Theoretical Performance of NASA's SGSLR System Ranging to GNSS Satellites

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ILRS Technical Workshop

Matera, Italy

October 26-30, 2015

SLR Link Equation

$$n_s = \frac{E_t}{h\nu}\eta_t \frac{2}{\pi(\theta_d R)^2}\exp\left[-2\left(\frac{\Delta\theta_p}{\theta_d}\right)^2\right]\left[\frac{1}{1+\left(\frac{\Delta\theta_j}{\theta_d}\right)^2}\right]\left(\frac{\sigma A_r}{4\pi R^2}\right)\eta_r\eta_c T_a^2 T_c^2$$

n_s = detected satellite photoelectrons per pulse
E_t = laser pulse energy
$h\nu$ = laser photon energy = 3.73×10^{-19}J @ 532 nm
η_t = transmitter optical throughput efficiency
θ_d = beam divergence half angle
R = slant range between station and satellite
$\Delta\theta_p$ = laser beam pointing error
$\Delta\theta_j$ = RMS tracking mount jitter
σ= satellite optical cross-section
A_r = Telescope Receive Area
η_r= receiver optical throughput efficiency
η_c = detector counting efficiency
T_a = one way atmospheric transmission
T_c = one way cirrus cloud transmission

Reference: J. Degnan, "Millimeter Accuracy Satellite Laser Ranging: A Review", in Contributions of Space Geodesy to Geodynamics: Technology Geodynamics, 25, pp. 133-162, 1993.

SGSLR Optical Bench

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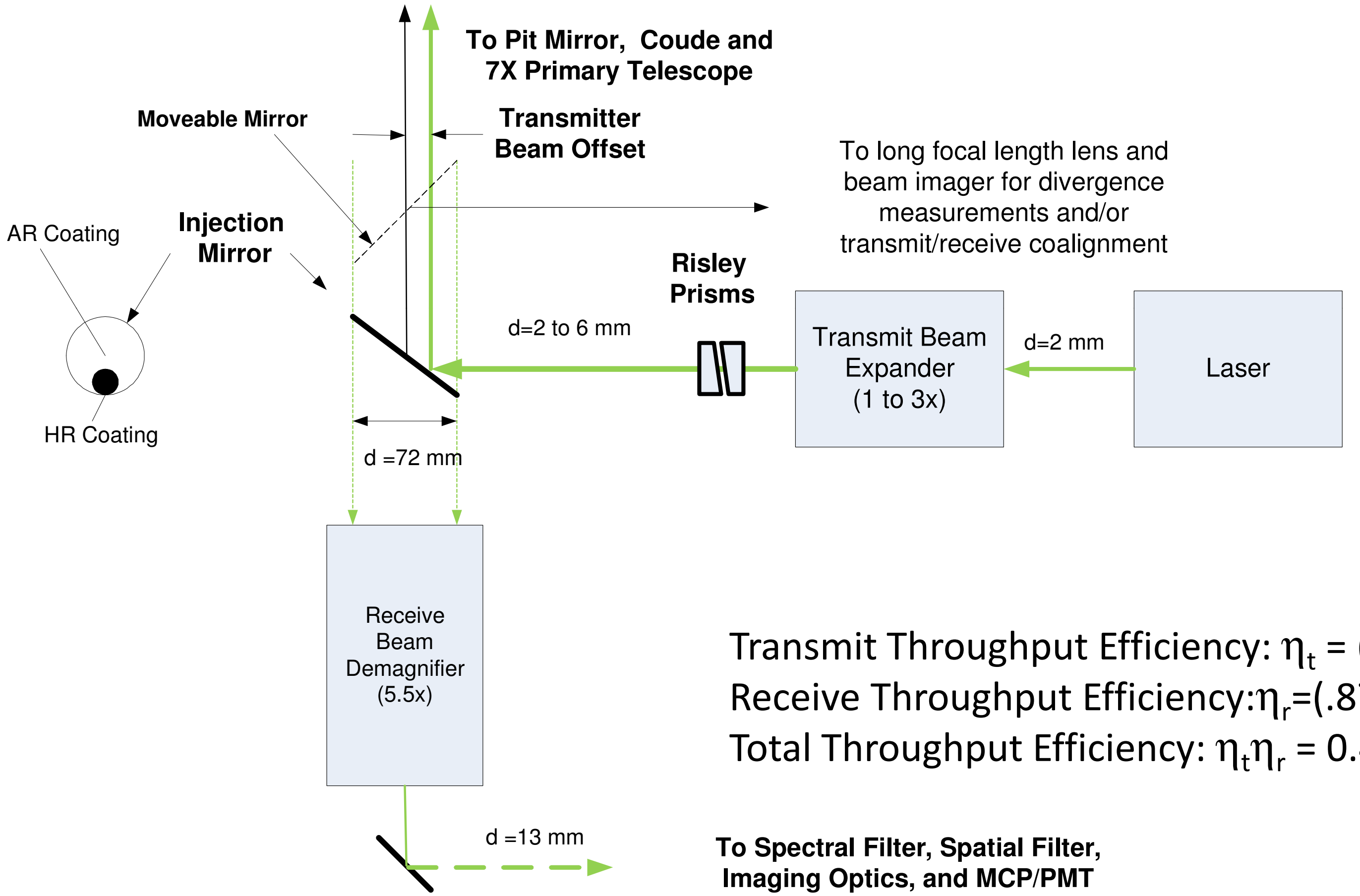
Simplified optical bench provides greater transmit and receive optical throughput and near-zero instrument laser backscatter into sensitive detector



Transmit Throughput Efficiency: $\eta_t = (.99)^{11}(.87) = .78$
Receive Throughput Efficiency: $\eta_r = (.87)(0.7)(.99)^{12} = 0.54$
Total Throughput Efficiency: $\eta_t\eta_r = 0.42$

Detection Probability and Normal Point Precision

For a SLR system with a single photon detection threshold, the probability of detecting the satellite signal is

$$P_d = 1 - \exp(-n_s) \cong n_s$$

where the approximation holds for $n_s \ll 1$. In general, the number of range measurements contributing to a satellite "normal point" is

$$N = P_d f_L \tau_{np} = \left(1 - e^{-n_s}\right) f_L \tau_{np}$$

where
f_L = the laser repetition rate ,
τ_{np} = the normal point time interval
and the normal point precision is equal to

$$\sigma_{np} = \frac{\sigma_{ss}}{\sqrt{N}}$$

where σ_{ss} is the satellite-dependent , single shot range precision.

GNSS Link vs Atmospheric Quality
(mean cirrus cloud cover, no pointing bias)

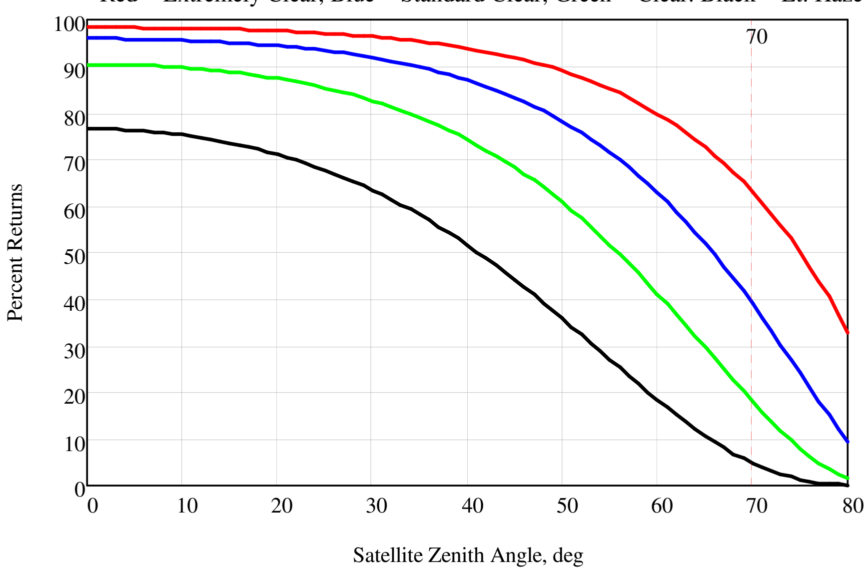
Effective Receive Area $A_r = 0.174m^2$

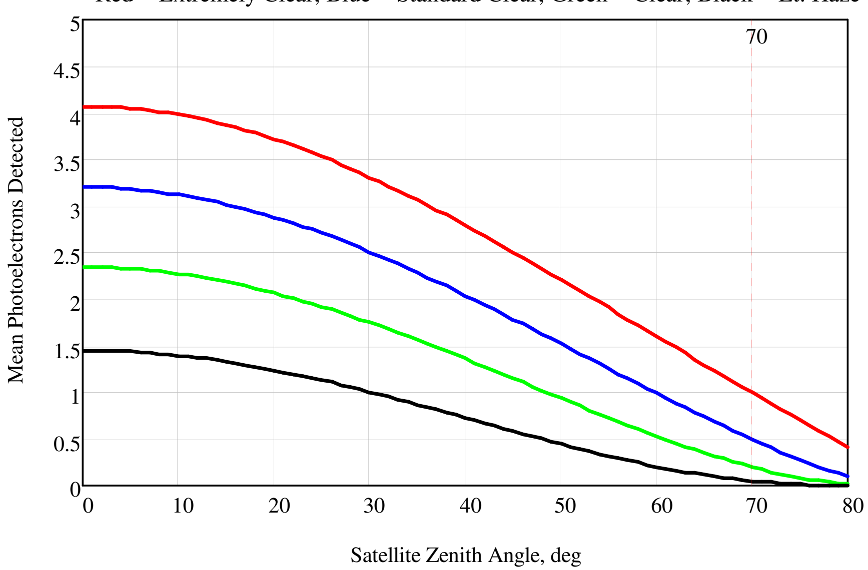
Optical Efficiencies: $\eta_c = 0.28$ $\eta_t = 0.746$ $\eta_r = 0.517$

Laser Parameters: $P_t = 3 \cdot W$ $f_{qs} = 2 \cdot kHz$ $E_t = 1.5 \cdot mJ$

Pointing $\theta_d = 3.5 \cdot arcsec$ $\Delta\theta_p = 0$ $\Delta\theta_j = 2 \cdot arcsec$

$\sigma = 10^8\ m^2$ = ILRS GNSS Standard





Pointing Errors

Pointing errors reduce the photoelectron count and decrease the accuracy of the satellite normal point. There are two types of pointing error:

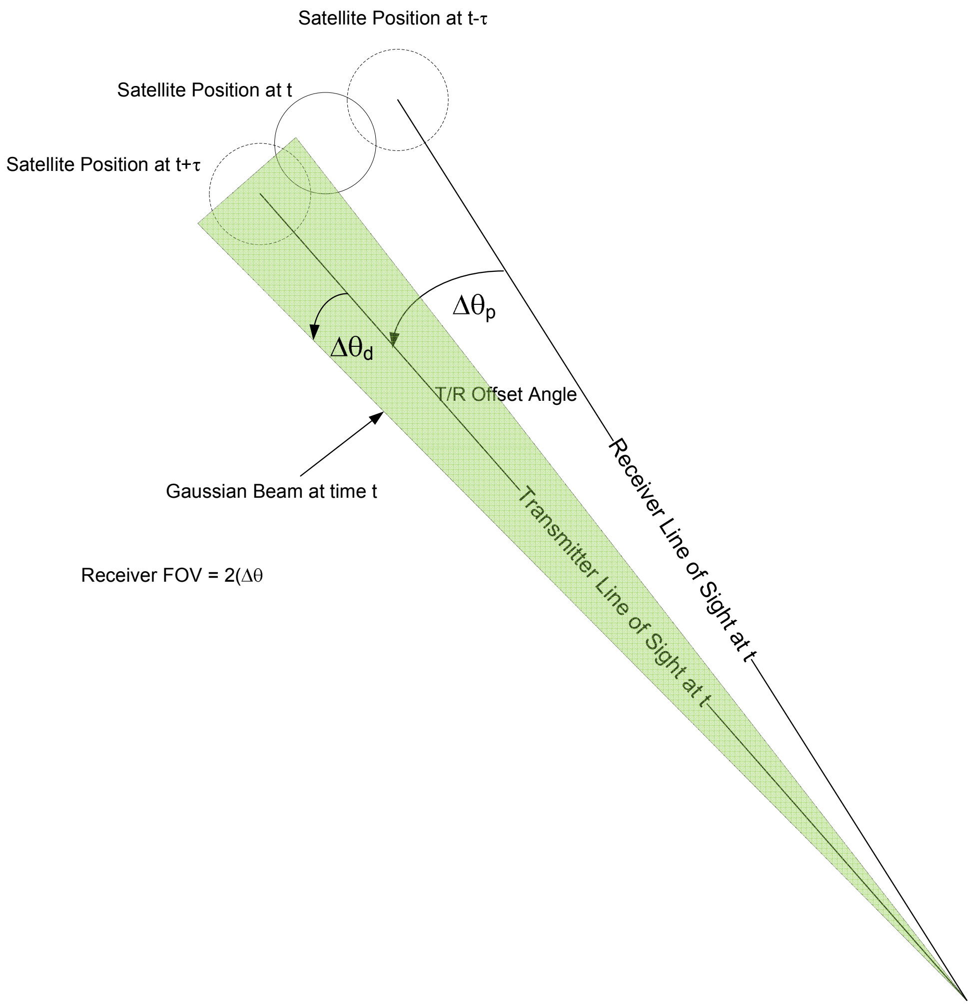
1. Biases introduced by poor satellite predictions , imperfect tracking of the satellite by the mount and/or the dual Risley prism point ahead mechanism.
2. Random errors with zero mean due to “jitter” in the tracking mount and/or atmospheric turbulence induced “beam wander”.

NGSLR Questions

The link analyses to date suggest SGSLR will have strong link margins to GNSS and LAGEOS but it also raises the question "Why were the return rates for NGSLR so low given roughly similar system characteristics?". Some possible explanations include:

- We have included two of the three effects induced by atmospheric turbulence (beam spread and beam wander) in our analyses but not scintillation. Could the latter cause large losses of signal?
- Did the poor NGSLR telescope/Coude alignment severely impact the quality of the transmit beam in the far field as it did the received star image?
- Was our telescope/ Risley prism pointing as good as we thought? No pointing biases were assumed in the initial calculations. Later field experiments indicated some angular oscillation in the tracking mount.
- Were our transmit and receive throughputs substantially decreased by some of the specialized components needed to implement eyesafe ranging in the original SLR2000 concept?
- Was our transmitter point ahead algorithm and hardware sufficiently accurate?

NGSLR Risley Prism Approach "Transmitter Look-Ahead"



•Receive optical axis at time t is aligned with the satellite position at time t-τ where τ is the one way pulse propagation time and sees the returning pulse in the center of receiver array.
•The narrow laser pulse leaving at time t is pointed ahead to where the satellite will be at time t+ τ (Transmitter Look Ahead).
•When the pulse returns at time t+2 τ, it will again fall on the center of the receiver array.
•If the error in the "look ahead angle" is comparable to the beam divergence half angle, the energy captured and returned by the satellite retroreflectors will be greatly reduced, leading to lower probabilities of detection and return rates and longer normal point integration times.
•This angular error can be caused by an error in telescope pointing, an error in the computation of the T/R offset angle, or an algorithm or mechanical error in dual Risley Prism pointing of the transmitter.

Optimum Divergence for No-Risley Case

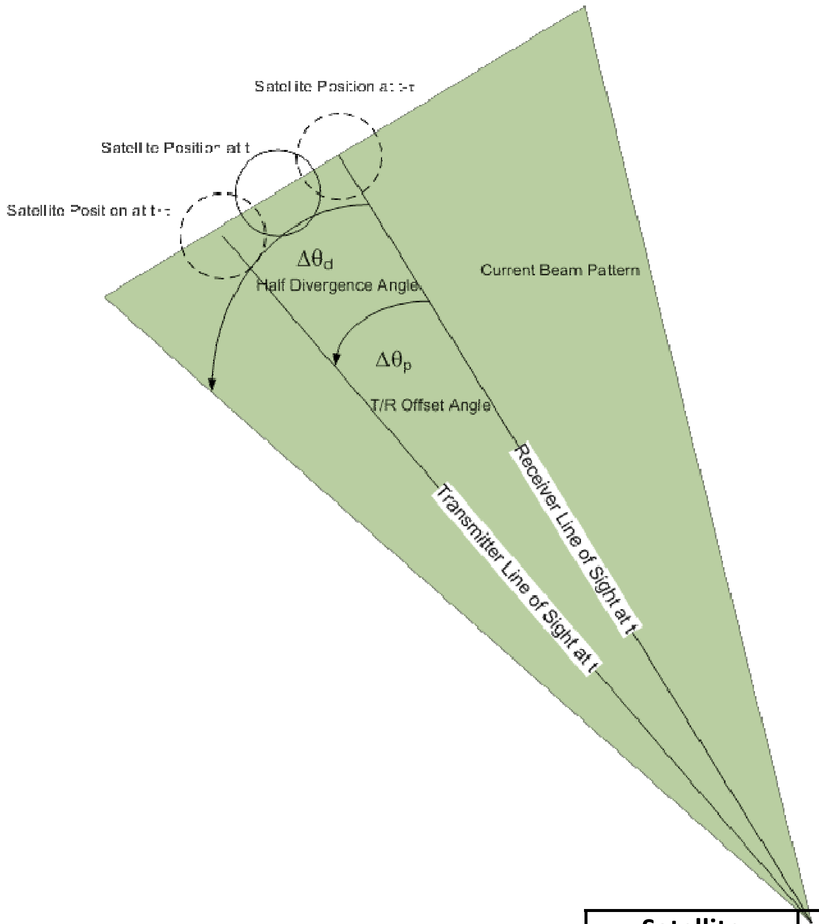
Let us first ask the question: " What beam divergence value , θ_d ,maximizes the signal return for an <u>uncorrected</u> transmitter beam having an angular offset , $\Delta\theta_p$,, relative to the receiver?" This is answered by setting the following derivative equal to 0:

$$\frac{d}{d\theta_d}\left[\frac{2}{\pi(\theta_d R)^2}\exp\left[-2\left(\frac{\Delta\theta_p}{\theta_d}\right)^2\right]\right]=\frac{4\left(2\Delta\theta_p^2-\theta_d^2\right)}{\pi R^2\theta_d^5}\exp\left[-2\left(\frac{\Delta\theta_p}{\theta_d}\right)^2\right]=0 \qquad (2)$$

which implies maximum signal for a beam half-divergence of

$$\theta_d=\sqrt{2}\Delta\theta_p$$

Alternate Risley-Free Approach



•Transmit and receive optical axes at time t are co-aligned with the satellite position at time t-τ where τ is the one way pulse propagation time.

•Under this constraint, the beam half divergence which maximizes the return signal is given by $\sqrt{2}\,\Delta\theta_d$ where $\Delta\theta_d$ is the instantaneous offset angle between the received and transmitted pulses.

•The satellite is no longer illuminated by the peak of the Gaussian pattern but by a side slope of the Gaussian with a lower intensity, i.e. 1/e of the peak intensity. Furthermore, the peak intensity falls due to the wider beam divergence.

Satellite	Altitude, h (km)	Maximum Offset (arcsec)	Maximum Opt. Divergence (arcsec)	Minimum Offset (arcsec)	Minimum Opt. Divergence (arcsec)
Starlette	950	10.13	28.65	5.22	14.76
LAGEOS	6,000	7.78	22.00	6.70	18.96
GNSS	20,000	5.33	15.08	5.18	14.64
GEO	35,790	4.21	11.91	4.16	11.78

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Range of the T/R Angular Offset

The maximum T/R pointing offset is given by [Degnan, 1993]

$$\Delta\theta_p(h,\theta_z,\omega) = \Delta\theta_p^{\max}(h)\sqrt{\cos^2\omega + \Gamma^2(h,\theta_z)\sin^2\omega}$$

$$\Gamma(h,\theta_z) = \sqrt{1 - \left(\frac{R_E\sin(\theta_z)}{R_E + h}\right)^2}$$

The Max and Min offsets occur at $\omega=\pi/2$ and the beam half divergence that maximizes the signal is given by $\sqrt{2}\,\Delta\theta_p$.

Satellite	Altitude, h (km)	Maximum Offset (arcsec) @ Zero degrees	Maximum Opt. Divergence (arcsec)	Minimum Offset (arcsec)	Minimum Opt. Divergence (arcsec) @ 80 degree zenith angle
Starlette	950	10.13	28.65	5.22	14.76
LAGEOS	6,000	7.78	22.00	6.70	18.96
GNSS	20,000	5.33	15.08	5.18	14.64
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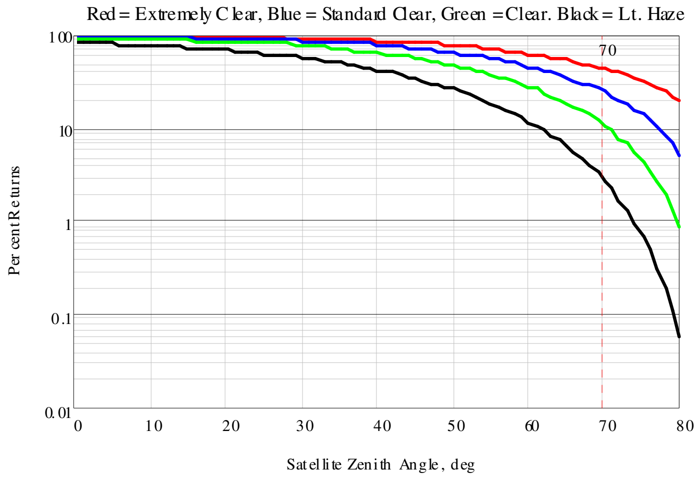
Table 1: Representative LEO to GEO satellites, their orbital altitudes, maximum and minimum angular offsets between the transmitter and receiver assuming a maximum zenith angle of 80 degrees, and corresponding optimum transmitter full divergence angles that maximize the signal strength in the absence of an angular offset compensation mechanism. The recommended minimum telescope FOV required is roughly equal to the maximum optimum divergence or about 30 arcsec for LEOs.

GNSS- No Pointing Error



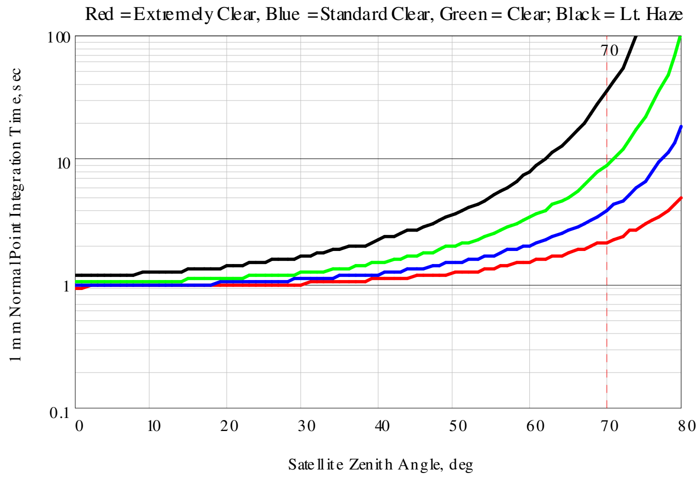
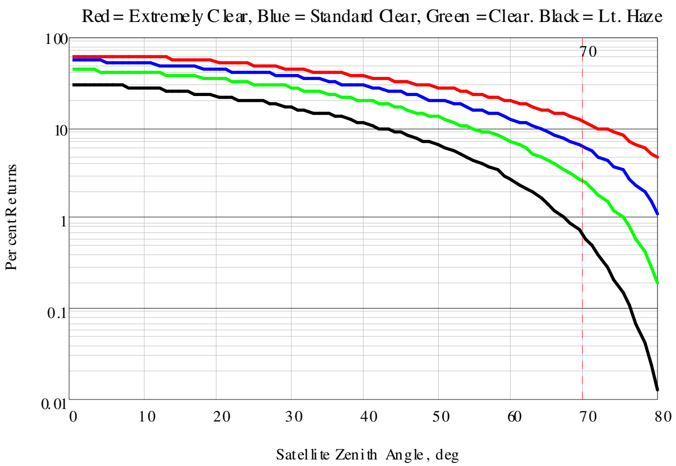
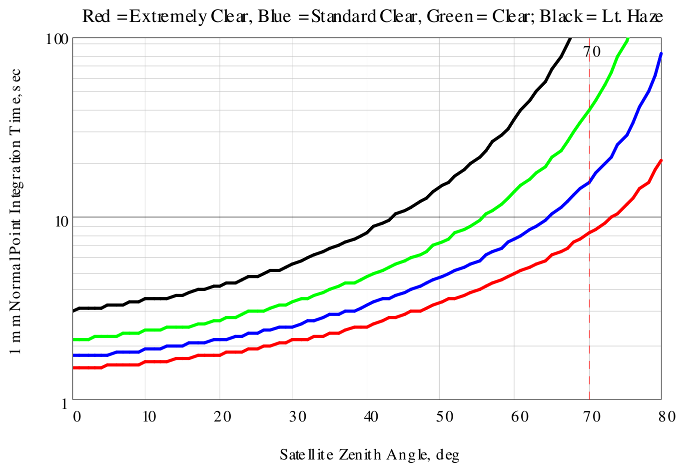


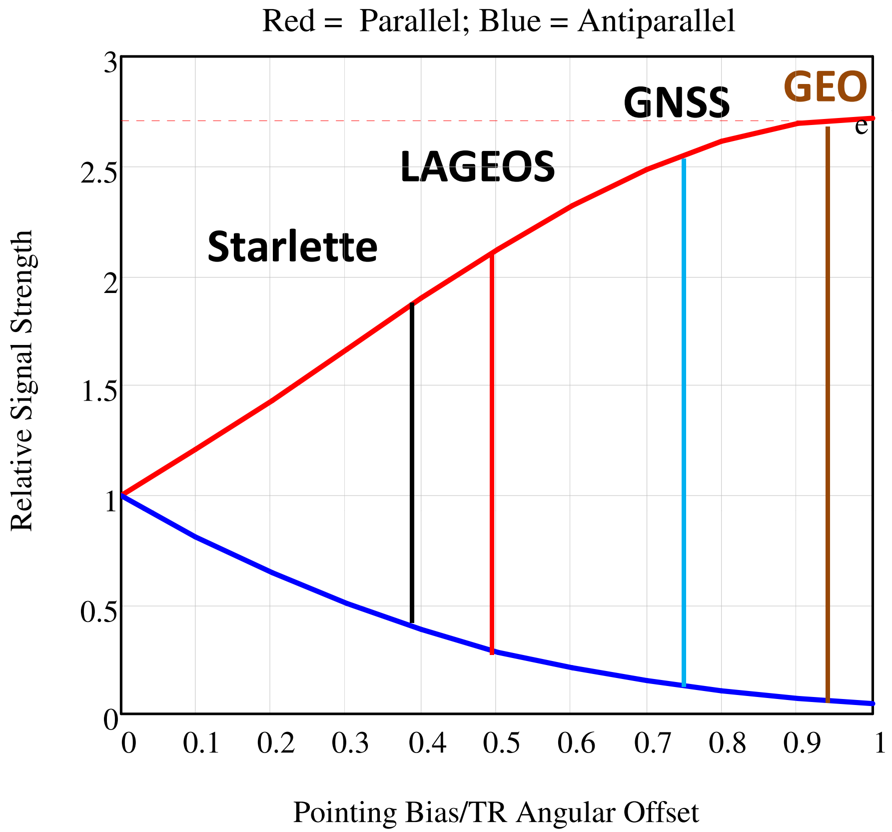
Figure 12: GNSS results for a 7 arcsec full width transmitter divergence as a function of satellite zenith angle and atmospheric quality (Extremely Clear to Light Haze) : (a) Percent Return Rate; (b) Time required to create 1 mm normal point in seconds.





WORST CASE

Risley-Free Signal Strength vs Pointing Bias



Transmitter Point-Ahead

Colored vertical lines correspond to a pointing bias of 4 arcsec and a worst case T/R angular offset for:

Starlette =10.1 arcsec

LAGEOS = 7.8 arcsec

GNSS = 5.3 arcsec

GEO = 4.2 arcsec

Signal strength, relative to zero bias, as a function of the ratio of pointing bias to the T/R angular offset. The red and blue curves correspond respectively to the cases where the pointing bias is parallel to and antiparallel to the angular offset between the transmitted and received beams. The relative signal strength for all other orientations of a specific pointing bias will fall on a vertical line between the red and blue curves. A ratio of 1 corresponds to the "transmitter point- ahead" case where the peak of the Gaussian transmit beam falls on the satellite during tracking.

Compare Risleys to Worst Case Risley Free

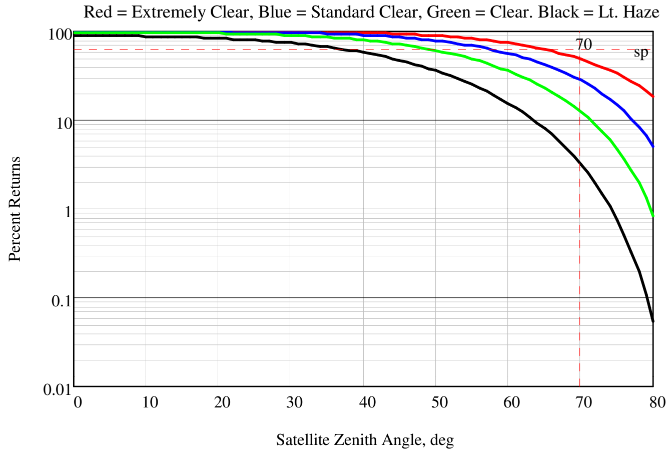
In the next two slides, we compare the Dual Risley configuration to the "Worst Case" Risley Free configuration for two satellites – LAGEOS and GNSS with the recommended ILRS cross-section. "Worst Case" implies we are using the larger of the two "optimum divergences" listed in the table earlier in this presentation.

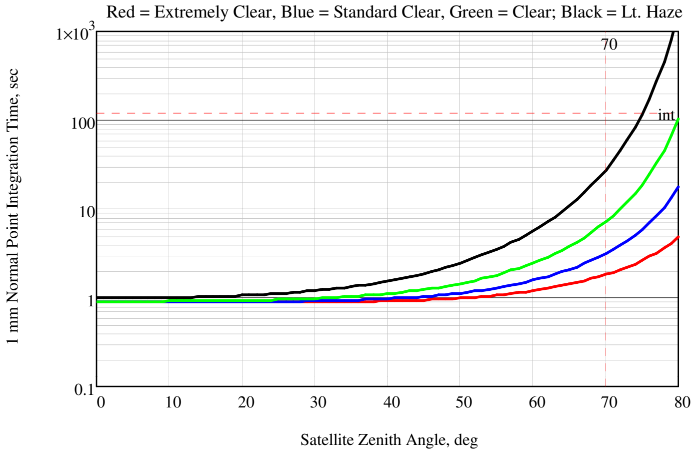
The horizontal dashed red line in the Percent Returns plot marks the expected 63% return rates of single photon detection assuming a single photon threshold, and the horizontal dashed line in the second 1 mm integration time plot indicates the traditional normal point time for the satellite under consideration, i.e. 120 seconds for LAGEOS, and 300 seconds for GNSS and GEO.

LAGEOS (Worst Case)

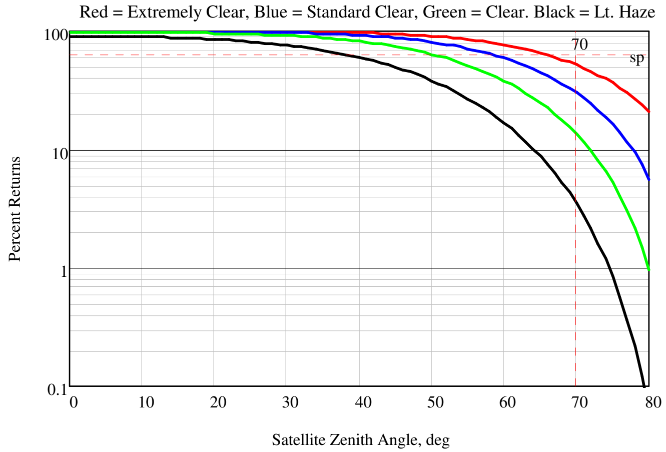
7 arcsec full divergence (Risleys) and 4 arcsec Pointing Bias

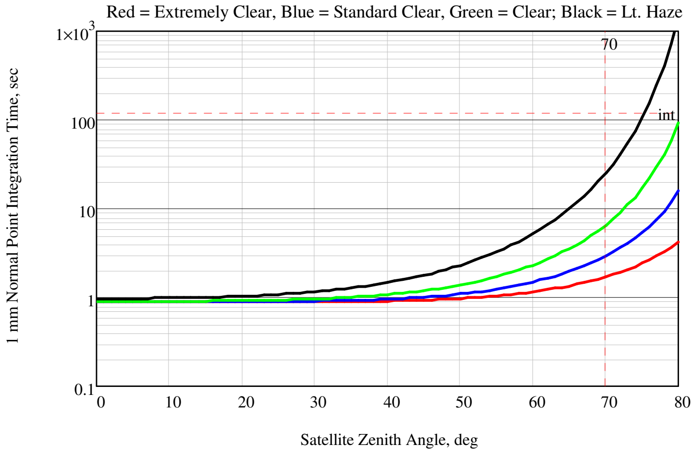
Risley Prisms





No Risley Prisms - Worst Case





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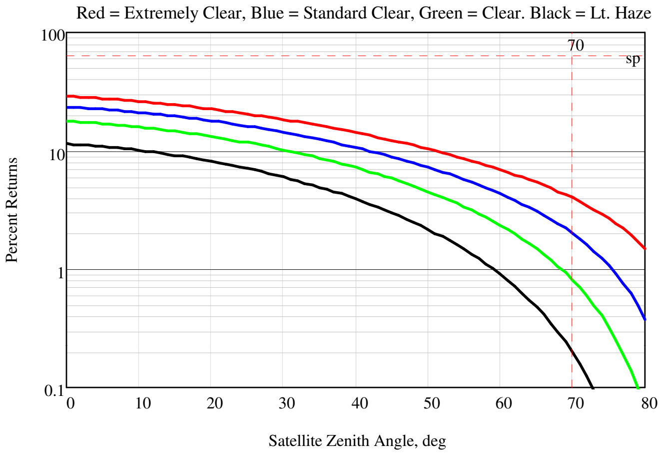
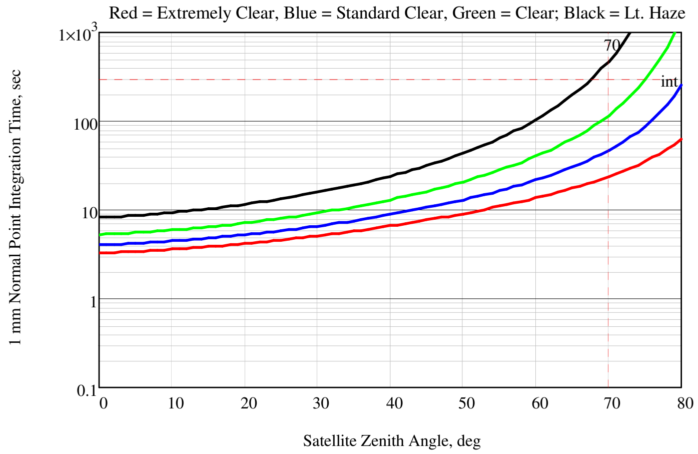
GNSS (Worst Case)

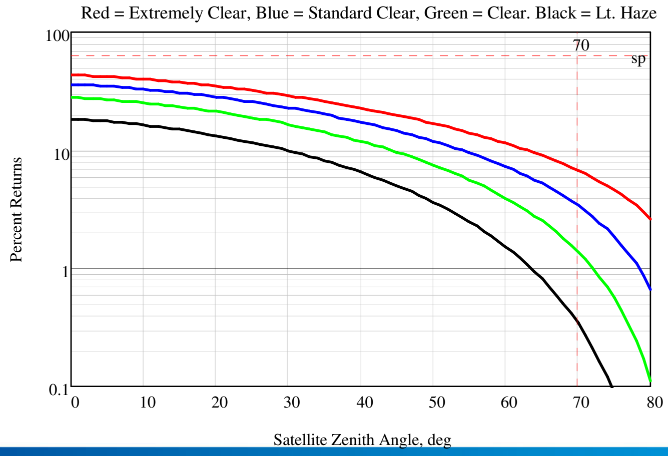
7 arcsec full divergence (Risleys) and 4 arcsec Pointing Bias

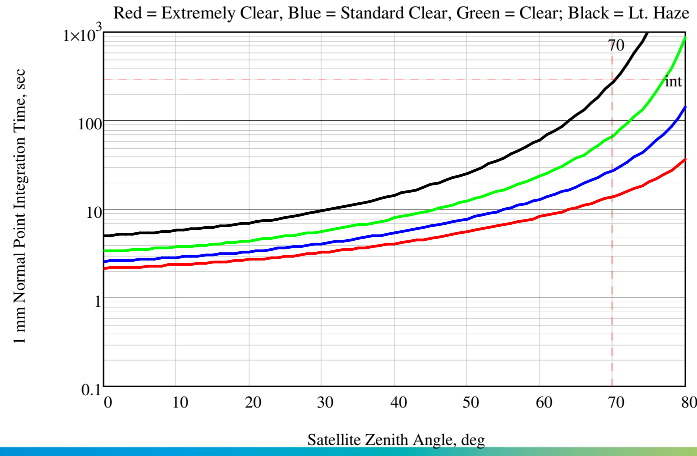
Risley Prisms





No Risley Prisms - Worst Case





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Implementation

- The overall SGSLR optical system can be designed to include both a COTS transmitter beam expander (with a somewhat larger range of magnification) and the Dual Risley Prism Point-Ahead mechanisms with the latter being eliminated if it is found through field experiments that they are not needed.

- Best results are obtained for LEO and LAGEOS at low elevations by allowing the beam divergence to change with elevation angle between best case (maximum optimum divergence) and worst case (minimum optimum divergence).

- The original NGSLR Special Optics Beam Expander with high power coatings has sufficient flexibility and quality to do this job.

Advantages of Risley-Free Approach

•It removes the requirement to coordinate the independent 2D tracking of both the telescope (azimuth and elevation) and the Risleys (phase angles of Prisms 1 and 2). This becomes especially difficult with narrow transmit beams at or near telescope keyhole regions where satellite angular rates are highest.

•Link analyses suggest that significantly wider beam divergences make the signal strength and normal point integration times less sensitive to pointing error without significantly sacrificing return rate or extending normal point integration times.

•The larger beam divergences relax the requirements on the coalignment of the transmit and receive optical axes.

•Lower transmitter optical throughput losses via the elimination of four optical surfaces associated with the Dual Risley Prisms.

•Lower cost and simplified tracking operations with the ability to correct for telescope pointing errors using the angle determination capabilities of the Sigma Multichannel Range Receiver.

•Telescope FOV requirements are not significantly different. Max FOV for Risley –Free is about 30 arcsec (LEO-see earlier table) but the temporal and spatial filtering of the Sigma receiver will limit noise collection to one pixel (4 to 6 arcsec for N=7 or 5).

And now you know why duck hunters use shotguns instead of rifles.

GEO - No Pointing Error

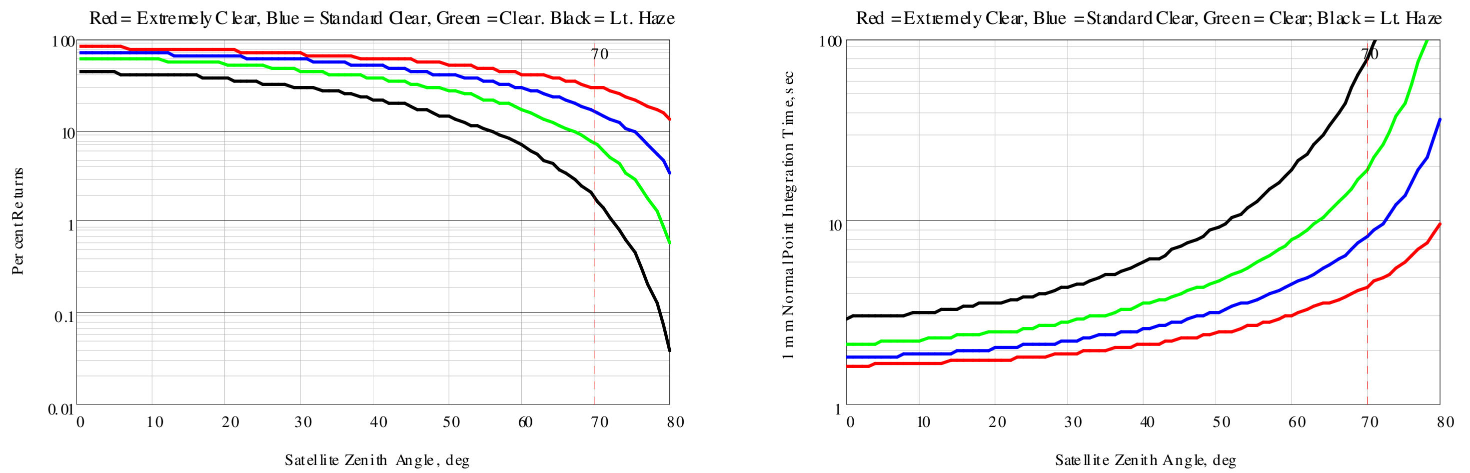
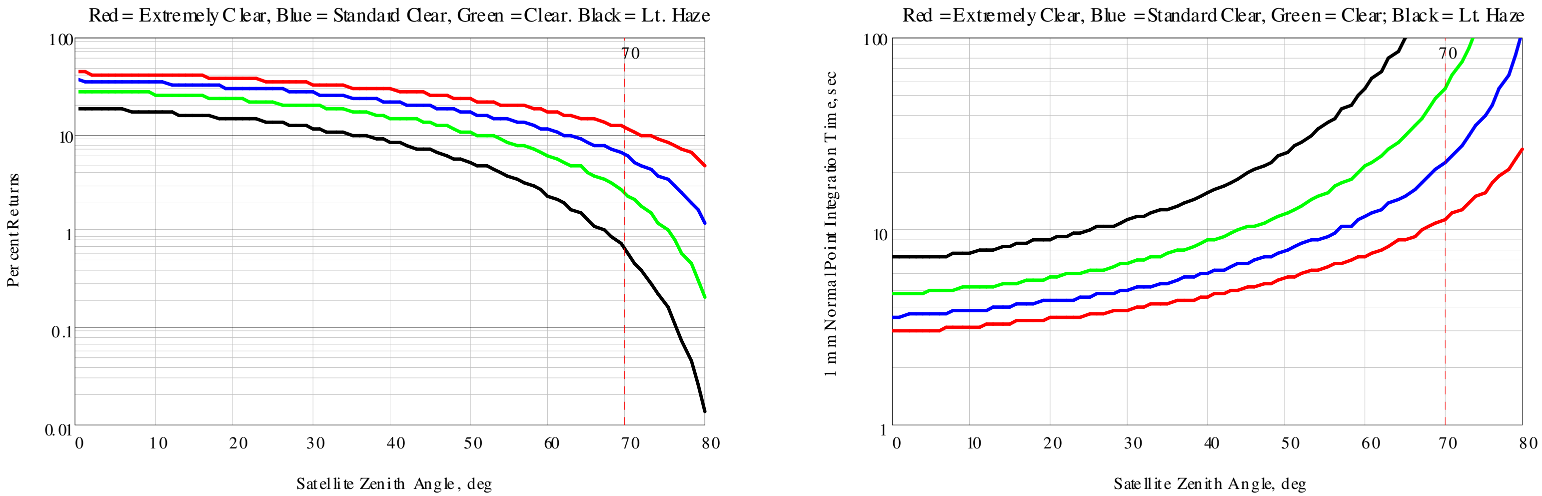


Figure 13: Geosynchronous results for a 7 arcsec full width transmitter divergence as a function of satellite zenith angle and atmospheric quality (Extremely Clear to Light Haze) : (a) Percent Return Rate; (b) Time required to create 1 mm normal point in seconds.



WORST CASE