np}} \right)^2 \qquad n \geq 1 \qquad (6)$$

$$\frac{1}{\left(1 - e^{-n}\right) f_L} \left(\frac{\sigma_1}{\sigma_{np}} \right)^2 \qquad n < 1$$

and is plotted in Figure 1 where it can be seen that the time required to generate a 1 mm precision normal point can be reduced by an order of magnitude or more by utilizing multiphoton returns.

The PDF of the ranging instrument is easily determined by ranging to a single cube calibration target which has a delta-function impulse response. The single photon instrument PDF for the NGSLR system, shown in Figure 1, has a nominal zero-to-zero width of 225 mm, from which we can estimate a nominal instrument single photon RMS of σ_{inst} ~225mm/6 = 38 mm [Clarke et al, 2013]. The variance of the instrument is equal to the sum of the variances introduced by the individual subsystems asdescribed in Figure 1. Since the computed RMS range errors contributed by the 50 psec FWHM laser pulsewidth (σ_L = 6.3mm), the Event Timer resolution (σ_{ET} = 3.4 mm), and the target retroreflector (σ_T =0 mm) are all quite small relative to the observed RMS, one can conclude that the MicroChannel Plate PhotoMultiplier Tube (MCP/PMT) is the largest contributor, i.e., σ_D = 36.8 mm The large single photon variance, requires longer Normal Point (NP) integration times to achieve 1 mm precision, thereby greatly reducing the number of satellites tracked and greatly extending the length of the satellite arc that the NP represents. Furthermore, in both legacy and kHz systems, the PDFs for photon events in the start and stop channels are different since the satellite signature affects only the PDF of the stop channel. Thus, a range bias in the normal point can result unless the start and stop times are both determined by the centroids of their respective distributions rather than a simple threshold crossing.

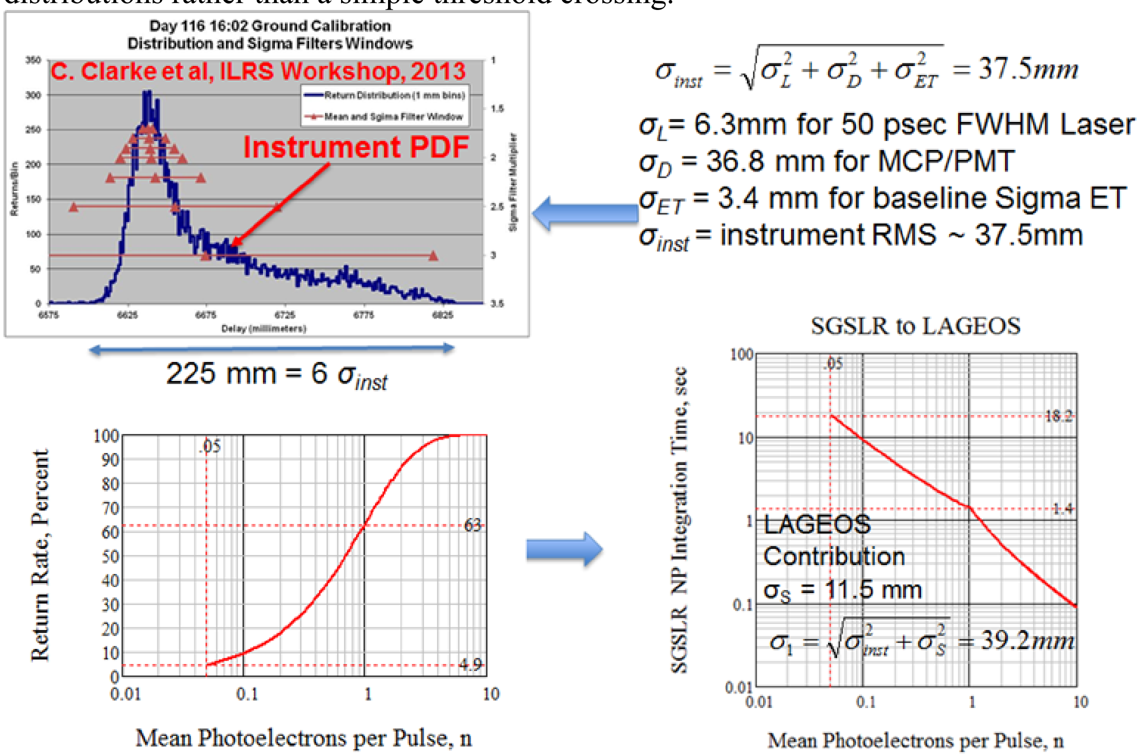

Figure 1. Top Left: Single Photon Instrument (Start) PDF for NASA's NGSLR prototype as determined by ranging to a single cube corner; Top Right: Individual contributions to the PDF variance/RMS due to the laser, detector, event timer, and LAGEOS impulse response; Bottom Left: Percent return rate as a function of the mean photoelectrons per pulse, n; Bottom Right: SGSLR Normal Point integration time required to achieve 1mm RMS on the LAGEOS satellite as a function of n.

4. Measuring the Pulse Centroid

According to Eq. (1), the multiphoton range measurement is bias-free provided one time tags all of the n photons in the start and stop pulses and computes the mean time-of-flight (TOF). Certain single photon sensitive detectors, such as MicroChannel Plate PhotoMultiplier Tubes (MCP/PMTs) and Silicon PhotoMultiplier Tubes (SiPMTs), are capable of recording multiple returns on a single anode. This is made possible since each detector respectively contains hundreds to thousands of closely spaced microchannels or single stop SiAPDs, which independently and jointly contribute to the final signal out of the anode. However, due to the short laser pulsewidths (measured in picoseconds and low satellite spread), the photons arrive almost simultaneously and recovery times in the Event Timers are too long (>1.6 nsec) to time tag the arrival of each individual photon. Thus, the electrical pulses created by individual photons tend to form narrow but irregularly shaped pulses at the detector anode that, with threshold detection, will produce biased range measurements. Over four decades ago, however, radar engineers developed relatively simple circuits (Figure 2) to detect the centroid of irregularly shaped pulses [Harris, 1975] although they undoubtedly worked with broader temporal pulse shapes. With more modern GHz bandwidth electronics components, it should be possible to accurately measure the centroid of fast optical pulses in the 1 to 2 nsec regime. .

Other SLR detectors currently in use, such as Compensated-Single Photon Avalanche Diodes (C-SPADs), can only record the arrival of a single photon (signal, dark count, or solar noise) before saturating. As a result, a prior dark or solar noise count prevents the C-SPAD from seeing the satellite return, thereby increasing the normal point integration time. If it does see a multiphoton satellite return, the measurement will suffer from the "first photon bias" which underestimates the actual range to the target.

According to the inventors, the circuit provides the zero crossing of the detected pulse plus a fixed offset The combined time offsets for the start (C_0) and stop (C_1) pulses can be determined by ranging to a single retroreflector as in Figure 3.

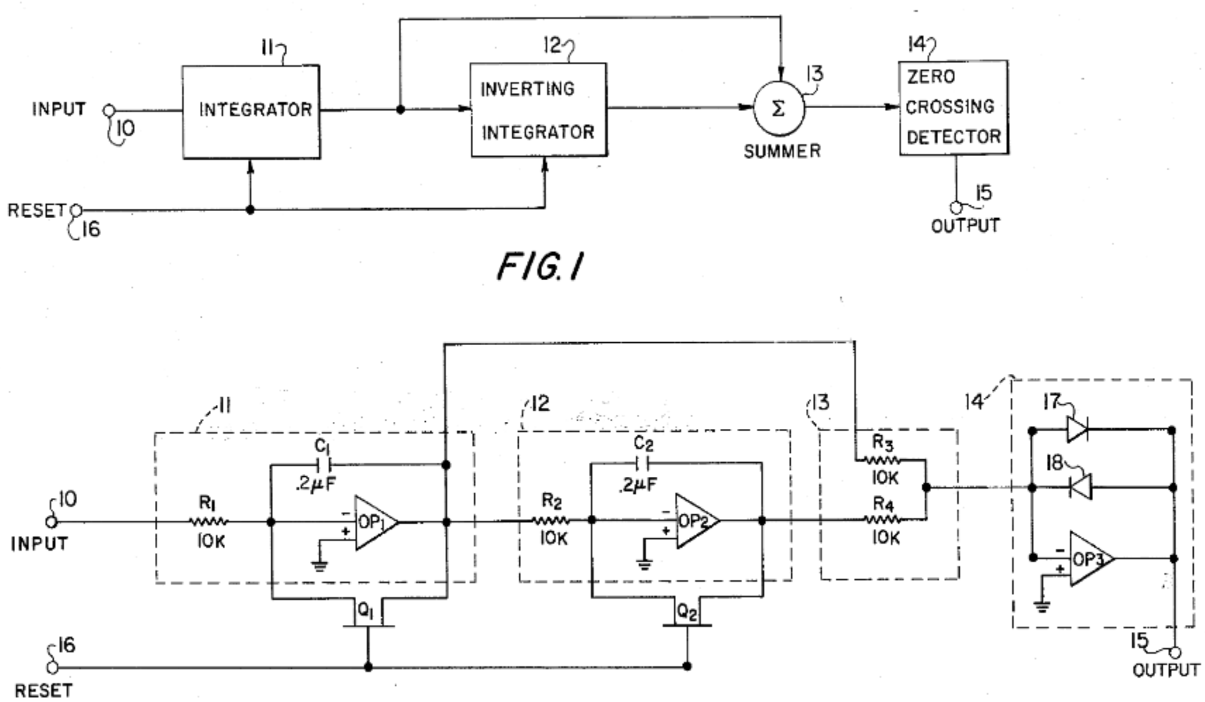

Figure 2: Pulse centroid detector developed by radar engineers (US Patent #3,906,377, Sept. 16, 1975)

The measured TOF to the calibration target is given by

$$TOF = (C_1 - C_0) + (t_1 - t_0) \tag{7}$$

where t_1 and t_0 are the centroids of the stop and start pulses respectively and $(t_1 - t_0)$ is the actual pulse TOF. The constant $(C_1\text{-}C_0)$ is determined by ranging to a single calibration cube corner at a known distance R_{cal} via the equation

$$\left(C_1 - C_0\right) = TOF - 2R_{cal} / c \tag{8}$$

and subtracted from all subsequent satellite TOF measurements to obtain a bias free range.

The aforementioned centroid detection scheme is easily incorporated into existing legacy multiphoton SLR systems which have single start /single stop timing channels. In such systems, the detection threshold is set high enough such that isolated solar noise and detector dark counts, not concurrent with multiphoton satellite returns, go undetected. For the newer kHz systems operating at the single photon level, however, the photon detector, centroid detector, and the timing receiver must all have fast recovery times if the likelihood of multiple noise counts within the range gate is high. In this situation, the zero crossing detector in the centroid circuit will output the satellite signal plus multiple noise signals per pulse to the Event Timer where the satellite return can be determined based on its temporal correlation with returns from other pulses. In the next section, however, we illustrate how the proposed SGSLR Multifunctional Receiver can largely eliminate the problem of concurrent noise in computing the centroid of the satellite return.

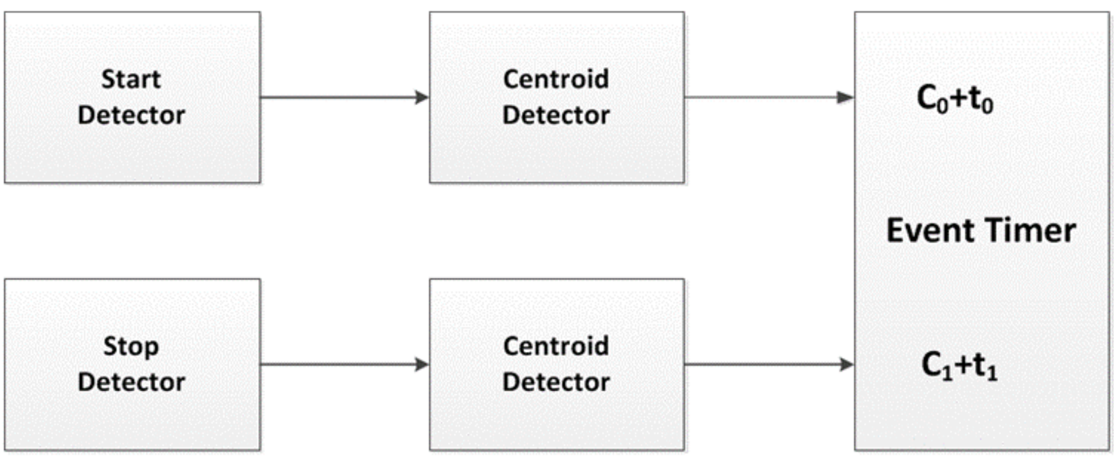

Figure 3: Field calibration using a single retroreflector placed a known distance from the SLR system origin or invariant point .

5. Centroid Detection in the SGSLR Multifunctional Receiver

The SGSLR Multifunctional Receiver in Figure 4 is designed to make satellite acquisition and tracking more autonomous via an array of 45 individual SiPMTs. The array monitors the receiver FOV to determine the angular offset of the satellite return from the telescope optical axis by identifying the SiPMT with the most counts. During satellite acquisition, this information is used by the system computer to drive the telescope mount such that the satellite returns are “always” captured by the central pixel. Since the transmitter point-ahead implemented by SGSLR’s Dual Risley Prism Assembly is computed relative to the telescope axis, maintaining the return signal on the central pixel ensures that the peak of the Gaussian laser beam always falls on the satellite, thereby maximizing return signal strength In addition, the array reduces the solar noise by a factor of 45 (97.8%) relative to a single anode detector monitoring the entire receiver FOV. Furthermore, with a typical range gate of 500 nsec, the nominal 100 kHz detector dark count rate produces only 1 dark count per pixel for every 20 shots and the likelihood that the dark count coincides with a 2 nsec signal pulse is further reduced by a factor of 250 to 1 per 5000 pulses! If the telescope pointing jitter can be largely contained within the FOV of the central pixel (typically a few arcsec) by using the receiver FOV adjustment to accommodate the jitter, the stop centroid detection circuit can be confined to the central pixel, thereby eliminating the need to provide such circuitry to multiple pixels.

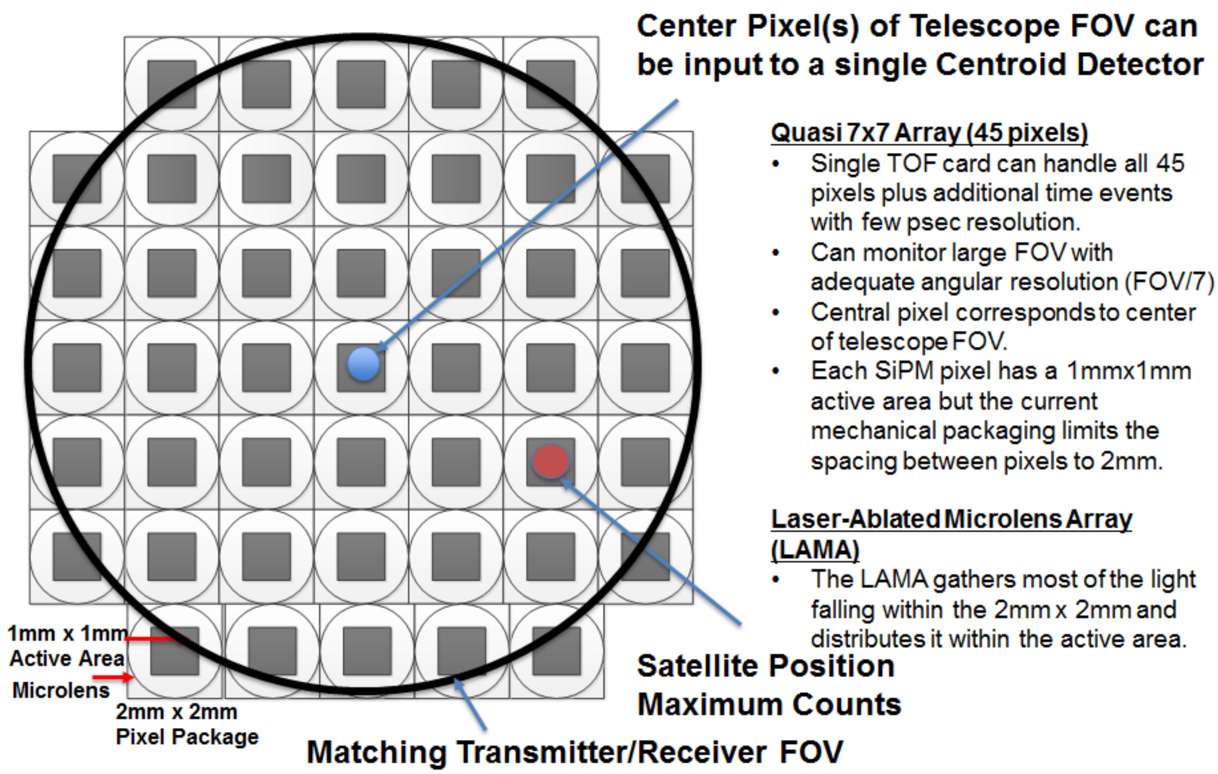

Figure 4. The 45 pixel Sigma Multifunctional Receiver proposed for use in NASA's SGSLR system.

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